Stanisław Domoradzki

Mathematics in Lwów before the Lwów Matematical School

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MATHEMATICS IN LWÓW BEFORE THE LWÓW MATEMATICAL SCHOOL

STANISŁAW DOMORADZKI

Abstract: The article reveals a less known Polish environment of Lwów in the period of Galician autonomy, its aspirations and achievements. This environment had a significant impact on the success of the wider understood Polish Mathematical School in the two interwar decades. The article uses archival material and rare books**.**

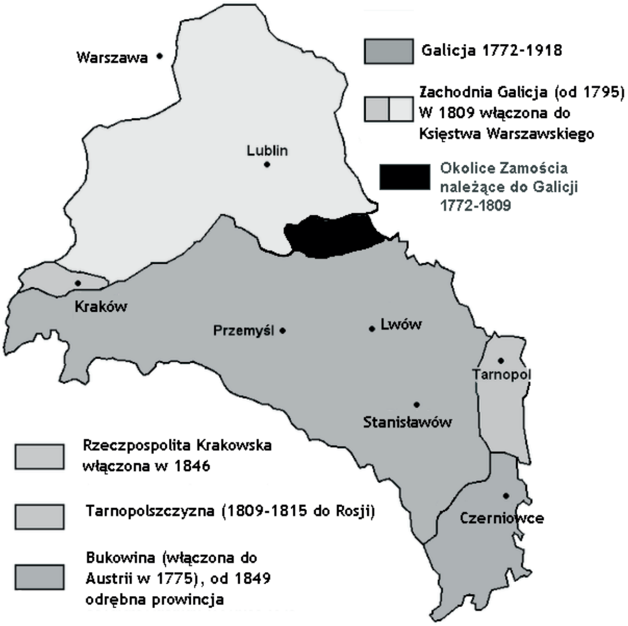
1 Introduction

The history of mathematics in Lwów before the Lwów Mathematical School is little known. For instance, professor Vishik from Moscow University writes: *In general, S. Banach was a genius-mathematician. And it is absolutely unknown how such a powerful school of functional analysis could appear in such a city like Lwów. The "Banach School" even published its own journal, "Studia Mathematica". 10 issues of this journal appeared.* In this article we try to answer the questions posed by Vishik. We describe the state of the mathematical education and other relevant information in the period before the Lwów Mathematical School.

One can learn the history of Polish lands from numerous textbooks and monographs dealing with the history of Poland. In 1861, Galicia obtained autonomy with Sejm Krajowy (Parliament) and the goverment was located in Lwów, the capital. In 1863, the January revolt broke out and, after its decay, in the Russian region there was a period of repressions and oppression. In Galicia, when the state of siege was interrupted (February 1864 – April 1865) there have been some signs for freedom in the Austrian part. In 1867, in the Austrian-Hungarian empire, the so called December constitution was promulgated which, inter alia, approved the activities of the Sejm Krajowy (Parliament) of Galicia. The Sejm worked once a year for six weeks, the meetings covered education, economy, transportation.

The following map serves to illustrate the places and times mentioned:[1]

[1] http://www.wikipedia.pl.



Galicia 1772–1918 (Galicja 1772–1918)

Western Galicia (since 1795), 1809 included into the Warsaw Duchy (Zachodnia Galicja (od 1795) od 1809 włączona do Księstwa Warszawkiego)

Zamość neighborhoods belonging to Galicia 1772–1809 (Okolice Zamościa należące do Galicji 1772–1809)

Cracov Republic included in 1846 (Rzeczpospolita Krakowska włączona w 1846)

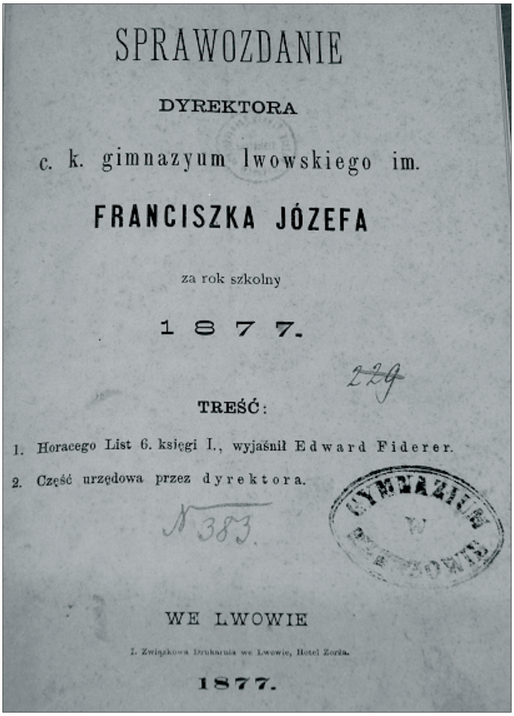
Tarnopol region, 1809–1815 to Russia (Tarnopolszczyzna – 1809–1815 w Rosji)

Bukowina (belonging to Austria since 1775), since 1849, a separate province Bukowina (Bukowina włączona do Austrii w 1775, od 1849 odrębna prownicja).

2 Mathematics in Gymnasiums in Lwów

During the years 1867–1914, the number of secondary schools in Galicia increased from 19 to 130 or nearly by the factor seven, while the number of teachers increased from about 309 to 2045 in state schools and about 1,000 teachers in private schools. The size of the staff of secondary state schools varied depending on the size and resilience of its territory. The staff of every high school (that is, with classes from I to VIII) consisted of one director and from 10 to up to 40 teachers, depending on the number of students.

Catechists of all faiths also belonged to the group of teachers, religion was a compulsory subject. By the end of the nineteenth century, the most numerous was the group of teachers of the Gymnasium of St. Anna in Kraków, now the I B. Nowodworski Lyceum, the second place belonged to the IV Gymnasium in Lwów.



SPRAWOZDANIE

DYREKTORA

c. k. gimnazyum lwowskiego im.

FRANCISZKA JÓZEFA

za rok szkolny

1877.

TREŚĆ:

1. Horacego List 6. księgi I., wyjaśnił Edward Fiderer.

2. Część urzędowa przez dyrektora.

WE LWOWIE

I. Związkowa Drukarnia we Lwowie, Hotel Żorża.

1877.

Report of the Gymnasium in Lwów named after Franz Josef.
This is one of the main sources for our knowledge of studying mathematics.

We present the teaching of mathematics in gymnasiums as an example of classical and real type education.

C. K. Gymnasium, named after Franz Jozef, of classical type, was founded in 1850 as parallel classes I–IV with Polish as teaching language, at II gymnasium. Since 1857, it was named the III King Franz Jozef I Gymnasium with Polish as teaching language, an expression of gratitude for the recognition of the Polish language; this happened in the period of establishing the foundations of the Galician autonomy.

Since 1919, the Gymnasium was under the auspices of King Stefan Batory. In 1919, after obtaining independence from Poland, it was named Państwowe III Gimnazjum Lwówskie im. Króla Stefana Batorego (King Stefan Batory State III Lwów Gymnasium). During 8 years of gymnasium, the weekly schedule in mathematics was the following: 3, 3, 3, 3, 4, 3, 3, 2 hours; in the subsequent years, this schedule was modified.

The following is an example of the final examination problems in C. K. Gymnasium named after Franc Jozef from 1877:

Solve the equation: $x^2 + x(a+b)(a-b) + a^2\sqrt{a+b} = \sqrt{a+b}(x+b)^2$.

- A ball with volume 275768·9 cm^3 is cut by a plane. What is the distance from the center to the cutting plane if the length of the intersection is 1769·5 cm?
- After how many years 7548 złr. (złoty per year) deposited on compound percent of 6 ¾, decreased by 547 złr. at the end of every year, will diminish to 2450 złr.?

As an example of a real gymnasium, let us mention the Mickiewicz Gymnasium in Lwów (functioned since 1908). From 1916, it was named the Adam Mickiewicz High and Real Gymnasium in Lwów.

Initially, in 1908 and 1909. instruction in the gymnasium took six years. In 1910, one more teaching year was added. The pupils graduated after the seventh class. In 1911, the gymnasium became an eight year school. In total, the schedule for mathematics included the following weekly hours:

21 hours in 6 years gymnasium,
23 hours in 7 years gymnasium,
25 hours in 8 years gymnasium.

Rozkład godzin.

(g=oddział klasyczny; rg=oddział realno-gimnazyalny).

	I	II	III		IV		V		VI		VII		VIII	
			g.	rg.	g.	rg	g.	rg.	g.	rg.	g	rg.	g.	rg.
Religia	2	2	2	2	2	2	2	2	2	2	2	2	2	2
Język polski	3	4	3	3	3	3	3	3	3	3	3	3	4	4
„ łaciński	6	6	6	6	6	6	6	6	6	6	5	5	5	5
„ grecki	—	—	5	—	4	—	5	—	5	—	4	—	5	—
„ francuski	—	—	—	3	—	3	—	3	—	3	—	3	—	3
„ niemiecki	5	4	4	4	4	4	4	4	4	4	4	4	4	4
Historya	2	2	2	2	2	2	3	3	3	3	4	4	3	3
Geografia	2	2	2	2	2	2	1	1	1	1	—	1	—	—
Matematyka	3	3	3	3	3	3	3	3	3	3	3	3	2	2
Geometrya wykreślna	—	—	—	—	—	—	—	2	—	2	—	—	—	—
Nauki przyrodnicze	2	2	—	—	3	—	3	3	3	3	—	1	—	1
Fizyka	—	—	2	2		3	—	—	—	—	3	3	3	4
Chemia	—	—	—	—	—	—	—	—	—	2	—	2	—	—
Propedeutyka filozof.	—	—	—	—	—	—	—	—	—	—	2	2	2	2
Rysunki	2	2	2	2	3	3	—	—	—	—	—	—	—	—
Kaligrafia	1	—	—	—	—	—	—	—	—	—	—	—	—	—

The weekly schedule in the classical and the real division (taken from the school annual reports). Notation: g concerns the classical type and rg the real type.

In the time of Galician autonomy, the gymnasial authorities published annual reports for every school year. In addition to an official part, they contained materials from different areas of science. Ludwik Horodyński (1882–1920), one of the founders of the Mathematical Society in Kraków, later the Polish Mathematical Society[2], published

[2] In 1917, the Mathematical Society was also founded in Lwów.

a mathematical article *Podstawowe twierdzenia rachunku całkowego*.[3] Here, the notion of the definite integral is introduced and numerous applications of integrals are considered. In the introduction we find the following text that concerns the acquaintance with the foundations of set theory in Poland:

Dr Józef Puzyna was the first in Poland who presented the foundatios of set theory which – since a few years – open every textbook in analysis. In his two-volume classical work „Teorya funkcyj analitycznych" (Lwów, 1898–1900) he includes a chapter „Z teoryi mnogości" („From the set theory"). The foundations of the theory are also, to a large extent, considered by S. Dickstein in the book „Pojęcia i metody matematyki" (Warszawa 1891). A year ago, „Zarys teoryi mnogości" by Dr. Wacław Sierpiński was published (Warszawa 1912); here everything which was created in this area of knowledge is presented in a precise, clear and fundamental way. The lack of applications to analysis and geometry is explained in the introduction with the heterogenity of these applications in all areas of mathematics. We conclude that it is time that the achievements of the set theory penetrated a wider area.

3 Polytechnic School in Lwów

In 1811, King Franz I agreed to open a Real School in Lwów. However, the school was opened only in November, 1817. The school was of a technical character and was regarded as the first step of establishing a technical high-school. As the result of efforts of the Galician Parlament (Sejm Stanowy), King Ferdinand I transformed the Real School into CK Real-Commerce Academy in 1835, in which the technical studies were even beyond the level of secondary school. Due to subsequent actions of Sejm Stanowy, the Academy was promoted to three-year subdivisions in technical and agricultural directions, and was named the CK Technical Academy, this time as a technical high-school in Lwów. In November 4, 1844, there were inauguration ceremonies of the Academy to which the Chair of Construction from the university was added.

There were two directions in the Academy: technical (three years) and trading (one year, replaced by secondary school in 1853–1854). A two-year Real School was offered at Technical Academy. Six chairs were opened: Mathematics, Physics, Mechanics, Chemistry, Construction and Geodesy. There were 7 Professors, 6 Associate Professors, 5 teachers of languages and pictures, and 3 Assistant Professors. The Court Education Commission in Vienna decided on programs and affairs of the Academy. Prof. Dr. Florian Schindler was appointed Director of the Technical Academy in 1844–1849.

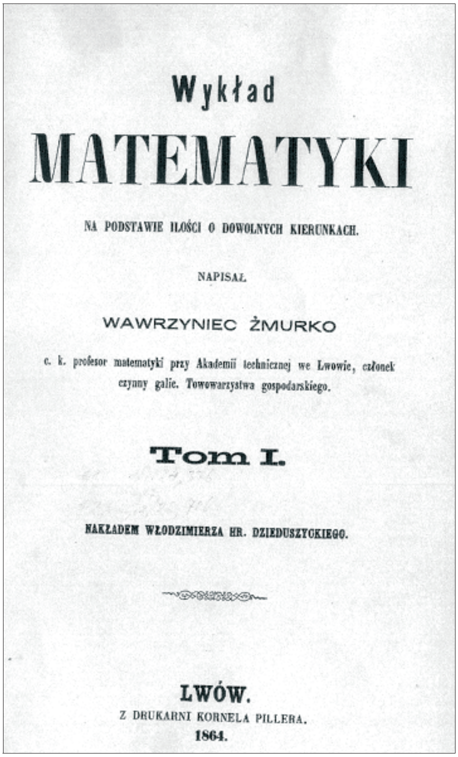
The acting director of the Academy, Professor Aleksander Reisinger, managed the reconstruction of the Academy (after bombing in the period of Springtime of the Nations) and the organization of teaching. He was also the head of the Chair of Mathematics and successfully served the Academy. After Prof. Reisinger was appointed to the director position, Prof. Dr Wawrzyniec Żmurko became head of the Chair of Mathematics (in 1852); he was the first Pole among the professors of the Academy and he is now recognized as the pioneer of the Lwów Mathematical School.

[3] Sprawozdania Dyrekcyi c. k. II Szkoły Realnej we Lwówie za r. sz. 1912/13.

3.1 Wawrzyniec Żmurko

Wawrzyniec Żmurko (1824–1889) graduated from the University of Lwów. Żmurko's biography can serve as an example of how the graduates from the University of Lwów could build their scientific career in the times when Poland did not exist on the maps of Europe.

Żmurko was born in July 9, 1824, in Jaworow, near Przemyśl. After graduating, 1844, in the gymnasium in Przemyśl, he started the philosophical studies in Lwów. Two years later, Żmurko moved to Vienna where he studied simultaneously at the University and Polytechnic School. In 1849, he published *Beitrag zum Integralcalcul*; this book was recognized as his habilitation thesis. In 1848, as an Associate Professor (Docent) of Mathematics, Żmurko started teaching in the Polytechnical School in Vienna. At his own request, he was moved to Lwów, where he received a nomination for professorship in Mathematics at the Polytechnical School. In 1851, Żmurko became the head of the Chair of Mathematics. In 1872, he received the II Chair of Mathematics at the University of Lwów. In 1872–1884, he headed both chairs. Żmurko published about 26 works including two texbooks. He published his results, in particular, in Viennese journals and also in *Pamiętnik Towarzystwa Nauk Ścisłych w Paryżu* [Memoirs of Society of Exact Sciences] and *Roczniki Towarzystwa Naukowego Krakowskiego* [Annals of Kraców Scientific Society].


Wykład

MATEMATYKI

NA PODSTAWIE ILOŚCI O DOWOLNYCH KIERUNKACH.

NAPISAŁ

WAWRZYNIEC ŻMURKO

c. k. profesor matematyki przy Akademii technicznej we Lwowie, członek czynny galic. Towowarzystwa gospodarskiego.

Tom I.

NAKŁADEM WŁODZIMIERZA HR. DZIEDUSZYCKIEGO.

LWÓW.
Z DRUKARNI KORNELA PILLERA.
1864.

Title page of the lecture in mathematics, it is worth noting that it was published at the cost of Count Dzieduszycki.

In 1864, he published in Lwów the textbook *Wykład matematyki na podstawie ilości o dowolnych kierunkach* [Exposition of Mathematics on the basis of quantities in different directions]. In 1873, he published (in German) an extensive book *Beitrag zur Erweiterung der Operationslehre der konstruktiven Geometrie ...*, in which he presented the description of his own examples.

Żmurko worked on certain problems of calculus and didactics; he wrote (with Oskar Fabian) also *Matematyka dla szkół średnich* [Mathematics for Secondary Schools, 1876]. He belonged to the group that founded the Polytechnical Society in Lwów (1865). Since 1872, Żmurko was a member of the Academy of Skills.

3.2 Entering and course examinations

Like in other polytechnical schools of the monarchy, mathematics was an essential discipline that was required for state examinations. The entering examinations in Descriptive Geometry and Manual Drawing were obligatory for those who had not studied these subjects in school; at least, a "satisfactory" mark was necessary. The requirements were the following: *Descriptive Geometry as a thorough knowledge of the methods of orthogonal projections, in particular, projections of points, lines and plane figures onto three planes. Traces of lines and planes. The edges of planes, the intersection point of the line and plane. Rotation of a point about a line. Intersection of planes. Projection of the circle. Projections, sections, and the penetration of prismatic pyramids. Drawings: the execution of a perspective picture from a model, representation by brush and in detailed form.*

In order to verify the acquired knowledge and skills at the Polytechnic School, in the scientific and technical profession, the governmental examinations took place. The first governmental examination or general examination in preparatory sciences included the mathematical disciplines. The second governmental examination or professional examination was on subjects in the chosen profession. The mathematical subjects were not obligatory. The following mathematical subjects were obligatory at the first governmental examination:

For Civil Engineering: knowledge of mathematics (courses I and II) and descriptive geometry.
For Construction: knowledge of mathematics (course I) and descriptive geometry.
For Machine Construction: knowledge of mathematics (courses I and II) and descriptive geometry.
For Technical Chemistry: knowledge of mathematics (course I).

Course I
The generalization of mathematical operations on the basis of vectors, operations, compositions and substitutions. Logarithms, powers, roots, determination of convergent and divergent series on the basis of Newton series. Solving equations of higher degree by Horner method representing their roots numerically, or series arranged according to the powers of the parameter, according to whether the coefficients are numbers or expressions organized according to the powers of the parameter.
Decomposition of rational functions into simple fractions by Horner method. The theory of determinants and applications. Analytic Geometry of the plane and the space, identification of curves and surfaces up to the second order. Arithmetic and inverse

series, the principles of differential and integral calculus. Separation of the roots of numerical equations by Fourier method, as well as by means of analytic geometry. The most important properties of the curvature of curves and surfaces.

Course II

Methods of integration and fundamental integration formulas by transformation, the parts and expansion, by reduction to the measurability, by reduction and by series. Definite integrals and their properties. Methods of calculating definite integrals. Euler integrals, Fourier series and integrals. Rectification and determination of the length of curves.

Volume and area of surfaces. First order ordinary differential equations. Some types of higher order equations. The theory of ordinary differential equations with the aid of infinite series. Integrating the system of simultaneous ordinary differential equations, especially linear. The theory of singular solutions of ordinary differential equations, integrating partial differential equations of higher order and linear equations. Theory of curvature on a surface. The first principles of variational calculus, theory of brachistochrone and geodesic curves.

We place here a biography of Lucyan Böttcher who was connected to the Polytechnic School for about 35 years.

3.3 Lucyan Böttcher (1872–1937)

Lucyan Böttcher was an interesting personality, little known, related to Lwów, but not to Lwów Mathematical School.

L. Böttcher was born on January 21, 1872 in Warsaw. In his Curriculum Vitae which supplemented his PhD thesis in Leipzig, he indicated that he belonged to the Evangelic-Lutheran confession. In Warsaw he finished a so called real school and in 1893 he graduated from the gymnasium in Łomża. Then Böttcher started his studies at the university in Warsaw, which was then a Russian university called the Tsar University. Because of his participation in demonstrations dedicated to J. Kilinski, Böttcher was forced to leave Warsaw and entered the Department of Machine Construction at the Polytechnical School in Lwów. Two years later, he interrupted his studies in Lwów and moved to the Leipzig University. In 1898, Böttcher was awarded PhD degree for his thesis *Beiträge zur Theorie der Iterationsrechnung*.

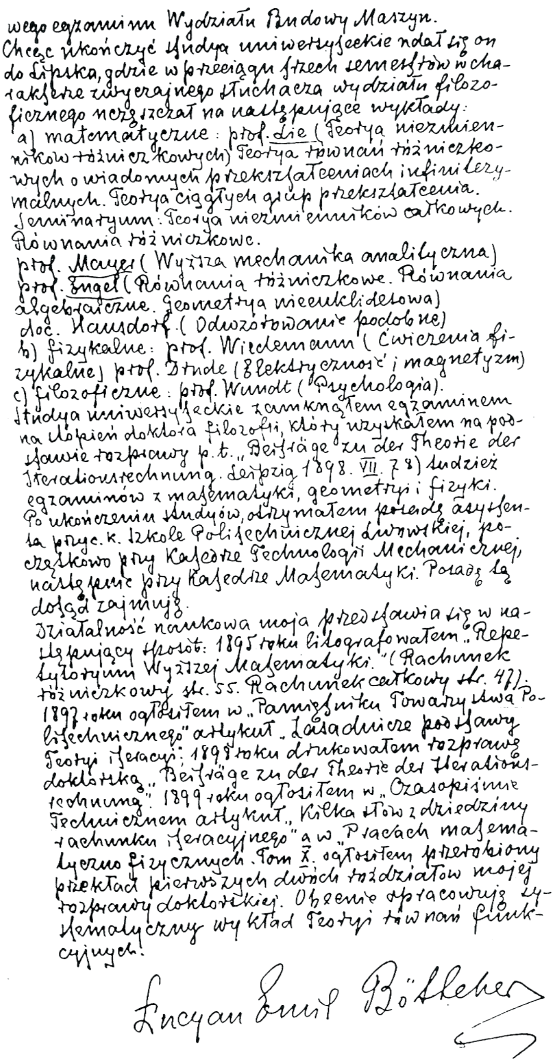
In the same year he returned to Lwów and started his activity at the Polytechnical School, initially as an Assistant Professor of the Chair of Mechanical Technology and, in the next year, of the renowned Chair of Mathematics. In 1910, Böttcher occupied an Associate Professor position and in 1912 a Privatdozent position at the Polytechnical School.

Summing up we see that Böttcher worked about 37 years (until his retirement in 1935) for the Polytechnical School. He taught applied mathematics (solving and discussing mathematical problems important for technical applications), vector theory (the course contained fundamental facts on vectors and operations on them as well as applications of vectors in geometry and mechanics), differential equations, notions and methods of elementary mathematics.

Since 1912, Böttcher got interested in spiritism and metapsychology (whatever this term means) as well as occultism. One may speculate whether this direction of Böttcher's interests caused that he did not contact the other mathematicians from the famous Lwów mathematical school.

Evidently, these interests also affected the style of Böttcher's mathematical life – his active participation in different mathematical events (e.g. congresses of Polish doctors and scientists: Kraków, July, 1899, the title of his talk was *Substitutional functional equation*, Lwów, July, 1907, talk *From the theory of functional equations*; Kraków, 1900, session of Academy of Skills, talk *Fundamental properties if grevian*) before this period, we did not notice any similar activity in the forthcoming years. In addition to presenting mathematical talks, Böttcher paid a lot of attention to the didactics of mathematics; this is witnessed by his numerous articles in Polish periodicals devoted to teaching mathematics.

Böttcher died on May 29, 1937 in Lwów.



wego egzaminu Wydziału Budowy Maszyn.
Chcąc ukończyć studya uniwersyteckie udałem się
do Lipska, gdzie w przeciągu trzech semestrów w cha-
rakterze zwyczajnego słuchacza wydziału filozo-
ficznego uczęszczał na następujące wykłady:
a) matematyczne: prof. Lie (Teorya niezmien-
ników różniczkowych) Teorya równań różniczko-
wych o wiadomych przekształceniach infinitezy-
malnych. Teorya ciągłych grup przekształcenia.
Seminaryum: Teorya niezmienników całkowych.
Równania różniczkowe.
prof. Mayer (Wyższa mechanika analityczna)
prof. Engel (Równania różniczkowe. Równania
algebraiczne. Geometrya nieeuklidesowa)
doc. Hausdorff (Odwzorowanie podobne)
b) fizykalne: prof. Wiedemann (Ćwiczenia fi-
zykalne) prof. Drude (Elektryczność i magnetyzm)
c) filozoficzne: prof. Wundt (Psychologia).
Studya uniwersyteckie zamknąłem egzaminem
na stopień doktora filozofii, który uzyskałem na pod-
stawie rozprawy p.t. „Beiträge zu der Theorie der
Iterationsrechnung. Leipzig 1898. VII. 78) tudzież
egzaminów z matematyki, geometryi i fizyki.
Po ukończeniu studyów, otrzymałem posadę asysten-
ta przy c. k. Szkole Politechnicznej Lwowskiej, po-
częstkowo przy Katedrze Technologii Mechanicznej,
następnie przy Katedrze Matematyki. Posadę tę
dotąd zajmuję.
Działalność naukowa moja przedstawia się w na-
stępujący sposób: 1895 roku litografowałem „Repe-
tytoryum Wyższej Matematyki.” (Rachunek
różniczkowy str. 55. Rachunek całkowy str. 47).
1897 roku ogłosiłem w „Pamiętniku Towarzystwa Po-
litechnicznego” artykuł „Zasadnicze podstawy
Teoryi Iteracyi”. 1898 roku drukowałem rozprawę
doktorską „Beiträge zu der Theorie der Iterations-
rechnung”. 1899 roku ogłosiłem w „Czasopiśmie
Technicznem artykuł „Kilka słów z dziedziny
rachunku iteracyjnego” a w „Pracach matema-
tyczno fizycznych. Tom X. ogłosiłem przerobiony
przekład pierwszych dwóch rozdziałów mojej
rozprawy doktorskiej. Obecnie opracowuję sy-
stematyczny wykład Teoryi równań funk-
cyjnych.
Lucyan Emil Böttcher

A part of CV of L. Böttcher.

That is how a short biography of Böttcher could look. We know from Böttcher's CV that he also attended lectures of the famous Russian professors Sonin and Anisimov.

In Leipzig, he also attended lectures of Professors: S. Lie (*Theory of differential invariants*, *Theory of differential equations*, *Theory of continuous transformation groups*; seminars *Theory of integral invariants* and *Differential equations*), Mayer (*Higher analytical mechanics*), Engel (*Differential equation*, *Algebraic equations*, *Non-Euclidean geometry*), F. Hausdorff (*Similarity maps*).

Böttcher applied three times for the Veniam legendi at the Lwów University, during the times of occupation as well as the Galician autonomy, and during the II Republic. Despite his efforts, Böttcher did not receive Venia legendi at the University of Lwów. Nowadays, his publications are widely cited by experts in differential equations and iteration theory.[4]

S. Lie's opinion was that Böttcher is a hard-working, talented student. Lie also emphasized that Böttcher himself chooses his direction of investigations.

After oral examination S. Lie found Böttcher to be *an intelligent mathematician having good and fundamental knowledge*.

4 Academy of Commerce

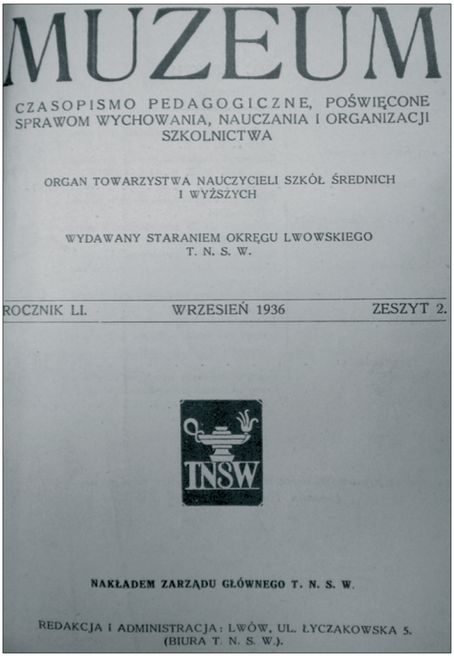
In 1817, a 3-year state real-commercial school was established, which was named the Real-Commerce Academy in 1845. From 1875, a high school called Academy of Commerce has been separated from the Polytechnic schools in Lwów in 1894 and transformed in the secondary-high 4-year National School of Commerce with the Polish language of teaching.

In 1922, on the basis of the National School of Commerce, there was established the 4-year Higher School of International Trade transformed into the Academy of International Trade (with the right of granting masters degrees) in 1937 Merchant accounts General and political arithmetic's, later algebra Geometry.

5 Universities Teachers' Association

Universities Teachers' Association for lower secondary schools, Galicia conservative Universities Teachers' Association for lower secondary teachers, teachers of seminars and universities, published in Lwów from January 1885 the monthly *Muzeum*, with a gap in 1914–1915. Following the union, on 28 December, 1919, of the Society with the the former Kingdom Teacher Association to Association of Polish universities and secondary school Teachers, the journal *Muzeum*, converted at the beginning of 1920 into a quarterly journal, was the organ of the Lwów district.

[4] Reich L.: *Generalized Böttcher equations in the complex domain*, Symp. On Complex Differential and Functional Equations. Univ. Joensuu Dept. Math. Rep. (2004), no. 6, pp. 135–137.
Balibrea F., Reich L., Smital J.: *Iteration theory: dynamical systems and functional equations*, Intern. Journal of Bifurcation and Chaos 13(2003), no. 7, pp. 1627–1647.
Bergweiler: *Iteration of meromorphic functions*, Bull. Amer. Math. Soc. 29(1993), no. 2, pp. 151–188.
Jones O.: Multivariate Böttcher equations for polynomials with nonnegative coefficients, Math. Research Report No. MRR 99.014.
Poggi-Corradini P.: *Canonical conjugations at fixed points other than the Denjoy-Wolff point*, Annales Academiae Scientiarum Fennicae Mathematica 25(2000), pp. 487–499.



MUZEUM

CZASOPISMO PEDAGOGICZNE, POŚWIĘCONE SPRAWOM WYCHOWANIA, NAUCZANIA I ORGANIZACJI SZKOLNICTWA

ORGAN TOWARZYSTWA NAUCZYCIELI SZKÓŁ ŚREDNICH I WYŻSZYCH

WYDAWANY STARANIEM OKRĘGU LWOWSKIEGO T. N. S. W.

ROCZNIK LI. WRZESIEŃ 1936 ZESZYT 2.

TNSW

NAKŁADEM ZARZĄDU GŁÓWNEGO T. N. S. W.

REDAKCJA I ADMINISTRACJA: LWÓW, UL. ŁYCZAKOWSKA 5. (BIURA T. N. S. W.).

The cover of the journal *Muzeum*

6 Pedagogical Overview

Covering, in the years of World War I, all secondary school teachers of the former Kingdom, the Polish Teachers Association published in Warsaw in January, 1917 the monthly *Pedagogical Overview*. In 1918/1919, it became a central body of the Society of Teachers of secondary and higher education. In 1922, it was transformed into the quarterly, whose topics no longer included the organizatorial matters.

The *Muzeum* published materials on the teaching of mathematics by Zaremba, Ingarden, Böttcher and others. In 1905, the Society established the Commission for Reforms in Secondary Schools in Austrian regions.

The program featured in the *Muzeum* initiated a long enough discussion lasting till almost the beginning of the World War II. L. Böttcher, A. Hoborski, L. Zarzecki, after 1918 S. Kulczycki, S. Zaremba, B. Bielecki e.a. contributed.

7 Congresses of natural scientists and doctors

At the end of the nineteenth century opportunities for natural scientists and doctors to meet were *Zjazdy* [Congresses]. In the course of these meetings the mathematical section was formed. The teaching of mathematics in secondary schools was also an object of consideration at the meetings of mathematicians. The materials from the Congresses appeared in the *Dzienniki* [Diaries], for example, in 1907, when the Congress took place in Lwów (from 21 July, for a week). It was the Xth Jubilee Congress. The following lectures in mathematics were presented in the Section of Mathematics and Physics:

- Lucyan Bottcher (Lwów) – From the field of the theory of functional equations;
- Władysław Gosiewski (Warszawa) – The value of induction from the point of view of probability;
- Stanisław Zaremba (Kraków) – New method of verification of the fundamental properties of Green functions;
- Józef Puzyna (Lwów) – reserved.[5]

In the introductory information on the Congress one could not find two lectures by K. Żorawski: *On a relationship concerning partial differential equations of the first order*; *On some research from the theory of differential forms of the second degree*.

8 University of Lwów

The opening of the Main School in Warsaw in 1862 coincided with the origin of the Polish science center, namely the University of Lwów. Founded, on leftovers of the Jesuit Academy in Lwów, by Jan Kazimierz in 1661, Austrian university had a changing fortune. In 1859, under the influence of certain political events, the authorities in Vienna agreed that the language of instruction was Polish. It is worth mentioning that this was also a proposal of the University Senate. Particular enthusiasts of this were the departments of law and philosophy. The mathematical resources in Lwów were formed slowly and gradually. The following mathematicians worked and created in Lwów: Wawrzyniec Żmurko (1824–1889; see its biography above), Władyslaw Zajączkowski (1837–1898), Władyslaw Kretkowski-Trzaska (1840–1910), Józef Puzyna (1856–1919), Placyd Dziwiński (1851–1936).

In 1851, Żmurko was appointed the Chair of mathematics at the Technical Academy in Lwów (transformed to the Polytechnic in 1877). In 1871 he received the Chair of Mathematics at the University of Lwów. He kept both positions at the Polytechnic School and the University until his retirement in 1884.

8.1 Władysław Zajączkowski (1837–1898)

Doctor, Assoc. Professor, Private doc. at the Jagiellonian University, a lecturer of the Principal School, from 1872 he was a professor at the Technical Academy in Lwów, a professor and, since 1881, the University of Lwów. (see A. Pelczar, Differential equations in Poland, WM ... Outline, 2001). Władysław Kretkowski graduated from the School of Roads and Bridges as well as from the Sorbonne in Paris. In 1879, an Associate Prof. (Docent) of the Polytechnical School in Lwów, and in 1881 an Ass. Prof. (Docent.) of the University of Lwów. The most well-known and appreciated work of Kretkowski was *Short information on determinants* (Paris, 1870). *Information* was an addendum to the textbook – Foundations *of differential and integral calculus*, Vol I. by Władysław Folkierski.

[5] From Chronicle News of Wiadomości Matematyczne (1907, vol. IX., p. 285), we know that Puzyna's topic was *Notes on a linear integral equation*.

8.2 Józef prince Puzyna (1856–1919)

Prof. Józef prince Puzyna was the head of the Seminar. Puzyna, a pupil of W. Żmurko, graduated from the University of Lwów.

Between 1883–1885, he studied mathematics in Berlin and attended, among others, lectures by C. Weierstrass, L. Kronecker and L. Fuchs. He was affiliated with the University of Lwów with his scientific and educational activities. He taught mathematics as a lecturer (1885–1889), Associate Professor (since 1889), Professor and Head of Department of Mathematics (from 1892 until his death). He was Dean of the Faculty of Philosophy (1894–1895) and Rector of the University (1904–1905). In 1900 Puzyna was elected a corresponding member of the AU? In 1917, he founded the Mathematical Society in Lwów, of which he was the president. The Mathematical Society existed till 1919. A. Pelczar wrote: *He* [Puzyna] *was an outstanding professional in the field of analytic functions, and has written a paper in the theory of differential equations. The most outstanding student of W. Żmurko.*

The fundamental monograph *Theory of analytic functions* by Puzyna was published In Lwów in 1898 (Volume I) and in 1900 (Volume II). Volume I consists of 6 parts (Chapters). Chapter 1 is of introductory character; it contains the material from arithmetics, theory of complex numbers and set theory.

In their Introduction to *Funkcje analityczne* published in 1938 S. Saks and A. Zygmund wrote that Puzyna's monograph *is a genuine encyclopedia of Analysis – partially exposed in beautiful Weierstrassian style – and contains the material from the areas of Set Theory and Topology (Analysis Situs), Group Theory, Algebra, Differential Equations, and Harmonic Functions. If it were published in any of the widespread foreign languages, the monograph would expect subsequent, more and more perfect editions, having all the properties to become a classical textbook.* However, Saks and Zygmund even do not attempt to provide a wider survey of Puzyna's monograph. The literature in the history of mathematics does not pay too many attention to it, therefore we decided to consider its contents as well as its significance in the present paper.

Puzyna's monograph is of encyclopedical character. Let us describe briefly the content of *Volume I.*

Part 1. On numbers, variable quantities and sets.

Chapter I. From arithmetic: consists of the following topics: Definition of real numbers, rational, irrational, and their systematic expansions. The expansion of a rational number is infinite periodic. Infinite non-periodic expansions. Definition of irrational numbers by limits of sequences of rational numbers. Arithmetic operations with irrational numbers. Series of positive numbers, their divergence and convergence. Arithmetic operations for series of positive summands. Quotient conditions of convergence. Series of positive and negative terms. Definition of unconditional convergence. Conditional convergence. Examples. Convergence of series with sign--changing terms. Oscillating series. Theorems of Abel and Dirichlet. Infinite products with real factors. A necessary condition of their convergence. Unconditionally convergent products. Coexistence of unconditional and absolute convergence. Conditionally convergent products. Expansion of real numbers into continued fractions. Finite and infinite continued fractions. Systematic expansion of Strauss.

Chapter II. *On complex numbers:* contains the following topics: Definition of complex numbers by means of arithmetic operations. Absolute value of a complex number. Theorems on absolute value of the sum, difference, product and ratio. Geometric representation of complex numbers. Complex plane. Geometric description of arithmetic operations. Introduction of the symbol 1_φ. Formulas of Moivre and Euler. Calculating of the n-th root of a complex numbers, for $n = 2q$. Algebraic forms that provide an approximation for arbitrary n. The n-th root of the unit +1. Prime roots of the unit. Two theorems on prime roots. Cases of algebraic solutions of the equation $wn - 1 = 0$. Geometric construction performed by means of ruler and compass. Remark on a general power $(a + bi)^{\alpha + \beta i}$. Infinite series with positive terms. Their conditional and unconditional convergence. Infinite products with complex factors. Various forms of their convergence.

Chapter III of Volume I contains the material from set theory. It is worth mentioning that this was the very first exposition of the fundamentals of set theory in Polish. (The name "teoria mnogości" for "set theory" belongs to Puzyna; in modern Polish terminology "zbiór" is used for "set"). The author introduces, in an informal manner, bounded and unbounded domains of real and complex numbers, real and complex n-dimensional spaces, neighbourhoods and points at infinity. The concept of cardinality is discussed in detail and is supplemented with numerous examples. It is proved, in particular, that the unit interval is not of countable cardinality. The following topological notions are also mentioned: accumulation point, the derivative of a set, closed set, dense set, isolated point. The author does not define compactness and uses the term "zwarty" ("compact") in the sense of "connected" or rather "arcwise connected". The connected domains are considered and it is proved that the complement in a connected domain of two variables to a countable set is again connected. The exposition here is not rigorous by modern mathematical standards, since some undefined notions are used. As an example, we present here the translation of a definition from this part of the book: *A "continuum" P such that from each of its places* $(x_1', \ldots, x_n')$ *to every other of its places* $(x_1'', \ldots, x_n'')$ *one can pass only via places belonging to the same "continuum" is called compact or a compact domain.* (See the remark above concerning the terminology.) Also, there are defined the derivatives of the set of transinite orders. It is interesting to note that, in Chapter III, a proof is given that the n-dimensional cube (in modern terminology) and the unit segment are of equal cardinality. This question was later asked by Sierpiński.

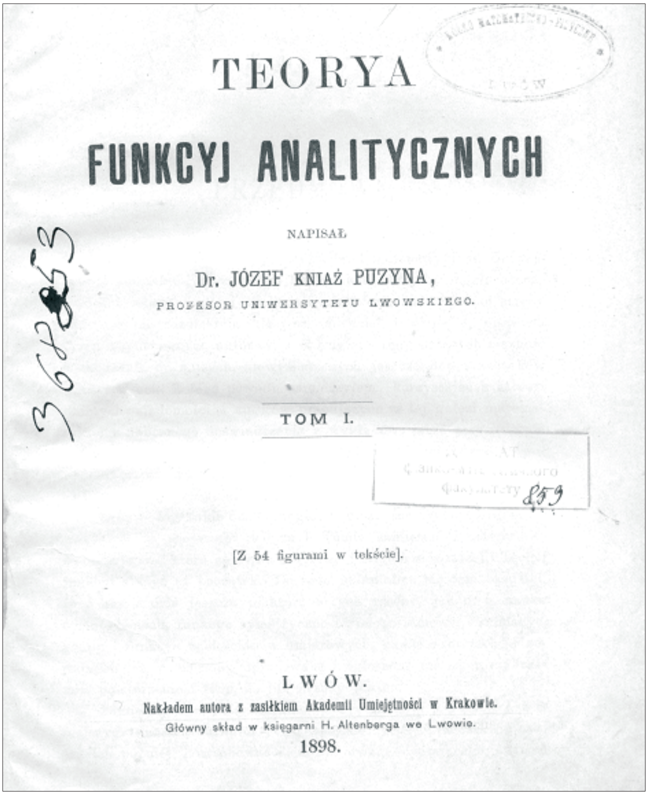
Part 2. *On rational functions:* consists of the following chapters:
IV: On rational entire functions of one variable.
V. On rational fractional functions of one variable.
VI. On rational entire and fractional functions of several variables.

Part 3 is devoted to the symmetric and multiformed functions as well as to the rotations of polyhedra and their functions. The author defines the symmetric functions and proves the fundamental property of symmetric functions, namely that every symmetric function of n variables is a rational entire functions $c_1, \ldots, c_n$, where c_i is the sum of all possible products of i variables. Every symmetric function can be represented by means of elementary symmetric functions in a unique manner. A part of this chapter is devoted to the permutation group. This group is investigated in details. The Galois type is also defined. In modern language, the fundamental statement of the Galois theory is demonstrated for one extension of $C(x_1, \ldots, x_n)$. The symmetry groups of polygons and regular polyhedra are considered. In particular, it is proved that the group of tetrahedron consists of twelve rotations.

Part IV. *On eliminations and the theory of binary forms* contains the theory of resultants and elements of invariant theory.
IX. On eliminations of two equations.
X. On eliminations of n equations ($n > 2$).
XI. On the theory of forms.

Part V. On power series.
XII. On power series and their convergence.
XIII. Arithmetic connection of power series.
XIV. Fractional expansions of rational functions.

Part VI. *Continuation of power series. Definition and the most general subdivision of analytic functions*. An algebraic approach to the theory of analytic functions can be seen from this part.
XV. Contiunuation of power series.
XVI. Definitions and general properties of analytic functions as well as sums of such functions.
XVII. Laurent theorem. The most general subdivision of the univalent analytic functions.


TEORYA

FUNKCYJ ANALITYCZNYCH

NAPISAŁ

Dr. JÓZEF KNIAŻ PUZYNA,

PROFESOR UNIWERSYTETU LWOWSKIEGO.

TOM I.

[Z 54 figurami w tekście].

LWÓW.

Nakładem autora z zasiłkiem Akademii Umiejętności w Krakowie.

Główny skład w księgarni H. Altenberga we Lwowie.

1898.

The title page of Puzyna's monograph published by the author's costs with support of the Academy of Skills in Kraków. The book contains about 1000 pages.

— 79 — [30

ROZDZIAŁ III.

Z teoryi mnogości.

30. Zmienna rzeczywista i zmienne rzeczywiste. Ograniczone i nieograniczone ich obszary. Miejsca i otoczenia. I. Wielkość rzeczywista x, która może przybierać wszystkie możliwe wartości od $-\infty$ do $+\infty$ nazywa się z m i e n n ą, a cały zbiór wartości tej zmiennej zawarty między $-\infty$, a $+\infty$ nazywamy n i e o g r a n i c z o n y m o b s z a r e m (z a k r e s e m) t e j z m i e n n e j, albo n i e o g r a n i c z o n y m o b s z a r e m j e d n o k r o t n y m. Naznaczamy go pisząc:

$$(x)=(-\infty\ldots+\infty). \qquad (a)$$

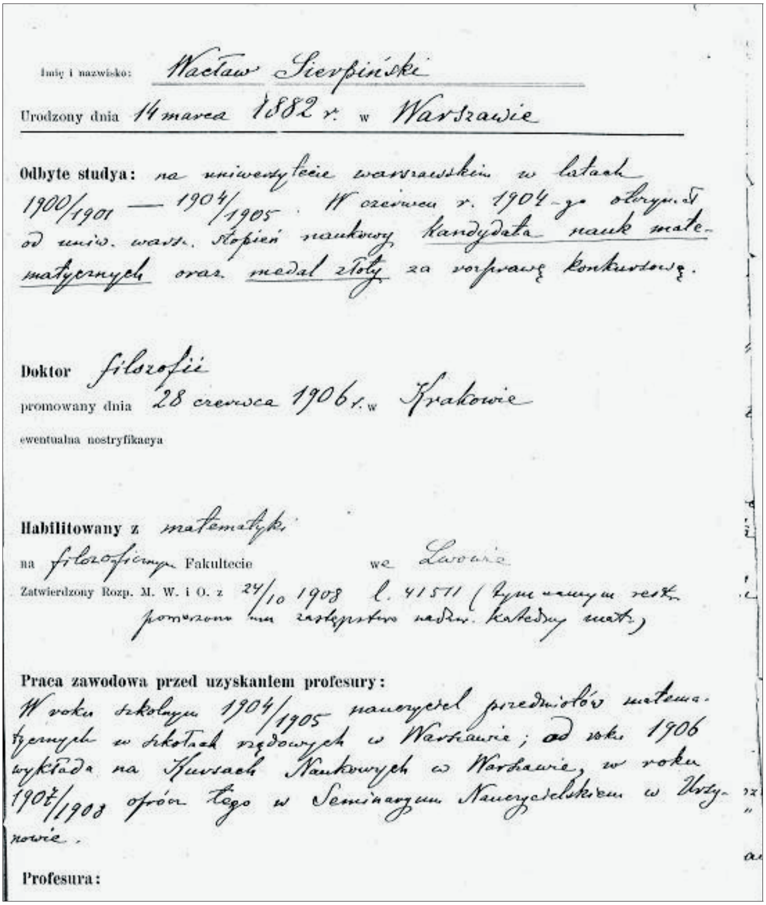
Rozdział III. **Z teoryi mnogości.**

A part of the content of Puzyna's monograph: one can see Charter III *From the set theory*.

The Mathematical Seminar at the University of Lwów was created on December 1, 1893. Since 1893/94, it worked in two divisions: lower and higher. In the winter semester of 1894, there were 18 students of the lower division, 10 in the higher division. In the summer semester the numbers were 13 and 9 respectively.

8.3 Wacław Sierpiński (1882–1969)

Since the beginning of 20-th century, the activity of Wacław Sierpiński was tightly connected with the University of Lwów. Sierpiński, after studying in Warsaw (Tsar University) and doctorate in Kraków, moved to Lwów; habilitation in 1908. The seminar of Sierpiński was attended by, among others, K. Ajdukiewicz, A. Łomnicki, O. Nikodym, S. Ruziewicz, Z. Zawirski, Z. Janiszewski came to Lwów (habilitation 1913), S. Mazurkiewicz, S. Ruziewicz were PhD students of Sierpiński. We would like to mention here that the sources of the Polish mathematical school were layed in Lwów, closely related with the activities of Sierpiński.



Imię i nazwisko: Wacław Sierpiński

Urodzony dnia 14 marca 1882 r. w Warszawie

Odbyte studya: na uniwersytecie warszawskim w latach 1900/1901 — 1904/1905. W czerwcu r. 1904-go otrzymał od uniw. warsz. stopień naukowy kandydata nauk matematycznych oraz medal złoty za rozprawę konkursową.

Doktor filozofii

promowany dnia 28 czerwca 1906 r. w Krakowie

ewentualna nostryfikacya

Habilitowany z matematyki

na filozoficznym Fakultecie we Lwowie

Zatwierdzony Rozp. M. W. i O. z 24/10 1908 l. 41571 (tymczasowym reskr. powierzono mu zastępstwo nadzw. katedry mat.)

Praca zawodowa przed uzyskaniem profesury: W roku szkolnym 1904/1905 nauczyciel przedmiotów matematycznych w szkołach rządowych w Warszawie; od roku 1906 wykłada na Kursach Naukowych w Warszawie, w roku 1907/1908 oprócz tego w Seminaryum Nauczycielskiem w Ursynowie.

Profesura:

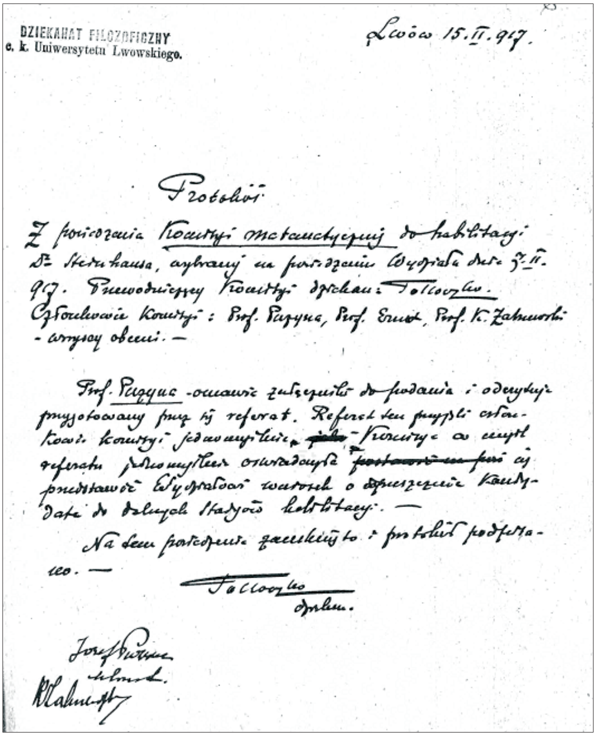
A part of Sierpiński's CV before his application for a professor's position. Note that earlier he worked in high school. He graduated from the Tsar University (a disciple of Voronoi) with gold medal.

A similar role like Sierpiński's one was played in Lwów by Hugo Steinhaus. It is worth noting that it was Puzyna who made possible hiring of Sierpinski and Steinhaus at the University of Lwów, therefore it is hard to overestimate Puzyna's merits in forming the Polish Mathematical School.

8.4 Hugo Steinhaus (1887–1972)

Hugo Steinhaus after studying in Göttingen under the advise of Hilbert and Klein, was awarded the PhD degree for his thesis *Neue Anvendungen des Dirichlet'schen Prinzips*.

He habilitated in Lwów in 1917 on the basis of his thesis *O niektórych własnościach szeregów Fouriera* [On some properties of Fourier series]. This work is considered as one of the first concerning the theory of operations, the direction that made famous the Lwów school of mathematics.


DZIEKANAT FILOZOFICZNY
c. k. Uniwersytetu Lwowskiego.

Lwów 15.II.917.

Protokół

The report of a meeting of the Commission in which Prof. Puzyna recommends admission of H. Steinhaus to further stages of the habilitation procedure. H. Steinhaus was the "inventor of S. Banach" and cocreator of the successes of the Lwów Mathematical School.

9 Conclusion

The Polish mathematicians in Lwów in the nineteenth and early twentieth century formed a unique mathematical culture. This required of them a great commitment, systematic work and far-sightedness in building the foundations of a mathematical center. Their actions were aimed at teaching activities, publishing, self-education, and popularization. An important role was ascribed to middle school teachers, school and academic textbooks. Analysis of academic textbooks and scientific papers shows that the environment to some extent was already formed, and a responsible approach to such a discipline as mathematics and the creation of a modern Polish mathematical terminology deserves the full attention and commitment. Noteworthy is the research, teaching, and organizational activity of professor Puzyna. He contributed to the mathematical successes in Poland immediately after regaining independence in 1918. Puzyna cooperated with and promoted mathematical centers in Kraków, Warsawand Polish mathematicians working in St. Petersburg, Paris and elsewhere. Among the others, Professors Stanislaw Zaremba (even from Paris, before taking the Chair in Kraków), Professors Jan and Ptaszycki, Julian Sochocki from St. Petersburg, were working intensively on the development of mathematics in the Polish lands, as well as Stanislaw Kepinski from Lwów (previously from Kraków). There was seen a successful development of Polish mathematics. It would be difficult to compete even in technical sciences with the European countries in view of the modest financial resources.

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