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Some problems on normality of products of spaces

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SOME PROBLEMS ON NORMALITY OF PRODUCTS OF SPACES

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Products of normal spaces are not always normal. The first problem which I would like to pose is:

(a) If $X \times Y$ is normal, for any normal space Y, is X discrete ?

Here and in the following X and Y are assumed to be Hausdorff spaces.

In case X is a k-space (= compactly generated space), (a) is answered in the affirmative; because, in this case any compact subset of X is proved to be finite by virtue of M. Rudin [5] .

In [4] I proved that a metric space is σ-locally compact if and only if its product with any countably paracompact normal space is normal and that X is a normal P-space if and only if $X \times T$ is normal for any metric space T. In this connection I would like to pose the following problems:

(b) If $X \times Y$ is normal for any countably paracompact normal space Y, is X metrizable and σ-locally compact ?

(c) If $X \times Y$ is normal for any normal P-space Y, is X metrizable ?

Since any normal P-space is countably paracomact, the affirmative answer to (c) (resp. (b)) for some class $\mathfrak{A}$ of spaces X implies the affirmative answer to (b) (resp. (a)) for the same class $\mathfrak{A}$ of spaces X . In [3] T. and K. Chiba proved that the answer to (c) is "yes" in case X is separable. Thus, (a) and (b) are answered in the affirmative in case X is separable. This result for (a) has been obtained by M. Atsuji [1] from another point of view.

Any space X whose product with every normal P-space is normal has a property such that the product of a finite number of copies of X is a paracompact P-space . A paracompact M-space has such a property and (c) is answered in the affirmative by K. Chiba [2] in case X is a paracompact M-space. This result, however, does not give a new result concerning problem (a) since a paracompact M-space is a k-space.

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[2] K. Chiba: On products of normal spaces. Rep. Fac. Sci. Shizuoka Univ. 9 (1974), 1-11.

[3] T. Chiba, K. Chiba: A note on normality of product spaces. Sci.Rep. Tokyo Kyoiku Daigaku Sect. A 12 (1974), 165-173.

[4] K. Morita: Products of normal spaces with metric spaces. I, II. Math. Ann. 154 (1964), 365-382. Sci. Rep. Tokyo Kyoiku Daigaku Sect. A 8 (1964), 87-92.

[5] M.E. Rudin: A normal space X for which $X \times I$ is not normal. Fund. Math. 73 (1971), 179-186.

Addendum (November 10, 1976). Quite recently problem (a) has been solved in the affirmative by Mary E. Rudin on the basis of M.Atsuji [1]; cf. her forthcoming paper " κ-Dowker spaces".