D. Zaremba

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ON PSEUDO-OPEN MAPPINGS

D. ZAREMBA

Wrocław

Let us call a mapping[1] $f: X \to f(X)$ of a topological space X to be *pseudo-open* if

(1) *the images $f(U)$ of open sets $U \subset X$ are open whenever U contains at least one component of the counter-image $f^{-1}(y)$, for which $f^{-1}(y) \cap U \neq \emptyset$.*

Now suppose that the topological spaces in further considerations are metric. It is not too hard to show that for a compact space X a mapping f is pseudo-open if and only if

(2) *the inequality $f^{-1}(y) \cap \operatorname{Ls} f^{-1}(y_n) \neq \emptyset$ implies the inequality $C \cap \cap \operatorname{Ls} f^{-1}(y_n) \neq \emptyset$ for all components C of $f^{-1}(y)$*[2].

In general, (1) is stronger than (2).

It follows from the definitions that for compact spaces the class of pseudo-open mappings contains the class of open ones as well as the class of monotone mappings, but it is strictly larger than the union of both of them. Some theorems which are true for monotone as well as for open mappings can be generalized to pseudo-open mappings. For example one can show that

a non-degenerate pseudo-open image of an arc is an arc.

The concept of pseudo-open mappings concerns the problem to give some sufficient conditions, for metric spaces $\{X, \varrho\}$ and for mappings $f: X \to f(X)$, such that the set $\{\varrho(x, y) : f(x) = f(y)\}$ is connected. It appears that for pseudo-open mappings of chainable continua the set $\{\varrho(x, y) : f(x) = f(y)\}$ is connected, and moreover, it is connected also for pseudo-open mappings of such continua X, for which

(3) *any continuum $Q \subset X \times X$ intersects the diagonal of $X \times X$ whenever $Q \cap \{x\} \times X \neq \emptyset$ and $Q \cap X \times \{x\} \neq \emptyset$*[3].

[1]) A mapping means in this paper a continuous function.

[2]) $\operatorname{Ls} f^{-1}(y_n)$ denotes here superior limit of the sequence $\{f^{-1}(y_n)\}$.

[3]) Property (3) holds for all continua X of span 0 (for definition of the span of a space see A. Lelek: Disjoint mappings and the span of spaces, Fundamenta Mathematicae *55* (1964), 199—214), thus for all chainable continua (ibidem).

Therefore the following theorem holds:

If a continuum X has the property (3) *and a mapping $f: X \to f(X)$ is pseudo-open, then the set $\{\varrho(x, y) : f(x) = f(y)\}$ is connected.*

While proving this theorem a stronger result is obtained, namely it appears that the set $\{(x, y) : f(x) = f(y)\}$ is also connected.

The following lemma is useful to prove the theorem:

If $f: X \to f(X)$ is a pseudo-open mapping of a compact space X onto a connected space $f(X)$, then $f(C) = f(X)$ for any component C of X.

Detailed proofs will be published in Colloquium Mathematicum.