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REMARKS ON DIMENSIONS OF MAPPINGS

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In the dimension theory, besides the dimension of spaces, the dimension of mappings is often examined. If X, Y are topological spaces, $f : X \to Y$ a continuous mapping, we put $\dim f = \sup \{\dim f^{-1}[y] \mid y \in Y\}$. Similarly for uniform spaces, in addition to the uniform dimension Δd of spaces (see [2]), a uniform dimension of mappings can be defined. If $(X, \mathscr{U})$, $(Y, \mathscr{V})$ are uniform spaces, $f : X \to Y$ a uniformly continuous mapping, then $\Delta d f \leqq n$ means that for each U in $\mathscr{U}$ there exist V in $\mathscr{V}$ and W in $\mathscr{U}$ such that for any V-small subset M of Y there exists a W-cover $\mathscr{K}$ of $f^{-1}[M]$ consisting of U-small sets and such that each point of $f^{-1}[M]$ belongs to at most $n + 1$ sets of $\mathscr{K}$. Some results are stated in [1], let us mention here three properties only.

(a) If f maps a uniform space X onto a one-point space then $\Delta d f = \Delta d X$ (therefore the same symbol Δd is used).

(b) If g is the restriction of f onto a dense subspace then $\Delta d\, g = \Delta d f$.

(c) If $f : X \to Y$, then $\Delta d X \leqq \Delta d Y + \Delta d f$.

If X, Y are normal (T_1) topological spaces, we may consider the spaces endowed with some uniformities such that every continuous mapping $f : X \to Y$ becomes uniformly continuous and search for a connection between $\dim f$ and $\Delta d f$. This also enables us to derive some results on dim from the properties of Δd. We have the following theorems.

Theorem 1. *Let X be a normal space, Y a paracompact space, $f : X \to Y$ a closed continuous mapping. If both spaces X and Y are endowed with the fine uniformity, then $\Delta d f = \dim f$.*

Theorem 2. *Let X be a normal space, Y a paracompact space, $f : X \to Y$ a closed continuous mapping. Suppose Y is compact or $\dim Y$ is finite. If both spaces X and Y are endowed with the Čech uniformity, then $\Delta d f = \dim f$.*

Using Theorem 1 or 2, the equality of Δd and dim for both spaces and the above property (c), we immediately obtain the following version of Hurewicz theorem: *If X is a normal space, Y a paracompact space, $f : X \to Y$ a closed continuous mapping, then* $\dim X \leqq \dim Y + \dim f$. This result was also obtained by Pasynkov

[3]. His proof is essentially based on the same theorem for both X, Y paracompact which was proved by Skljarenko by means of the theory of sheaves.

Let X, Y be spaces endowed with the Čech uniformity, $f: X \to Y$ a continuous mapping, βf the extension of f onto the Čech-Stone compactifications, which are the completions of the spaces X and Y. Then the above property (b) and Theorem 1 or 2 imply $\Delta d\, f = \Delta d\, \beta f = \dim \beta f$. Thus Theorem 2 also concerns the equality of $\dim f$ and $\dim \beta f$. The additional assumption on the space Y in Theorem 2 cannot be omitted, which can be shown by an example. In this example, we also construct a closed continuous mapping $f: X \to Y$ with $\dim f = 0$ (moreover with finite pre-images of points), but $\dim \beta f > 0$; the spaces X, Y are metric, locally compact and σ-compact. The construction essentially depends on the following lemma.

Lemma. *Let $G_1, \ldots, G_n$ be open sets which cover the n-dimensional cube I^n. Then at least one set G_j contains a component which joins two opposite faces of the cube I^n.*

A detailed paper containing the proofs of all assertions is intended for publication in Czechoslovak Mathematical Journal.

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