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FIXED-POINT THEORY ON NEIGHBORHOOD RETRACTS OF CONVEXOID SPACES

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The author sets up a fixed-point theory on neighborhood retracts of spaces introduced by JEAN LERAY in 1945 under the name of "convexoid spaces". Proofs and further details can be found in Bull. Soc. Math. France *87*, 235—243 and *89*, 223 – 226.

Leaning heavily on results of J. Leray, a fixed-point index is defined for the retracts of convexoid spaces. Then it is proved that each compact neighborhood retract of a convexoid space consists of a finite number of components, each of which is retract of a convexoid space. Using this fact, a fixed-point index $i_\xi(0)$ is defined for each map ξ of a compact neighborhood retract E of a convexoid space into itself and for each open subset 0 of E such that its boundary contains no fixed point of ξ. The fixed-point index thus defined posesses all the usual properties of this notion.

In this manner, the fixed-point theory of S. Lefschetz is generalized in two directions:

1° It may be applied to spaces which are not absolute neighborhood retracts in the sense of K. BORSUK (there are even acyclic convexoid spaces which are not absolute neighborhood retracts).

2° It studies not only the fixed points in the whole space, but, more generally, the fixed points which lie in an open subset of the space under consideration.

By making use of the notion of the fixed-point index, several fixed-point theorems are established. For instance:

A. *Any acyclic retract of a convexoid space has the fixed-point property.*

B. *Any map ξ of a retract of a convexoid space into itself such that there exists an integer $p > 0$ with the property that ξ^p is homotopic to a constant map has at least one fixed point.*

C. *Let C be a compact space, which is neighborhood retract of a convexoid space, and let ξ be a map of C into itself. If there exists a compact acyclic subset K of C such that*

$$\bigcap_{n>0} \xi^n(C) \subset K,$$

then ξ has at least one fixed point.