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## THE WEAK RADON-NIKODYM PROPERTY

by

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Def. A Banach space  $X$  has the weak RNP iff given any finite complete measure space  $(S, \Sigma, \mu)$  and a  $\mu$ -continuous measure  $\nu : \Sigma \rightarrow X$  of (G)-finite variation there exists a weakly measurable  $f : S \rightarrow X$  such that

$$\forall E \in \Sigma \quad \nu(E) = \text{Pettis} - \int_E f d\mu .$$

Theorem. If  $X$  is separable then  $X^*$  has WRNP iff  $X \not\cong \mathcal{L}_1$  (isomorphically).

Corollary 1. If  $X$  is separable and  $X^{**} = \bigcup_{\alpha \in \omega_1} X_\alpha$  ( $X_\alpha = w^*$ -sequential closure of  $X_{\alpha-1}$  whenever  $\alpha$  is non limit and  $X_\omega = \bigcup_{\beta < \omega} X_\beta$  if  $\omega$  is limit,  $X_0 = X$ ), then  $X$  is weak\* sequentially dense in  $X^{**}$ .

Corollary 2. If  $X$  is separable,  $X \not\cong \mathcal{L}_1$  and  $X^*$  is non-separable, then given a non purely atomic finite measure space  $(S, \Sigma, \mu)$  there exists a bounded Pettis integrable function  $f : S \rightarrow X^*$  which is not weakly equivalent to any strongly measurable  $g : S \rightarrow X^*$ .

Corollary 3 (Rybakov). If the set of extreme points of the unit ball of  $X^*$  is norm separable, then  $X^*$  is separable.

Remark. There exists  $X$  with WRNP and without RNP.