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THE WEAK RADON-NIKODYM PROPERTY

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Def. A Banach space X has the weak RNP iff given any finite complete measure space (S, Σ, μ) and a μ-continuous measure $\nu : \Sigma \longrightarrow X$ of (σ)-finite variation there exists a weakly measurable $f \cdot S \longrightarrow X$ such that

$$\forall E \in \Sigma \qquad \nu(E) = \text{Pettis} - \int_E f d\mu .$$

Theorem. If X is separable then X^* has WRNP iff $X \not\supset \ell_1$ (isomorphically).

Corollary 1. If X is separable and $X^{**} = \bigcup_{\alpha < \omega_1} X_\alpha$ ($X_\alpha = w^*$-sequential closure of $X_{\alpha-1}$ whenever α is non limit and $X_\alpha = \bigcup_{\beta < \alpha} X_\beta$ if α is limit, $X_0 = X$), then X is weak* sequentially dense in X^{**}.

Corollary 2. If X is separable, $X \not\supset \ell_1$ and X^* is non-separable, then given a non purely atomic finite measure space (S, Σ, μ) there exists a bounded Pettis integrable function $f: S \longrightarrow X^*$ which is not weakly equivalent to any strongly measurable $g: S \longrightarrow X^*$.

Corollary 3 (Rybakov). If the set of extreme points of the unit ball of X^* is norm separable, then X^* is separable.

Remark. There exists X with WRNP and without RNP.