Luděk Zajíček

On the singlevaluedness and differentiation of metric projections

In: Zdeněk Frolík (ed.): Abstracta. 7th Winter School on Abstract Analysis. Czechoslovak Academy of Sciences, Praha, 1979. pp. 104--105.

Persistent URL: http://dml.cz/dmlcz/701158

SEVENTH WINTER SCHOOL /1979/

ON THE SINGLEVALUEDNESS AND DIFFERENTIATION OF METRIC PROJECTIONS

L. Zajíček, Praha

Let X be a real Banach space and $M \subset X$ a closed subset of X. For $x \in X$ denote by $d_M(x)$ the distance from the point x to the set M. The metric projection P_M of the space X on the set M is defined as the /possibly/ multivalued operator

$$P_M(x) = \{ y \in M \; ; \; \|x-y\| = d_M(x) \} .$$

Denote by A_M the set of the multivaluedness of P_M.
The sets A_M were investigated e.g. in [2] , [5] and [3] .
<u>Definition.</u> Let $o \neq v \in X$ and Z be a topological complement of $\mathrm{Lin}\{v\}$. Let f be a Lipschitz function defined on Z. Then the set $M = \{ z + f(z)\, v \; ; \; z \in Z \}$ is termed a Lipschitz hypersurface.
<u>Theorem</u> 6 If X is a separable strictly convex Banach space then A_M can be always covered by countably many of Lipschitz hypersurfaces.

In the following the Frechet differentiability of multivalued operators is consider in the natural generalized sense. By N_M we denote the set of all points at which P_M is not Frechet differentiable. The sets N_M were investigated e.g. in [4] and [1] .
<u>Theorem [7]</u> There exists a compact convex set $M \subset R^2$ such that $R^2 - (M \cup N_M)$ is a set of the first category.
<u>Theorem [7]</u> If X is a two dimensional strictly convex Banach space then N_M is always a set of /Lebesgue/ measure zero.
<u>Theorem [7]</u> Let X be a finite dimensional space with a norm q which belongs to the class $C^2(X - \{o\})$ and for which $D^2 q(x)(h,h) > 0$ for any linearly independent $x \neq o, h \neq o$. Then N_M is always a set of /Lebesgue/ measure zero.

R E F E R E N C E S

[1] E.Asplund: Differentiability of the metric projection in finite-dimensional Eucledian space,Proc.Amer.Math.Soc. 38 /1973/,218-219.

[2] P.Erdös: On the Hausdorff dimension of some sets in Euclidean space,Bull.Amer.Math.Soc. 52/1946/,107-109.

[3] S.V.Konjagin: Approximation properties of arbitrary sets in Banach spaces, Dokl.Akad.Nauk. SSSR, 239/1978/,No.2,261-264.

[4] J.B.Kruskal: Two convex counterexamples: A discontinuous envelope function and a nondifferentiable nearest-point mapping,Proc.Amer.Math.Soc. 23/1969/,697-703.

[5] S.Stečkin: Approximation properties of sets in normed linear spaces, Rev.Math.Pures Appl. 8/1963/,5-18 /Russian/.

[6] L.Zajíček: On the points of multivaluedness of metric projections in separable Banach spaces, Comment.Math.Univ.Carolinae 19/1978/,513-523.

[7] L.Zajíček: On the differentiation of metric projections in finite dimensional Banach spaces, to appear.