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HORIZONTAL LIFTS AND FOLIATIONS

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The purpose of this paper is to adapt the concept of a horizontal lift of a connection to a natural vector bundle to foliated mamanifolds. Firstly we recall the basic properties of transverse natural bundles. Secondly we define the horizontal lift of basic connections of order r to a transverse natural vector bundle and we study its properties. Next we prove that for any basic connection of order r on a foliated manifold M there is exactly one horizontal lift to transverse natural vector bundle, which fulfills the satisfactory conditions. The results presented here generalize the results obtained by K.Yano, S.Ishihara, E.M.Patterson and J.Gancarzewicz (see [3], [4], [10], [11]).

I. TRANSVERSE NATURAL BUNDLES

Let M be a smooth manifold of dimension n, and F a codimension q foliation on M defined by a cocycle $\{U_i, f_i, g_{ij}\}$ where $\{U_i\}_{i\in I}$ form an open covering of M. Assume that $f_i:U_i \longrightarrow \mathbb{R}^q$ is a submersion, and $g_{ij}:f_j(U_i\cap U_j)\longrightarrow f_i(U_i\cap U_j)$ is a diffeomorphism such that

$$f_j|U_i\cap U_j = g_{ji}f_i|U_i\cap U_j$$

If M_F is a smooth q- dimensional manifold equal to $\bigcup f_i(U_i)$, then the mappings g_{ij} can be regarded as local diffeomorphisms of the manifold M_F and the foliation F as modelled on M_F.

Let $\{(U_i, \varphi_i)\}$ be an adapted atlas of the manifold M to the foliation F. Thus $\varphi_i:U_i \longrightarrow \mathbb{R}^{n-q}\times\mathbb{R}^q$ and if we denote the first n-q coordinates by $y_1,\dots,y_{n-q}$ and the last q coordinates by $x_1,\dots,x_q$, then $\varphi_j\circ\varphi_i^{-1}:\mathbb{R}^{n-q}\times\mathbb{R}^q \longrightarrow \mathbb{R}^{n-q}\times\mathbb{R}^q$ is of the

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form $(\xi_0(y,x), \xi_1(x))$, where $\xi_0 : \mathbb{R}^{n-q} \times \mathbb{R}^q \longrightarrow \mathbb{R}^{n-q}$ and $\xi_1 : \mathbb{R}^{n-q} \times \mathbb{R}^q \longrightarrow \mathbb{R}^q$.

By FOL_q denote the category of smooth manifolds foliated by smooth codimension q foliations with smooth, foliation preserving transverse mappings to the foliation.

Definition 1. A covariant functor $\mathcal{N}$ on the category FOL_q into the category of locally trivial fibre bundles and their fibre mappings is called a transverse natural bundle if the following conditions are satisfied:

i) for any foliated manifold (M,F), $\mathcal{N}(M,F)$ is a fibre bundle over the manifold M;

ii) if $f:(M_0,F_0) \longrightarrow (M_1,F_1)$ is a transverse mapping such that $f^*F_1 = F_0$, then $\mathcal{N}(f): \mathcal{N}(M_0,F_0) \longrightarrow \mathcal{N}(M_1,F_1)$ covers f and maps the fibre $\mathcal{N}(M_0,F_0)_x$ over x diffeomorphically onto the fibre $\mathcal{N}(M_1,F_1)_{f(x)}$ over $f(x)$;

iii) $\mathcal{N}$ is a regular functor i.e. if $f: U \times M_0 \longrightarrow M_1$ is a differentiable mapping (where U is an open subset of $\mathbb{R}^k$), such that for any point t of U, the mapping $f_t: (M_0,F_0) \longrightarrow (M_1,F_1)$ $f_t(x) = f(t,x)$ is a transverse mapping to the foliation and $f_t^*F_1 = F_0$, then the mapping

$$U \times \mathcal{N}(M_0,F_0) \ni (t,y) \longmapsto \mathcal{N}(f_t)(y) \in \mathcal{N}(M_1,F_1)$$

is of class C^∞.

Using metods analogous to the metods presented in the paper [1] one can prove that the third condition is a consequence of the first and the second conditions of the definition.

Examples.

1. Let A be an associative algebra over the field $\mathbb{R}$ with the unit The algebra A is called local if it is commutative, of finite dimension over $\mathbb{R}$, and if it admits the unique maximal ideal $\mathfrak{m}$ of codimension 1 such that $\mathfrak{m}^{h+1} = 0$ for some non-negative integer h. Let $\mathbb{R}[[p]] = \mathbb{R}[x_1,\dots,x_p]$ be the algebra of all formal power series in $x_1,\dots,x_p$, and let $\mathfrak{m}_p$ be the maximal ideal of $\mathbb{R}[[p]]$ of all power series without constant terms. Let $\mathfrak{a}$ be a non-trivial ideal of $\mathbb{R}[[p]]$ such that $\mathbb{R}[[p]]/\mathfrak{a}$ is of finite dimension. Then $A = \mathbb{R}[[p]]/\mathfrak{a}$ is a local algebra with the maximal ideal $\mathfrak{m} = \mathfrak{m}_p/\mathfrak{a}$. Any local algebra is isomorphic to such a local algebra.

Let $A = \mathbb{R}[[p]]/\mathfrak{a}$, $\mathfrak{m} = \mathfrak{m}_p/\mathfrak{a}$ and $\xi_A : \mathbb{R}[[p]] \longrightarrow A$ be the natural

projection. Let us denote by N the dimension of $\mathfrak{m}$.
Let $\tau: C^{\infty}(\mathbb{R}^p;0) \longrightarrow \mathbb{R}[[p]]$ be the natural mapping, where $C^{\infty}(\mathbb{R}^p;0)$ denotes the set of germs of smooth functions on $\mathbb{R}^p$ at 0.

Let f,g be two smooth mappings of $\mathbb{R}^p$ into M such that $f(0) = g(0) = m$. We say that f is A-equivalent to g at m if $\tau(hf) = \tau(hg) \bmod \mathfrak{a}$ for any $h \in C^{\infty}_m(M,F)$ where $C^{\infty}_m(M,F)$ is the algebra of the germs at m of smooth functions constant on the leaves of the foliation F.

By $[f]_A$ (or $j^A f$) we denote the equivalence class of f, by $A_m(M,F)$ (or $T^A_m(M,F)$) all equivalence classes at m, and

$$A(M,F) = \bigcup_{m\in M} A_m(M,F) \quad (= T^A(M,F)).$$

It is easy to prove that A(M,F) is a smooth manifold. Moreover $\pi_A: A(M,F) \longrightarrow M$ is a fibre bundle over M with the fibre A. The mapping π_A is a natural projection defined by the formula

$$\pi_A([f]_A) = f(0).$$

On the manifold A(M,F) there is canonically defined foliation F_A of the same dimension as the foliation F. The projection π_A maps leaves of the foliation F_A onto leaves of the foliation F. For any transverse mapping $f:M_1 \longrightarrow M_2$ to the foliation F_2 of the manifold M_2, f defines a smooth mapping $A(f) : A(M_1,F_1) \longrightarrow A(M_2,F_2)$ by the formula

$$A(f)([h]_A) = [f\cdot h]_A$$

The functor $(M,F) \longrightarrow A(M,F)$ will be called the transverse Weil functor (see [6], [8] , [9]).

2. Analogously as A-bundles we can define transverse bundles of p^r-velocities. The bundle of p^r-velocities is a special case of a transverse A-bundle for a satisfactory local algebra A. If we take $\mathfrak{a} = (\mathfrak{m}_p)^{r+1}$ and $A = \mathbb{R}[[p]]/\mathfrak{a}$, then we see that the notion of A-points is the same as the notion of p^r-velocities.

By $N^{p,r}(M,F)$ we denote the set of all transverse p^r-velocities and by π^r_p the natural projection of $N^{p,r}(M,F)$ onto M.

Summing up $N^{p,r}(M,F)$ is locally trivial fibre bundle, whose total space admits a dimension n-q foliation (denoted by F^r_p) projecting by π^r_p onto the initial foliation F.

If p=n=dim M and we take only local diffeomorphisms of $\mathbb{R}^n$ into M, then the construction of n^r-velocities gives a bundle called the transverse frame bundle of the foliated manifold (M,F) and denoted by $L^r(M,F)$, which is a principal fibre bundle with the fibre L^r_q (see [5], [6], [9]).

More generally, let $\mathcal{N}(M,F)$ be a transverse natural bundle on the foliated manifold (M,F). Then the manifold $\mathcal{N}(M,F)$ admits a foliation $F_{\mathcal{N}}$ of the same dimension as F modelled on $\mathcal{N}(M_F)$ with transformations $\mathcal{N}(g_{ij})$.

Now we give the definition of the order of a transverse natural bundle and we present the theorem that each transverse natural bundle has finite order.

Definition 2. A transverse natural bundle has finite order r if for any two morphisms $f,g:(M_0,F_0) \longrightarrow (M_1,F_1)$ the integer r is the smallest one for which the followig implication is true:

if $j^r_x f = j^r_x g$ then $\mathcal{N}(f)(y) = \mathcal{N}(g)(y)$ for any point y of the fibre $\mathcal{N}(M_0,F_0)_x$.

The following theorem is true for transverse natural bundles (see [7], [9]).

Theorem 1. Let $\mathcal{N}$ be a transverse natural bundle. Then there exists an integer r and an L^r_q-space W such that $\mathcal{N}$ is isomorphic to the fibre bundle associated to the transverse r-frame bundle with the standard fibre W. The sallest number with this property is the order of the transverse natural bundle $\mathcal{N}$.

II. HORIZONTAL LIFTS OF BASIC CONNECTIONS

Let $\pi : \mathcal{N}(M,F) \longrightarrow M$ be a transverse natural vector bundle. We denote by $F_{\mathcal{N}}$ the induced foliation on $\mathcal{N}(M,F)$ and put $E = \mathcal{N}(M,F)$, $\pi = \pi_E$.

We denote by $\underline{E}$ the module of all transverse sections of class C^∞ of E and by $\mathfrak{X}(M,F)$ (resp. $\mathfrak{X}(E,F_{\mathcal{N}})$) the module of all transverse vector fields of class C^∞ on M (resp. on E).

Let r be the order of E. We assume that $r \geqslant 1$. By the theorem 1 the vector bundle E is isomorphic to the fibre bundle associated with $L^r(M,F)$. Let W be the standard fibre of E. We denote by

$$\Phi : L^r(M,F)\times W \longrightarrow E$$

the canonical mapping for the associated fibre bundle E.

Let Γ be a basic connection in the principal fibre bundle $L^r(M,F)$. This connection is called a basic connection of order r on M. Γ determines a horizontal distribution on E. If $c=\Phi(p,z)$ is a point of E , then

$$H_c = d_p\Phi_z(\Gamma_p) \qquad (*)$$

where $\Phi_z : L^r(M,F)\longrightarrow E$, $\Phi_z(p) = (p,z)$.

The connection Γ determines the covariant derivation of transverse sections of associated fibre bundles with $L^r(M,F)$. In particular we have the covariant derivation

$$D: \mathfrak{X}(M,F)\times \underline{E} \ni (X,s) \longmapsto D_X s\in \underline{E}$$

of transverse sections of E.

The derivation D satisfies the following conditions:

$$\text{(II.1)}\qquad \begin{aligned} &1)\ D_{fX+gY}s = fD_Xs + gD_Ys\\ &2)\ D_X(s+s') = D_Xs + D_Xs'\\ &3)\ D_X(fs) = X(f)s + fD_Xs \end{aligned}$$

for all transverse vector fields X, Y on M, all transverse sections of E and all functions f, g of class C^∞ on the manifold M locally constant on the leaves of the foliation F.

This covariant derivation D is called a connection in E. The distribution H defined by (*) is the horizontal distribution for the connection D. More precisely, H_c is a vector subspace of T_cE and

$$T_cE = V_cE \oplus H_c$$

where $V_cE = \ker d_c\pi = T_c(E_{\pi(c)})$ is the subspace of the space of vertical vectors. In particular, $d_c\pi\big|_{H_c} : H_c \longrightarrow T_{\pi(c)}M$ is is an isomorphism.

Let X be a transverse vector field on (M,F). We can define its horizontal lift X^D by the formula

$$X^D(c) = (\, d_c\pi\big|_{H_c})^{-1}(X_{\pi(c)}) \qquad \text{(II.2)}$$

If X and Y are vector fields on M, and f,g $C^\infty(M,F)$ then

$$(fX + gY)^D = f^V X^D + g^V Y^D \qquad \text{(II.3)}$$

where $f^V = f\circ\pi$ and $g^V = g\circ\pi$ are vertical lifts of f and g.

We have the simple proposition.

Proposition 1. If X is a transverse vector field on M and D is a basic connection on E then the horizontal lift X^D is the transverse vector field on $(E, F_{\mathcal{V}})$.

Proof. Let $(U, y_1,\dots,y_{n-q},x_1,\dots x_q)$ be an adapted local coordinate system on M. Let $\varphi : E|_U \longrightarrow U\times \mathbb{R}^K$ be a trivialization of E and $e_1,\dots,e_K$ be the canonical base of $\mathbb{R}^K$. We consider sections $\xi_1,\dots,\xi_K$ of $E|_U$ defined by

$$\xi_\alpha(z) = \varphi^{-1}(z,e_\alpha), \quad \alpha = 1,\dots,K. \tag{II.4}$$

These sections are called the edapted sections to the trivialisation φ. It is easy to verify that these sections are transverse to the foliation F_E.

If $(U,y_1,\dots y_{n-q},x_1,\dots,x_q)$ is an adapted chart on U, then there are functions $\Gamma^\alpha_{i\beta}$ on U constant on the leaves such that

$$D_{\partial_i}\xi_\alpha = \Gamma^\beta_{i\alpha}\xi_\beta \tag{II.5}$$

for $\alpha,\beta = 1,\dots,K$, $i = 1,\dots,q$ where $\partial_1 = \frac{\partial}{\partial x_1},\dots,\partial_q = \frac{\partial}{\partial x_q}$ is the canonical frame constant on the leaves associated to (U,y_j,x_i)

Now we define an induced chart on E. The induced chart is a chart $(\pi^{-1}(U),y_1,\dots,y_{n-q},x_1,\dots,x_q,z^\alpha)$ on E where

$$y_j(z) = y_j(\pi(z)),\quad x_i(z) = x_i(\pi(z)) \quad \text{and} \quad z = z^\alpha(z)\xi_\alpha \tag{II.6}$$

for all $z\in \pi^{-1}(U)$.

Using tese local representations we have

$$X^D(z) = X^i(\pi(z))\partial_i - X^i(\pi(z))\Gamma^\beta_{i\alpha}(\pi(z))z^\alpha\delta_\beta \tag{II.7}$$

where $\partial_1,\dots,\partial_q,\delta_1,\dots,\delta_K$ is the canonical frame associated to the induced chart. From this formula we have that X^D is transverse to F_E. Q.E.D.

Since $E_{\pi(z)} = \pi^{-1}(\pi(z))$ is a vector space, then for each point $z\in E$ there exists a natural isomorphism

$$\Psi_z : V_zE = T_zE_{\pi(z)} \longrightarrow E_{\pi(z)} \tag{II.8}$$

If $s:M\longrightarrow E$ is a transverse section of E, then we can define a vector field s^V on E by the formula

(II.9) $$s^V(z) = \varphi_z^{-1}(s_{\pi(z)})$$

This vector field is called the vertical lift of s to E.

Using an adapted induced chart on E we have

(II.10) $$s^V = s^\alpha \delta_\alpha$$

where $s = s^\alpha \xi_\alpha$ and $\xi_1, \ldots, \xi_K$ are the adapted transverse sections.

Let $\pi_s^r : L^r(M,F) \longrightarrow L^s(M,F)$, $s \leq r$ be the natural projection. Using this projection, for a given basic connection of order r on M we can induce a basic connection of order s, $s \leq r$. In particular, the given connection Γ of order r induces a linear basic connection on M called the linear part of Γ. We denote by ∇ the covariant derivation of foliated vector fields with respect to the linear part of Γ.

Now we present the main theorem of this paper.

<u>Theorem 2</u>. Let Γ be a basic connection of order r on M. By ∇ we denote the covariant derivation of vector fields on M with respect to the linear part of Γ and by D we mean the covariant derivation of transverse sections of $E = \mathcal{N}(M,F)$ with respect to Γ. If $\pi : E \longrightarrow (M,F)$ is a transverse natural vector bundle of order r, then there is exactly one linear basic connection $\tilde{\nabla}$ on the manifold E such that

(II.11)
$$\tilde{\nabla}_{X^D} Y^D = (\nabla_X Y)^D$$
$$\tilde{\nabla}_{X^D} s^V = (D_X s)^V$$
$$\tilde{\nabla}_{s^V} X^D = 0 \qquad \tilde{\nabla}_{s^V} s'^V = 0$$

for all foliated vector fields X,Y on M and all transverse sections s, s′ of E.

<u>Proof</u>. It is easy to notice the uniqueness of a basic connection $\tilde{\nabla}$ on E satisfying the conditions (II.11), as the Christoffel symbols of $\tilde{\nabla}$ are uniquely determined by the basic connection Γ of order r.

We need to prove the existence of $\tilde{\nabla}$. Let (U, y_j, x_i) be an adapted chart on M and $\xi_1, \ldots, \xi_K$ be sections of E adapted to the induced chart. First we observe that

(II.12) $$\xi_\alpha^V = \delta_\alpha$$

Now from the formula (II.7) we have

$$\partial_i^D = \partial_i - \Gamma_{i\alpha}^{\beta} z^\alpha \delta_\beta \tag{II.13}$$

We can define a basic connection $\tilde{\nabla}$ on $E|U$ such that its Christoffel symbols with respect to the induced chart are given by the Christoffel symbols of the linear part of Γ and the symbols $\Gamma_{i\beta}^{\alpha}$. For example we put:

$$\begin{gathered} \tilde{\Gamma}_{ij}^{k} = \Gamma_{ij}^{k} \qquad \tilde{\Gamma}_{i\alpha}^{\beta} = \Gamma_{i\alpha}^{\beta} \\ \tilde{\Gamma}_{ij}^{\alpha} = \left(\partial_i \Gamma_{j\beta}^{\alpha} + \Gamma_{i\gamma}^{\alpha} \Gamma_{j\beta}^{\gamma} - \Gamma_{ij}^{k} \Gamma_{k\beta}^{\alpha}\right) z^\beta \\ \tilde{\Gamma}_{\alpha i}^{j} = \tilde{\Gamma}_{j\alpha}^{i} = \tilde{\Gamma}_{\alpha\beta}^{i} = \tilde{\Gamma}_{\alpha\beta}^{\gamma} = 0 \end{gathered} \tag{II.14}$$

This basic connection $\tilde{\nabla}$ on $E|U$ satisfies the conditions:

$$\begin{gathered} \tilde{\nabla}_{\partial_i^D} \partial_j^D = \left(\nabla_{\partial_i} \partial_j\right)^D \\ \tilde{\nabla}_{\partial_i^D} \delta_\alpha^V = \left(D_{\partial_i} \delta_\alpha\right)^D \\ \tilde{\nabla}_{\delta_\alpha^V} \partial_i^D = \tilde{\nabla}_{\delta_\alpha^V} \delta_\beta^V = 0 \end{gathered} \tag{II.15}$$

for $i,j = 1,\dots,q$, $\alpha,\beta = 1,\dots,K$.

Now using the previous formulas it is easy to prove that

$$\begin{gathered} \tilde{\nabla}_{X^D} Y^D = \left(\nabla_X Y\right)^D \\ \tilde{\nabla}_{X^D} s^V = \left(D_X s\right)^V \\ \tilde{\nabla}_{s^V} X^D = \tilde{\nabla}_{s^V} s'^V = 0 \end{gathered} \tag{II.16}$$

for all foliated vector fields X,Y on U and all transverse sections s, s' of $E|U$. We show the proof of the second formula.

Let X be a foliated vector field on U and s be a transverse section of $E|U$. If we denote by

$$X = X^i \partial_i \qquad s = s^\alpha \delta_\alpha$$

the coordinates of X and s with respect to the adapted chart (U, y_j, x_i), then we have

$$X^D = (X^i)^V \partial_i^D \qquad s^V = s^\alpha \delta_\alpha$$

Hence we obtain

$$\tilde{\nabla}_{X^D} s^V = \tilde{\nabla}_{(X^i)^V \partial_i^D} \left(s^\alpha \delta_\alpha\right) =$$

$$= (X^i)^V(\partial_i^D(s^\alpha)\delta_\alpha + s^\alpha \tilde{\nabla}_{\partial_i^D} \delta_\alpha) =$$
$$= (X^i)^V(\partial_i^D(s^\alpha)\xi_\alpha^V + s^\alpha \tilde{\nabla}_{\partial_i^D} \xi_\alpha^V) =$$
$$= (X^i(\partial_i(s^\alpha)\xi_\alpha + s^\alpha D_{\partial_i}\xi_\alpha))^V =$$
$$=(D_X s)^V$$

If (U, y_j, x_i) and (U', y'_j, x'_i) are two adapted charts on M, then we can define two linear connections $\tilde{\nabla}$ and $\tilde{\nabla}'$ on $E|U$ and $E|U'$ respectively. We have

$$\tilde{\nabla}_{X^D}Y^D = (\nabla_X Y)^D = \tilde{\nabla}'_{X^D}Y^D$$
$$\tilde{\nabla}_{X^D}s^V = (D_X s)^V = \tilde{\nabla}'_{X^D}s^V$$
$$\tilde{\nabla}_{s^V}X^D = \tilde{\nabla}'_{s^V}X^D = 0 = \tilde{\nabla}_{s^V}s'^V = \tilde{\nabla}'_{s^V}s'^V$$

for all foliated vector fields X,Y on $U\cap U'$ and all sections s,s' of $E|U\cap U'$. Thus the linear connections $\tilde{\nabla}$ and $\tilde{\nabla}'$ coincide on $E|U\cap U'$

Using an adapted atlas on M we can define a basic connection $\tilde{\nabla}$ on E which satisfies the conditions (II.11).

The theorem is proved.

The following corollaries are the natural consequences of the above theorem.

Corollary 1. If ∇ is a basic connection on (M,F) then there is exactly one basic connection $\tilde{\nabla}$ on the normal bundle $\mathcal{N}(M)$ such that

$$\tilde{\nabla}_{X^H}Y^H = (\nabla_X Y)^H$$
$$\tilde{\nabla}_{X^H}Y^V = (\nabla_X Y)^V$$
$$\tilde{\nabla}_{X^V}Y^H = 0$$
$$\tilde{\nabla}_{X^V}Y^V = 0$$

for all foliated vector fields X and Y on M, where X^H is the horizontal lift of X to $\mathcal{N}(M)$ with respect to ∇. ([10])

Corollary 2. If ∇ is a basic connection on (M,F) then there is exactly one linear connection $\tilde{\nabla}$ on $\mathcal{N}^*(M)$ (the dual vector bundle to the normal bundle $\mathcal{N}(M)$) such that

$$\tilde{\nabla}_{X^H} Y^H = (\nabla_X Y)^H$$
$$\tilde{\nabla}_{X^H} \xi^V = (\nabla_X \xi)^V$$
$$\tilde{\nabla}_{\xi^V} X^H = 0$$
$$\tilde{\nabla}_{\xi^V} \eta^V = 0$$

for all foliated vector fields X,Y on M and all basic 1-forms ξ, η on M, where X^H is the horizontal lift of X to $\mathcal{N}^*(M)$ with respect to ∇ (see [11]).

Corollary 3. If E is a vector bundle associated to the transverse frame principal fibre bundle L^1(M,F) and ∇ is a basic connection on (M,F), then there is exactly one basic connection $\tilde{\nabla}$ on E such that

$$\tilde{\nabla}_{X^H} Y^H = (\nabla_X Y)^H$$
$$\tilde{\nabla}_{X^H} s^V = (\nabla_X s)^V$$
$$\tilde{\nabla}_{s^V} X^H = 0$$
$$\tilde{\nabla}_{s^V} s'^V = 0$$

for all foliated vector fields X,Y on M and all transverse sections of E, where X^H is the horizontal lift of X to E with respect to ∇ (see [4]).

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