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ORNSTEIN-UHLENBECK PROCESS IN QUANTUM MECHANICS
AND THE SUB-QUANTUM COHERENCE EFFECT

Jiří Souček

1. Introduction

In this paper we propose the more refined version of Quantum Mechanics (QM), which contains QM as a limiting case. After claryfying the mathematical and physical basis of this version we discuse the possible observational differences from QM, mainly the so-called subquantum coherence effect.

From equivalent formulations of QM the Feynman's formulation [1] serves best for our goal. By this we mean the following. At first, to each possible trajectory of a particle there corresponds its amplitude $\exp iS$. At second, it is supposed that each trajectory contributes with the equal weight to the total amplitude of the transition (of a particle from one space-time point to another). Then the probability of a transition is calculated as a squered modulus of the total amplitude of the transition. In fact, we shall take the first part of the principle (plus the rule $P = |\phi|^2$) as valid but we shall change the second part of the principle.

We shall propose a physical mechanism for the explanation of the fact that the quantum particle can follow different paths. The mechanism is based on the use of a concept of a random force acting on the particle; mainly we shall construct a quantum analog of the Ornstein-Uhlenbeck process (which itself is the more realistic description of the Brownian motion than the usual Wiener process).

At the second part of the paper we shall propose the physical origine of this random force using the idea of a medium composed of tachyons inside which a particle moves. As a consequence we obtain as a typical phenomenon the subquantum coherence effect. It says that the time-correlated group of particles (for example a very short lazer puls) can behave, in certain circumstances, as a unique quantum particle. It means that in the diffraction-like experiment all the group (ideally -practically only the main part of the group) enters in the same scattering channel.

All this can be seen differently. We divide QM into two parts. We take as a granted the Feynman's rules for the probability amplitudes. After this there is

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still the 'second mystery of QM' - the non-deterministic (diffusion-like) behavior of the evolution of the probability amplitude (Schroedinger equation). We construct the appropriate origin of this 'diffusion' (in terms of the probability *amplitude*, of course) and make hypotheses on the corresponding medium.

From the third point of view we introduce a new sort of hidden parameters with statistical properties governed by the distribution of a probability amplitude. We obtain that this sort of hidden parameters is quite possible (there are no Bell inequalities) showing that our theory may be arbitrarily close to QM.

In our theory particles are point-like objects with wave-like behavior attributed completely to the fact that their statistics is governed by the Feynman's laws for probability amplitudes instead of the usual Kolmogorov's axioms for probability. This is in an agreement with the trivial observation that the particle -like properties of a quantum particle can be found in the experiment with the *single particle* (for example the observation of its trace on a screen) while the wave-like properties can be found *only* in the experiment with the *ansamble of particles* (for example the interference pattern). This shows that the wave-like properties can be attributed correctly to the *statistical properties of an ansamble* (in the sence of Feynman's rules for amplitudes). In our theory there is no wave packet reduction during the measurement.

In the second chapter we give the description of the Ornstein-Uhlenbeck process using the path integral and in the third chapter we give its quantum version. In the next chapter we discuse the possible tachyonic background in QM. Then we introduce the subquantum coherence effect and propose its possible observation. At the end we sum up our interpretation of QM.

2. Ornstein-Uhlenbeck process and path integral

The Brownian motion can be conveniently described using the path integral. Let $x(t)$ be a trajectory of a Brownian particle determined by the Newton equation

$$m\ddot{x} + \gamma\dot{x} = F(t) \tag{2.1}$$

where $\gamma\dot{x}$ is the friction term and $F(t)$ denotes the random forse executed by the medium on the Brownian particle. In the Wiener process we neglect the inertial term $m\ddot{x}$ and (2.1) reduces to the Langevin equation

$$\gamma\dot{x} = F(t) \ . \tag{2.2}$$

The probability $K(x_1 \mid x_0;\, t)$ of the transition from the point x_0 into x_1 during the time-interval t can be calculated knowing the distribution of the random force. The white-noise distribution is usually used

$$e^{-\frac{a}{2}\int_0^t F(\tau)^2\, d\tau}\ \mathscr{D}F \tag{2.3}$$

where

$$\mathscr{D}F = \prod_{\tau \in (0,t)} dF(\tau)$$

is the 'Feynman measure' on the space of paths. Only trajectories obbeying the boundary conditions

(2.4) $$BC : \begin{array}{l} x(0) = x_0 \\ x(t) = x_1 \end{array}$$

contribute to the transition probability (with the weight (2.3) where the corresponding random force is given by (2.2)). Using the discrete time approximation to the path integral we see that

(2.5) $$\mathscr{D}x = \mathscr{D}F .$$

It follows that

$$K(x_1 \mid x_0;\ t) = N \int_{BC} e^{-S[x]} \mathscr{D}x$$

where 'the action' is given by

$$S[x] = \frac{a}{2} \int_0^t (\gamma\dot{x})^2 .$$

This Gaussian integral can be calculated by

(2.6) $$K = Ne^{-\bar{S}} ,$$

where $\bar{S} = S[\bar{x}]$ is the action evaluated for the classical path $\bar{x}(t)$ with boundary conditions (2.4). We obtain, of course, the heat kernel

$$K(x_1 \mid x_0;\ t) = \text{const.}\ \left(\frac{a\gamma^2}{t}\right)^{1/2} e^{-a\gamma^2 \frac{(x_1-x_0)^2}{t}} .$$

(The normalization factor N was calculated from the conservation of the total probability.)

The Ornstein-Uhlenbeck process starts from the complete Newton equation (2.1). By the same argument the path integral to be calculated is

(2.7) $$\int_{BC} e^{-\frac{a}{2} \int_0^t (m\ddot{x} + \gamma\dot{x})^2\, dt} \mathscr{D}x$$

assuming the same distribution (2.3) for the random force. Now 'the action'

(2.8) $$S[x] = \frac{a}{2} \int_0^t (m\ddot{x} + \gamma\dot{x})^2\, dt$$

contains the second time derivative $\ddot{x}$, so that the natural boundary conditions are

$$\text{(2.9)} \qquad \text{BC}: \begin{array}{ll} x(0) = x_0 , & \dot{x}(0) = v_0 \\ x(t) = x_1 , & \dot{x}(t) = v_1 . \end{array}$$

Consequently the transition probability depends on initial and final velocities, too

$$K(x_1, v_1 \mid x_0, v_0; t) .$$

Using the formula (2.6) we see again that the equation $\delta S/\delta x = 0$ determining the classical trajectory $\bar{x}(t)$ needs the boundary conditions (2.9). In this way we obtain

$$\text{(2.10)} \qquad K(x_1, v_1 \mid x_0, v_0; t) = N_t \exp \left\{ - a \left[\frac{(v_1 - v_0 e^{-2\beta t})^2}{1 - e^{-4\beta t}} + \right. \right.$$

$$\left. \left. + \frac{\left(\beta(x_1 - x_0) - \frac{v_0 + v_1}{2} \operatorname{th}\beta t \right)^2}{\beta t - \operatorname{th}\beta t} \right] \right\}$$

where th = tangens hyperbolicus,

$$N_t = (\beta t - \operatorname{th}\beta t)^{-1/2} (1 - e^{-4\beta t})^{-1/2} .$$

Here the natural time unit $1/\beta$ is given by

$$\text{(2.11)} \qquad 1/\beta = m/\gamma .$$

For long times $t >> 1/\beta$ the transition probability is close to the transition probability of the Wiener process times the Maxwell distribution in velocities

$$\text{(2.12)} \qquad K(x_1, v_1 \mid x_0, v_0; t) \simeq t^{-1/2} e^{-av_1^2} e^{-a\beta \frac{(x_1 - x_0)^2}{t}}$$

where the initial velocity is completely forgetten.

For small times $t << 1/\beta$ we have a picture completely different from the Wiener process. We have $\beta t - \operatorname{th}\beta t \approx (\beta t)^3$ so that the mean distance and the mean velocity behave like $\bar{x} \approx t^{3/2}$, $\bar{v} \approx t^{1/2}$ (for the Wiener process we have $\bar{x} \approx t^{1/2}$, $\bar{v} \approx t^{-1/2}$ for all t). The Ornstein-Uhlenbeck trajectories are more regular than the Wiener trajectories. The small time singularity $\bar{v} \approx t^{-1/2}$ is the mathematical artefact, because the Brownian particle must be described by (2.1) and not by (2.2). The origin of this singularity lies in the neglecting the inertial term $m\ddot{x}$ in (2.1) ($m \approx 0$ implies that the arbitrarily small force gives the particle the large velocity limited only by the friction force).

3. The QM-analog of the Ornstein-Uhlenbeck process

The similarity between QM and the Brownian motion is well known [2]; the difference lies mainly in the use of the probability amplitude instead of the probability (the Feynman approach to QM). We have seen that the Ornstein-Uhlenbeck process is the more realistic description of the Brownian motion than the Wiener process.

Our aim is to construct the quantum analog of the Ornstein-Uhlenbeck process. We consider the quantum particle as a point-like object with the behavior described by the distribution of the probability amplitude.

Of course, we cannot use the concept of the friction force, because it breaks the Galilean invariance of the theory. Instead of the friction term we have the Feynman principle in QM:

(i) probability amplitude for the path $x(t)$, $0 \leqq t \leqq t_1$ is given by

$$\text{ampl.}\ [x] = \text{const}\cdot e^{i/\hbar\ S_0[x]} ,$$

$$S_0[x;0,t_1] = \frac{m}{2}\int_0^{t_1} \dot{x}^2\, dt . \tag{3.1}$$

We interpret it as an amplitude of the presence of a particle. Given that the particle is surely (i.e. with prob. = 1) at the point, say x, we have still the freedom of a choice of a fase $\phi = e^{i\alpha}$, α real, of this presence. The principle (i) than says that the amplitude of the presence of the particle $\phi(t)$ changes when the particle moves along the path $x(t)$ by the rule

$$\phi(t) = e^{i/\hbar\ S_0[x;0,t]}\cdot\phi(0) . \tag{3.2}$$

The second part of the Feynman's principle says that having (i) than every trajectory contributes with the same weight to the transition amplitude.

Instead of this we shall suppose:

(ii) There is a random force acting on the particle so that instead of the inertial motion $\ddot{x}(t) = 0$ the particle moves along $x(t)$ given by the Newton equation without the friction term

$$m\ddot{x} = F(t) . \tag{3.3}$$

We shall suppose that the distribution of the random force is governed by the probability amplitude

$$e^{i\frac{a}{2}\int_0^{t_1} F(t)^2\, dt}\ \mathcal{D}F . \tag{3.4}$$

We have $\mathcal{D}F = \text{const}\ \mathcal{D}x$ as above.

So that the resulting amplitude of an event [traj. of part. $\equiv x(t)$] is given by (see (3.1), (3.4))

$$e^{i/\hbar\, S[x;0,t_1]}\,\mathfrak{D}x\ ,$$

(3.5)
$$S[x;0,t_1] = \int_0^{t_1} \left(\frac{m}{2}\,\dot{x}^2 + \frac{a}{2}\,m^2\,\ddot{x}^2\right) dt\ .$$

The transition amplitude

$$K(x_1,\ v_1 \mid x_0,\ v_0;\ t) = N_t \int_{BC} e^{i/\hbar\, S[x;0,t]}\mathfrak{D}x \tag{3.6}$$

depends naturally on the initial and final positions and velocities as before, so that boundary conditions BC should be (2.9). Using the classical trajectory $\bar{x}(t)$ (defined by (3.3) and (2.9)) in the calculation of (3.6) we obtain

$$K(x_1,\ v_1 \mid x_0,\ v_0;\ t) = N_t e^{i/\hbar\, \bar{S}(x_1,v_1|x_0,v_0;t)}\ , \tag{3.7}$$

$$\bar{S}(x_1,\ v_1 \mid x_0,\ v_0;\ t) = \frac{m}{2\beta}\left[\mathrm{th}\beta t\cdot(v_0^2 + v_1^2) + \frac{(v_1 - v_0)^2}{\mathrm{sh}2\beta t}\right] + \tag{3.8}$$

$$+\ m\,\frac{\left(x_1 - x_0 - \dfrac{v_0 + v_1}{2\beta}\,\mathrm{th}\beta t\right)^2}{t - \dfrac{1}{\beta}\,\mathrm{th}\beta t}\ ,$$

$$N_t = \mathrm{const}\cdot(1 - e^{-4\beta t})^{-1/2}(\beta t - \mathrm{th}\beta t)^{-1/2} \tag{3.9}$$

where the natural time unit $1/\beta$ is given by

$$\beta^2 = \frac{1}{am}\ . \tag{3.10}$$

Now it is natural to consider the amplitude distrubution

$$\psi(x,v;t)$$

as a "subquantum wave function" with the law (expressing (3.2))

$$\psi(x_1,v_1;t) = \int K(x_1,\ v_1 \mid x_0,\ v_0;\ t)\psi(x_0,v_0;0)\ dx_0\ dv_0\ . \tag{3.11}$$

The corresponding Schroedinger (or evolution or Focker-Planck) equation reeds

$$ih\partial_t\psi = \left(-\ mv^2 - i\hbar v\partial_x - \frac{\beta^2}{m}\,\hbar^2\partial_v^2 + i\hbar\beta\right)\psi\ . \tag{3.12}$$

We see that this equation has a deterministic character in ∂_x (∂_x enters in (3.12) in the first degree - not like ∂_x^2 in the usual Schroedinger equation), but the origine of the diffusion is in ∂_v^2 . Initially the diffusion takes place in the velocity and then enters also in position by the term $iv\partial_x$.

Contrary to the situation in the preceeding Chapter the initial velocity v_0 is not forgotten; moreover, the process (3.7) is time - reversible

(3.13) $$K(x_1, v_1 \mid x_0, v_0; t) = K(x_0, -v_0 \mid x_1, -v_1; t) .$$

For long times $t >> 1/\beta$ we have (with $\hbar = 1$)

(3.14) $$\bar{S} \approx \frac{m}{\beta} \frac{v_0^2 + v_1^2}{2} + m \frac{(x - v/\beta)^2}{t - 1/\beta} , \quad v = \frac{v_1 + v_2}{2}$$
$$\approx \frac{m}{\beta} \frac{v_0^2 + v_1^2}{2} + m \frac{x^2}{t}$$

because the typical orders are $\bar{v} \approx \sqrt{\beta}$, $\bar{v}/\beta \approx 1/\sqrt{\beta}$, $\bar{x} \approx \sqrt{t} >> 1/\sqrt{\beta}$. We obtain the Schroedinger propagator times the "amplitude" Maxwell distribution in v_0 and v_1 .

QM is the limiting case of our theory. This can be seen simply from the begining. If $a \approx 0$, than in (ii) every random force $F(t)$ contribute equally and (ii) comes back to the original Feynman assumption that every trajectory comes with the same weight.

4. Tachyonic background in QM

Now we came to the question of the possible origine of the random force $F(t)$. By the analogy we could suppose that the point-like particle moves in a certain medium (governed by the statistical law given by the probability *amplitude)*. Clearly, this medium cannot be formed from observable particles. Another important property is that the random force $F(t)$ depends on time, but *does not depend* on the position $x(t)$ of the particle.

We think that for this purpose serves very well the idea of a medium composed from tachyons. This is a rather strange idea, but something really strange is needed in this situation.

Now we shall report some properties of tachyons from [3]. The most important property is the impossibility to localize a tachyon. This means that the usual localized wave packet cannot be constructed. The classical approximation and other arguments [3] show that the "classical trajectory" of a tachyon (which should enter in the path integral for tychyons) is given by

(4.1) $$t = t(\vec{x}) .$$

Say $t = \vec{v}\cdot\vec{x}$ for the inertial motion. Things are clearer in the non-relativistic approximation which gives infinite velocity to each tachyon. The trajectory of such a tachyon in the space-time $(\vec{x},t)$ is given by

(4.2) $$t = t_0 .$$

The strength of the interaction of any object with the particle in our laboratory is proportional to the part of its wave function entering the laboratory, so that we can suppose that the interaction is proportional to $1/N_0$, where

$$(4.3) \qquad N_0 = \frac{\text{volume of the Universe}}{\text{volume of the laboratory}} .$$

Thus the effect of the interaction with any particular tachyon can be considered as negligibly small : a tachyon is not observable. This allows us to suppose that there may be, in fact, many tachyons in the Universe. But the *collective effect* of the interaction with many (say N_0) tachyons need not be small.

On the other hand we must not forget that *each* tachyon in the Universe (with the trajectory (4.2), where $0 \leqq t_0 \leqq t_1$) enters partially (like $1/N_0$) into our laboratory.

In conclussion : the influence of any particular tachyon is negligibly small, but the collective influence of many tachyons (all tachyons in the Universe with $0 \leqq t \leqq t_1$) can be considerable. The non-relativistic trajectory (4.2) corresponds well to the assumption that the random force $F(t)$ does not depend on $\vec{x}$.

We are not able to propose a more concrete form of the interaction between a tachyon and a particle. We shall represent it by the random force term as above. A possible consequence of our assumption on the tachyonic medium will be considered in the next chapters.

A natural question is how many tachyons there may be in Universe for a unit of time (say for 1 meter in units with $c = 1$). An idea of an order of this quantity may come (by analogy) from the fact that there are roughly 1 baryon for meter^3 in the Universe. In this way we 'obtain' an estimate 1 tachyon for meter/c .

Generally, having the fact of the non-observability of any individual tachyon, there is no obstacle to suppose there are many tachyons in the Universe. Thus the collective effect of many tachyom can be, in principle, observable. The question is : what sort of an effect it could be? Our proposal is the following : it is not a new effect, it is an effect well known from the begining of QM, namely, the phenomenon of the indeterminism of QM. (For the discussion of hidden parameters, see Chap. 7.)

5. The subquantum coherence effect

We shall consider now the system of n independent particles. We can suppose generally that the random forces $F_i(t) = m\ddot{x}_i$ are independent. We have their distribution

$$(5.1) \qquad \exp\left[ia \sum_i \int F_i^2 \, dt\right] \mathcal{D}F_1 \ldots \mathcal{D}F_n .$$

Clearly, they cannot create correlations between particles, because from (5.1) we have

(5.2) $$K(x_1', v_1', \ldots, x_n', v_n' \mid x_1, v_1, \ldots, x_n, v_n;\ t) = \prod_i K(x_i', v_i' \mid x_i, v_i;\ t) .$$

The situation is more interesting if we consider the physical origine of the random forces $F_i(t)$ proposed in the preceding chapter. The forces $F_i(t)$, $1 \leq i \leq n$ have their common root in the interaction with the tachyons (4.2). We can make then a reasonable assumption that forces $F_i(t)$ are correlated, so that we shall suppose, instead of (5.1) the following (amplitude) distribution for them

(5.3) $$\exp i\{\frac{a_1}{2} \sum_i \int (F_i - \overline{F})^2\, dt + \frac{a_0}{2} \int \overline{F}^2\, dt\} ,$$

where $\overline{F}$ denotes the mean force

(5.4) $$\overline{F}(t) = \frac{1}{n} \sum_i F_i(t) .$$

We shall suppose that

(5.5) $$a_1 >> a_0 ;$$

this means that typical forces $F_i(t)$ are (in a given instant) closed one to each other.

Consequently we have two time scales

(5.6) $$1/\beta_0 << 1/\beta_1 , \quad \beta_i = \frac{1}{a_i m} , \quad i = 1,2 .$$

During the time interval $1/\beta_0$ the initial mean velocity is forgotten, while at $1/\beta_1$ the initial correlation of velocities (if there exists) is lost.

We shall study the evolution of the system during the intermediate time intervals Δt

(5.7) $$1/\beta_0 << \Delta t << 1/\beta_1 .$$

(In fact, only $\beta_1 t << 1$ is essential in the following argument.)

It is clear that forces distributed by (5.3) will create a correlation in the 'sub-quantum' wave function

(5.8) $$\psi(x_1, v_1, \ldots, x_n, v_n;\ t)$$

of the system during time intervals of order Δt (this correlation will be lost at longer times). Equivalently, the formula (5.2) does not hold in this case.

For the simplicity we shall describe only the limit case

$$a_1 = \infty$$

for the evolution during Δt. *We shall show that the repeated localization of positions of our particles at instants*

(5.9) $\qquad t_0' > t_0 \,, \quad t_0' - t_0 \approx \Delta t$

will create the correlation of velocities of particles.

We shall consider the selection by two slits like in Fig. 1.

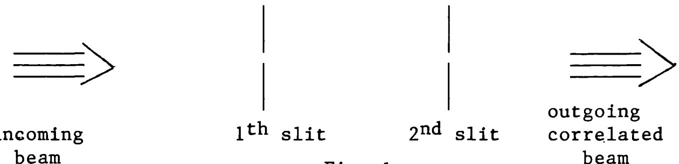

Fig. 1

We use, in fact, the '*space-time slits*' (y,t_0) , (y',t_0') which select particles with trajectories fulfilling

(5.10) $\qquad x_i(t_0) = y \,, \quad x_i(t_0') = y' \,.$

(We consider the ideal experiment with the exact localization of positions of particles at instants t_0 , t_0' .)

Let us suppose that two particles (say first and second ones) have passed through both slits (i.e. were selected in the preparation part). From (5.3), (5.7) and $a_1 = \infty$ we have $F_1(t) = F_2(t)$, $\ddot{x}_1(t) - \ddot{x}_2(t) = 0$ and using $x_1(t_0) = x_2(t_0) = y$ we obtain

(5.11) $\qquad x_1(t) - x_2(t) = [\dot{x}_1(t_0) - \dot{x}_2(t_0)](t - t_0) \,, \quad t_0 \leqq t \leqq t_0' \,.$

By the condition $x_1(t_0') = x_2(t_0') = y'$ we obtain

$$\dot{x}_1(t_0) = \dot{x}_2(t_0)$$

and, as a consequence, also

(5.12) $\qquad x_1(t) = x_2(t) \,, \quad t_0 \leqq t < t_0 + \Delta t \,.$

(This holds only in the approximation $a_1 = \infty$, of course.)

After the first slit the wave function (5.8) must have the form

(5.13) $\qquad \prod_i \delta(x_i - y)\phi(v_1,\dots,v_n;\, t_0) \,,$

while after the second slit the wave function will be

(5.14) $\qquad \prod_i \delta(x_i - y') \prod_{i\geq 2} \delta(v_i - v_1)\phi(v_1;t_0')$

with the correlated positions and velocities.

Now we shall measure the positions of our particle at the instant $t_0'' > t_0'$, $t_0'' - t_0' \approx \Delta t$. By (5.12) we see that in t_0'' the wave function will be of the type

$$\prod_{i\geq 2} \delta(x_i - x_1) \prod_{i\geq 2} \delta(v_i - v_1)\phi(x_1,v_1;t_0'') \ . \tag{5.15}$$

So we shall find the correlation of the particles' positions in t_0''. *After passing the preparation part of our experiment the group of particles moves coherently, like a unique quantum object* (during time intervals of order Δt).

Since $a_1 = \infty$ is the rather crude approximation, we shall give a more exact treatment for the case of two particles. Let the amplitude for the path $x_1(t)$, $x_2(t)$ in the time interval $t = t_0' - t_0$ is given by

$$\exp i\{\frac{a_0}{4} \int(F_1 + F_2)^2 \, dt + \frac{a_1}{4} \int(F_1 - F_0)^2 \, dt + \frac{m}{2} \int(\dot{x}_1^2 + \dot{x}_2^2) \, dt\}$$

where $F_1 = m\ddot{x}_1$, $F_2 = m\ddot{x}_2$.
Using substitutions $\sqrt{2}\, z_0 = x_1 + x_2$, $\sqrt{2}\, z_1 = x_1 - x_2$, $\sqrt{2}\, w_0 = v_1 + v_2$, $\sqrt{2}\, w_1 = v_1 - v_2$ the path integral reduces to (3.7). The resulting formula simplifies if we use the assumptions $1 \ll \beta_0 t$, $\beta_1 t \ll 1$ and their consequences $\mathrm{th}\beta_0 t = 1$, $\mathrm{sh}2\beta_0 t = \infty$, $(\mathrm{th}\beta_0 t)/(\beta_0 t) = 0$, $(\mathrm{th}\beta_1 t)/(\beta_1 t) = 1$ and

$$1 - \frac{\mathrm{th}\beta_1 t}{\beta_1 t} = \frac{1}{3}\beta_1^2 t^2 \ . \tag{5.16}$$

In this way we obtain (using the notation $\sqrt{2}\, z_0' = x_1' + x_2'$ etc.)

$$K(x_1',v_1',x_2',v_2' \mid x_1,v_1,z_2,v_2;\ t) = N_t \exp i/\hbar\, \bar{S} \ , \tag{5.17}$$

$$\bar{S} \approx \frac{m}{2\beta_0}(w_0^2 + w_0'^2) + mt^{-1}\left(z_0' - z_0 - \frac{w_0 + w_0'}{2\beta_0}\right)^2 +$$
$$+ \frac{m}{2\beta_1}\left[\beta_1 t(w_1^2 + w_1'^2) + \frac{(w_1' - w_1)^2}{2\beta_1 t}\right] +$$
$$+ \frac{3m}{\beta_1^2 t^3}\left[z_1' - z_1 - \frac{t}{2}(w_1 + w_1')\right]^2 \ .$$

In the case of our two-slit preparation we have $z_0 = \sqrt{2}\, y$, $z_1 = 0$, $z_0' = \sqrt{2}\, y'$, $z_1' = 0$ and thus

$$\bar{S} \approx \frac{m}{2\beta_0}(w_0^2 + w_0'^2) + mt^{-1}\left[\sqrt{2}(y' - y) - \frac{w_0 + w_0'}{2\beta_0}\right]^2 + \tag{5.18}$$
$$+ \frac{m}{2\beta_1}\frac{1}{\beta_1 t}\left[w_1'^2 + w_1^2 + (w_1' - w_1)^2\right] \ .$$

We see that for $\beta_1 t \ll 1$ a good correlation is obtained for velocities $v_1' - v_2' = w_1'$

$$w_1'^2 \approx \frac{2\beta_1\hbar}{m}\beta_1 t \,. \tag{5.19}$$

In the case of the measuring part of our ideal experiment we must consider the propagator in the time interval

$$t = t_0'' - t_0' \approx \Delta t \lesssim 1/\beta_1 \tag{5.20}$$

with condition $z_0' = \sqrt{2}\, y'$, $z_1' = 0$. From (5.17) (written for t_0' , t_0'') we have $(w_1'' - w_1')^2 \approx 4\beta_1 hm^{-1}\beta_1 t$ and thus also $w_1' + w_1'' = (hm^{-1}\beta_1^2 t)^{1/2}$. From the last term in (5.17) we obtain (using also (5.20))

$$\begin{aligned} z_1'' &\approx t/2(w_1' + w_1'') + \left(\frac{ht}{3m}\right)^{1/2}\beta_1 t \\ &\approx \text{const}\left(\frac{ht}{m}\right)^{1/2}\beta_1 t \lesssim \text{const}\left(\frac{h}{m\beta_1}\right)^{1/2}\beta_1 t \,. \end{aligned}$$

6. Possible observational differences between QM and the proposed theory

The main difference is in the short time behavior of the propagator (3.7). This needs measurements repeated in the short time interval of order $1/\beta$. But in the quantum theory the short time behavior is obscured by many effects and it is not a simple task to decide that a certain short time behavior contradicts QM .

We think that the subquantum coherence effect introduced above may serve better for this purpose. *It is not difficult to see that this effect contradicts strictly* QM . In fact, after the second slit the QM-wave function is of the type

$$\psi(x_1,\dots,x_n;t_0') = \prod_i \delta(x_i - y') \tag{6.1}$$

and does not depend on the existence of the first slit (only the intensity of the beam is lower). There is no possibility to introduce a correlation among particles into the wave function (6.1); QM predicts the independent amplitude distribution for particles and this excludes any sort of the coherence. In QM the amplitude diffusion (described by the Schroedinger equation) is absolute, without any underlying mechanism, and thus it is independent for different particles. There is another advantage of this type of an effect, namely that the time constant is $1/\beta_1$, which may be quite greater than $1/\beta_0$.

We propose to make *a systematic search for effects of the type of the sub--quantum coherence*. The main problem is to insure the 'space-*time*' character of the slits in the preparation part of the experiment (this is necessary for creation of the coherence). The time localization of particles at the slits may be obtained using *a very short lazer puls* and assuming that the 'sub-quantum' velocity of photons is always c .

The main ingredients of such a type of an experiment are:

(i) the shortest possible puls of the beam of particles (the duration of the puls should be smaller than $1/\beta_0$),

(ii) the two slits creating the sub-quantum correlation,

(iii) any usual instrument separating particles into different channels.

The result loocked for is the following. QM predicts the probability p_i for a particle to enter into the i-th channel. Let us suppose that n_i particles were found in the i-th channel, $n = \Sigma n_i$ being to total number of particles. From the mutual independence of particles we know (the normal distribution) that fluctuations are of order

(6.2) $$\left|\frac{n_i}{n} - p_i\right| \approx \frac{1}{\sqrt{n}} .$$

The usual puls contains an enormous number of particles, so that (6.2) is very small.

The pure coherence effect says that all particles of the puls will enter into one channel, i.e. that $n_{i_0} = n$, $n_i = 0$ for $i \neq i_0$ for some i_0 . Of course, we can expect only a partial coherence effect. So we suggest

(iv) to measure the number n_i of particles entering into the i-th channel and to calculate the observed fluctuations

(6.3) $$\tilde{\Delta p}_i = \left|\frac{n_i}{n} - p_i\right|$$

(v) to repeate the experiment and to find that the mean value of $\tilde{\Delta p}_i$ exceeds the limit given by (6.2).

In fact, the total number of particles need not be known with the sufficient accuracy, so that we prefer to use the relative probability between the i-th and j-th channels

(6.4) $$g_{ij} = \frac{p_i}{p_j} , \quad \tilde{g}_{ij} = \frac{n_i}{n_j} .$$

We have similarly to (6.2)

(6.5) $$|\tilde{g}_{ij} - g_{ij}| \approx n^{-1/2} .$$

The goal is then to find that the observed mean fluctuations

(6.6) $$\tilde{\Delta g}_{ij} = |\tilde{g}_{ij} - g_{ij}|$$

exceed the bound (6.5). The observation of the fluctuations exceeding their quantum mechanical values indicates clearly the tendency of particles to enter into the same channel.

There can be also another indication: the dependence of fluctuations (6.3) and (6.6) on

(a) the duration of the puls,

(b) the existence of the first slit.

Such a dependence is in the contradiction with the principles of QM (assuming that the interaction among particles can be neglected).

A possible form of the proposed experiment is in Fig. 2.

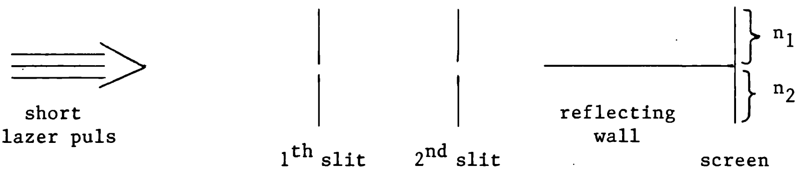

Fig. 2.

The simplified form of it using the ratio g_{12} is in Fig. 3.

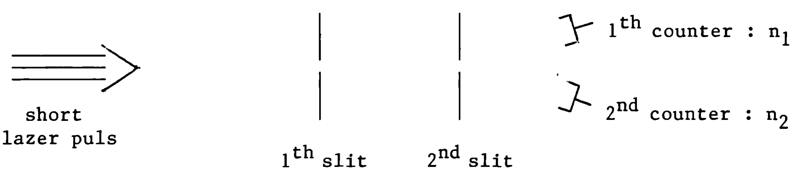

Fig. 3.

Note that the axial symmetry of Fig.s 2 and 3 imply in QM that $n_1 = n_2$ with the accuracy $n_1^{1/2}$.

7. The interpretation of Quantum Mechanics

Here we shall discuss the interpretation of QM underlying the ideas developed in the preceeding Chapters of the paper. The best way to do this is to develop the analogy between QM and the Brownian motion. For this purpose we shall introduce an 'orthodox' interpretation of the Brownian motion which corresponds to the orthodox interpretation of QM. We shall consider, for the simplicity, only the one-dimensional case.

1. The state of the Brownian particle is described by the distribution function $f(x)$ with properties

$$(7.1) \qquad \int_{R^1} f(x)\,dx = 1 \ , \quad f \geq 0 \quad \text{on} \quad R^1 \ .$$

2. The evolution of the state of the particle is *deterministic,* described by the equation

$$\partial_t f(x;t) = c\cdot\partial_x^2 f(x;t) \ , \quad c = \text{constant}. \tag{7.2}$$

3. *The measurement postulate.* The result of the measurement of the position of the particle is described by $x \in R_1$. The probability to find the particle in the interval (x_1,x_2) is given by

$$\int_{x_1}^{x_2} f(x)\, dx \ , \tag{7.3}$$

where the distribution $f(x)$ describes the state of the particle before the measurement. The state of the particle *changes discontinuously and un-predicably* during the measurement process (by the 'un-controllable' influence of the 'classical' measuring apparatus) so that after the measurement the particle will be in the state

$$f_{x_0}(x) = \delta(x - x_0) \ , \tag{7.4}$$

where x_0 is the observed position of the particle.

(Note that there is, contrary to QM, the only one decomposition of the unit in the space of probability distributions (7.1), composed of vectors (7.4), so that there is the only possibility to measure the position.)

This interpretation is correct if one considers only the phenomenon of the Brownian motion without any relations to other branches of Physics (the molecular theory etc.)

QM differs from the Brownian motion theory in the following :

(i) The state is described by the amplitude distribution $\psi(x)$, which takes values in the complex numbers and has the property

$$\int_{R^1} |\psi(x)|^2\, dx = 1 \ . \tag{7.5}$$

(ii) The evolution of the state is described by the Schroedinger equation for the amplitude $\psi(x;t)$

$$i\partial_t \psi(x;t) = c\cdot\partial_x^2 \psi(x;t) \ . \tag{7.6}$$

(iii) The Feynman probability amplitude theory [1] (a slightly more developed formulation of it can be found in [4]) is used instead of the usual Kolmogorov probability theory. Mainly, the probability is calculated by the formula $P = |\phi|^2$, where ϕ is the amplitude of an event.

(Note that (i) and (ii) are completely natural if we assume (iii).) The main disatvantage of the orthodox interpretation is that (in both cases) the change of the state during the measurement is completely mysterious.

We think that the fundamental importance of the Feynman probability amplitude theory (instead of the Kolmogorov probability theory) is the *main consequence of* QM confermed strongly by the experiment and it should be used without any doubts. On the other hand, the exact properties of the probability amplitude do not follow from the general assumptions. Namely, it is usually supposed that the amplitude distribution *can depend only on the particles' positions;* we have shown above that there is the theory with the amplitude depending on the *positions and velocities of particles* which agrees observationally with QM (the case with $1/\beta$ sufficiently small - QM is equal to the $1/\beta = 0$ case of our theory).

Now we shall discuss the so-called particle-wave dualism. It is assumed usually that there is a complete symmetry or duality between these two descriptions. We shall show that this is not true. It is possible to see the position of the particle (by seeing the point-like trace of it on a screen) in the experiment with the single particle. But it is not possible to see the interference pattern in the experiment with the single particle - this pattern can be developed only using the large number of particles. Thus there is not the complete symmetry between particle-like and wave-like descriptions.

We may conclude that quantum particles are point-like objets with the evolution described by the probability amplitude distribution. In this way *the paradox of the 'reduction of the wave function' disappears.* All this is in the complete analogy with the two (usual and orthodox) possible interpretations of the Brownian motion - we can observe the probability distribution $f(x)$ only in the experiment with many particles.

Clearly, assuming the point-like nature of particles, the description given by the probability amplitude distribution (or by the probability distribution in the Brownian motion case) is incomplete. In fact, *in the theory described above there are 'hidden parameters' - they are the random forces* $F_i(t)$. But these are the hidden parameters *different* from hidden parameters usually considered (so that the name 'hidden parameters' is rather misleading). The usual hidden parameters are described by the *probability distribution* and they are aimed to 'explain' classically the Feynman probability amplitude theory. Such an explanation cannot agree with QM (Bell inequalities). On the contrary, our hidden parameters *live inside* QM (i.e. they are the parameters introduced *into* QM); they are described by the *probability amplitude distribution.* The resulting theory *may agree with* QM with an arbitrary accuracy (there are no Bell inequalities).

Our hidden parameters probably cannot be observed directly. If we assume moreover the hypotheses of the tachyonic background from Chap. 4, these parameters principially cannot be observed, because any particular tachyon cannot be observed. Only the collective effect of many of them (the sub-quantum coherence) could be seen.

In conclusion, our theory is based on the following assumptions:

(a) The Feynman theory of the probability amplitude is used as the general and firmly established basis of any possible generalization of the quantum theory.

(b) The wave function (interpreted as the probability amplitude distribution) is considered as an incomplete description of the point-like particle.

(c) This incompleteness is attributed to the random forces acting on the particles. These forces are introduced in such a way that the resulting theory generalizes QM and reduces to QM if $1/\beta \to 0$.

(d) The special hypotheses on the (amplitude) distribution of random forces is introduced (implying the possibility of the subquantum coherence effect). The mechanism creating these random forces is proposed using the idea of a sub-quantum medium composed from tachyons.

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