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Wolfgang Ruess; Dirk Werner

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Structural Properties of Operator Spaces

WOLFGANG RUESS,*)

Essen, B.R.D.,

DIRK WERNER,**)

Berlin

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We present some observations on the problem of which topological or geometrical properties of Banach spaces X and Y are inherited by spaces of compact operators between X and Y, and we note some of the open problems in this context.

In 1957 L. Schwartz [21] introduced the ε-product $X\varepsilon Y$ of two locally convex spaces as the space $L_e(X_c^*, Y)$ of continuous linear operators from X_c^*, the dual of X endowed with the topology of uniform convergence on compact convex sets, into Y, with the topology of uniform convergence on the equicontinuous subsets of X^*. He noted that $L_e(X_c^*, Y)$ reduces to the space $K_{w^*}(X^*, Y)$ of compact weak*-weakly continuous linear operators with the operator norm topology in the Banach space setting and that it coincides with the completed injective tensor product $X \mathbin{\hat{\otimes}_\varepsilon} Y$ for complete locally convex space X and Y in case X or Y enjoys the approximation property.

The decisive feature of $L_e(X_c^*, Y)$ is its canonical embeddability into the space of continuous functions $C(U^0 \times V^0)$, U and V being zero neighbourhoods in X and Y, respectively. For Banach spaces, this embedding reads $K_{w^*}(X^*, Y) \hookrightarrow C(B_{X^*} \times B_{Y^*})$, where B_{X^*} denotes the closed dual unit ball with its relative weak* topology.

It is this embedding that makes it possible to tackle the basic problem in analyzing the operator space $L_e(X_c^*, Y)$, namely to connect its properties with the corresponding properties of X and Y. We are going to present a sample of propositions where this technique is employed, see [4], [5], [19], and the literature cited there for related results. We also point out some open problems.

In the first section we shall be concerned with compactness properties. The point here is that the Arzelà-Ascoli theorem quickly yields a precompactness criterion

*) Fachbereich Mathematik, Universität Essen, Universitätsstr. 3, D-4300 Essen, B.R.D.

**) I. Mathematisches Institut FU Berlin, Arnimallee 3, D-1000 Berlin 33.

for $L_e(X_c^*, Y)$ [21, p. 22]. By means of this tool (in the somewhat more elaborate version of [18]) we give fairly elementary proofs of some stability results. In the second section we observe that the injective tensor product of two weakly compactly generated Banach spaces is weakly compactly generated. Also, variants of this result are obtained in a rather trivial manner. The third section contains a discussion of geometrical property of Banach spaces, namely, of the M-ideal structure in $X \hat{\otimes}_\varepsilon Y$.

Finally, it should be mentioned that results on K_{w^*} can be parlayed into results on spaces of (a) compact operators, (b) vector-valued continuous functions, and (c) compact range vector measures via the isometric isomorphisms

(a) $K(X, Y) = K_{w^*}(X^{**}, Y)$, $T \to T^{**}$,

(b) $C(K, X) = C(K) \hat{\otimes}_\varepsilon X = K_{w^*}(C(K)^*, X)$,

(c) $cca(\Sigma, X) = cca(\Sigma) \hat{\otimes}_\varepsilon X = K_{w^*}(cca(\Sigma)^*, X)$.

For details and further examples, cf. [4], [5], and [19].

1. The Gelfand-Phillips property and the Schur property

In a series of papers [7–11], the following result was established (for the definitions see below).

1.1. Theorem. For Banach spaces X and Y with the Gelfand-Phillips property, the space $K_{w^*}(X^*, Y)$ has the Gelfand-Phillips property.

We shall give an elementary proof of 1.1 which, actually, yields a general locally convex version.

A bounded set B of a locally convex space X is called *limited* if every equicontinuous weak* nullsequence converges uniformly on B. X is said to have the *Gelfand-Phillips property* if every limited set is precompact. (Conversely, precompact sets are limited.)

In view of section 2 it is worth mentioning that a Banach space with a weak* sequentially compact dual unit ball has the Gelfand-Phillips property. (This is not hard to show.)

1.2. Theorem. For locally convex spaces X and Y with the Gelfand-Phillips property, the space $L_e(X_c^*, Y)$ has the Gelfand-Phillips property.

Proof. Let $H \subset L_e(X_c^*, Y)$ be limited. In order to show that H is precompact we use Theorem 1.5 of [18]. First, for $y^* \in Y^*$, $H^*(y^*) = \{T^*y^*: T \in H\}$ is limited in X and thus precompact. It remains to show that $H(U^0)$ is precompact in Y for any zero neighbourhood U in X. Let $(h_n x_n^*)_n$ be any sequence in $H(U^0)$ and $(y_n^*)_n$ any equicontinuous weak* nullsequence in Y^*. Note that $(x_n^* \otimes y_n^*)_n$ is an equicontinuous weak* nullsequence in $L_e(X_c^*, Y)$. Hence

$$\langle h_n, x_n^* \otimes y_n^* \rangle = \langle h_n x_n^*, y_n^* \rangle \to 0$$

as $n \to \infty$, and this shows that $H(U^0)$ is limited and, consequently, precompact.

Next, we consider the Schur property. (A locally convex space has the *Schur property* if every weakly convergent sequence converges w.r.t. the original topology.)

1.3. Proposition. If X and Y are locally convex spaces with the Schur property, then $L_e(X_c^*, Y)$ has the Schur property.

Proof. Let $(h_n)_n$ be a weak* nullsequence in $L_e(X_c^*, Y)$ and assume that $(h_n)_n$ is not convergent. Then there exist zero neighbourhoods U and V in X and Y, respectively, a sequence $(x_n^*)_n$ in U^0 and a subsequence $(h_{n_k})_k$ of $(h_n)_n$ such that

$$(*) \qquad h_{n_k}(x_k^*) \notin V \quad \text{for all} \quad k \in \mathbb{N}\,.$$

Note that for any $y^* \in Y^*, h_n^* y^* \to 0$ weakly in X and thus in the original topology of X. Therefore

$$\langle h_{n_k}(x_k^*), y^* \rangle = \langle h_{n_k}^*(y^*), \quad x_k^* \rangle \to 0$$

for all $y^* \in Y^*$. By assumption on Y this means

$$h_{n_k}^*(x_k^*) \to 0$$

in Y which contradicts $(*)$.

For Banach spaces, 1.3 was proved in [17] in a different manner. It follows from Eberlein's theorem that a Banach space has the Schur property if and only if every weakly compact subset is actually compact. We shall consider a variant of this property in the next result.

1.4. Theorem. Suppose X and Y are locally convex spaces with the property:

$(*)$ Every relatively weakly compact set is precompact.

Then $L_e(X_c^*, Y)$ has property $(*)$.

Proof. Let $H \subset L_e(X_c^*, Y)$ be relatively weakly compact. In order to show that H is precompact we again use [18, Th. 1.5.] First note that $H^*(Y^*)$ is relatively weakly compact in X and hence precompact for all $y^* \in Y^*$. Next, let U be any zero neighbourhood in X and $(h_\alpha x_\alpha^*)_\alpha$ any net in $H(U^0)$. Then there exist subnets $(h_\beta)_\beta$ and $(x_\beta^*)_\beta$, $h_0 \in L_e(X_c^*, Y)$ and $x_0^* \in U^0$ such that

$$h_\beta \to h_0 \quad \text{weakly}\,, \quad x_\beta^* \to x_0^* \quad \text{weak}^*\,.$$

We conclude

$$|\langle h_\beta x_\beta^* - h_0 x_0^*, y^* \rangle| \leqq |\langle (h_\beta - h_0)\, x_0^*, y^* \rangle| + |\langle h_\beta^* y^*, x_\beta^* - x_0^* \rangle| \to 0$$

since

$$\langle h_\beta - h_0, x_0^* \otimes y^* \rangle \to 0$$

and since $H^*(y^*)$ is precompact in X. Consequently, $(h_\alpha x_\alpha^*)_\alpha$ has a weakly convergent subnet, and $H(U^0)$ is relatively weakly compact, hence precompact.

2. WCG spaces and weaker notions

Recall that a Banach space is called *weakly compactly generated* (*WCG* for short) if it contains a weakly compact total subset. (cf. [6, Chapter V] for a discussion of this property.)

2.1. Theorem. If X and Y are *WCG* Banach spaces, then so is $X \hat{\otimes}_\varepsilon Y$.

Proof. Suppose $X = \overline{\text{lin}}\, V$, $Y = \overline{\text{lin}}\, W$ with V and W weakly compact. Then $V \otimes W \subset X \hat{\otimes}_\varepsilon Y$ is weakly compact (this is a consequence of the Dominated Convergence Theorem), and it is readily verified that $V \otimes W$ is total.

Actually, by [5, Theorem 1.4] the same proof extends to locally convex spaces X, and Y (cf. [3]).

2.2. Corollary. If X^* and Y are *WCG* and one of them has the approximation property, then $K(X, Y)$ is *WCG*.

It is not clear if the approximation assumption can be dispensed with, that is if the following is true.

2.3. Problem. Is $K_{w^*}(X^*, Y)$ *WCG* if X and Y are?

One may, however, prove easily:

(∗) If X and Y are *WCG* (or merely subspaces of *WCG* (spaces), then $K_{w^*}(X^*, Y)$ is a subspace of a *WCG* space.

(The point is, of course, that subspaces of *WCG* spaces need not be *WCG*, [6, p. 190 ff.].) Now, (∗) is entirely trivial if one uses the following well-know facts:

- If X is *WCG*, then $C(B_{X^*})$ is *WCG*. [6, p. 148]
- $C(K)$ is *WCG* iff K is Eberlein compact. [6, p. 152]
- If K_1 and K_2 are Eberlein compact, then $K_1 \times K_2$ is Eberlein compact.

It is left to observe (for $X \subset E$, $Y \subset F$)

$$K_{w^*}(X^*, Y) \subset K_{w^*}(E^*, F) \subset C(B_{E^*} \times B_{F^*})$$

which is *WCG* for *WCG* spaces E and F.

In his paper [25] (see also [23] and [26]) Talagrand studied the class of weakly k-analytic (w.k-a.) Banach spaces. (X is w.k-a. if X, in its weak topology, is the continuous image of a $\mathscr{K}_{\sigma\delta}$.) By [24] and [27], every *WCG* space is w.k-a. In the discussion of weak k-analyticity of $K_{w^*}(X^*, Y)$ we shall make use of the following non-trivial results of Talagrand's:

- X is w.k-a. iff $C(B_{X^*})$ is w.k-a. [25, Theorem 3.6]
- A closed subspace of a w.k-a. space is w.k-a.
- The class of cpmpacts K yielding w.k-a. $C(K)$-spaces is closed under formation of countable products. [25, Theorem 5.2]

In the same way as above, we now obtain a slight improvement of [25, 5.1. (iv)]. Moreover, this method also works for the even larger class of weakly countably determined (w.c.d.) spaces (discussed in [25] and [27]).

2.4. Proposition. If X and Y are w.k-a. (resp. w.c.d.), then $K_{w^*}(X^*, Y)$ is w.k-a. (resp. w.c.d.).

Also, the tensor stability result [23, 8.52] for the class $\mathscr{X}$ introduced by Stegall [23, Chapter 8] can be reformulated in the context of K_{w^*}-spaces.

According to [25, Theorem 6.4] w.c.d. (and thus w.k-a.) spaces have weak* angelic dual unit balls. In particular, these spaces have weak* sequentially compact dual unit balls, and weak* sequentially continuous functionals on the dual space are actually continuous. Hence, by 2.4 these properties are inherited by $K_{w^*}(X^*, Y)$ for w.c.d. spaces X and Y.

The latter property, sometimes called *Mazur's condition*, has been studied in [15]. Let us note that the proof of [15, Prop. 5.1] even yields the following result, proving the approximation assumption in [15] to be superfluous.

2.5. Proposition. Suppose X and Y fulfill Mazur's condition and

(i) the canonical operator $j: X^* \hat{\otimes}_\pi Y^* \to (K_{w^*}(X^*, Y))^*$ has weak* sequentially dense range,

(ii) $ex\, B_{X^*}$ or $ex\, B_{Y^*}$ is weak* sequentially relatively compact.

Then $K_{w^*}(X^*, Y)$ fulfills Mazur's condition.

2.6. Corollary. If X and Y fulfill Mazur's condition and X^* or Y^* has RNP, then $K_{w^*}(X^*, Y)$ fulfills Mazur's condition.

In fact, in this case j is onto [4], and (ii) is fulfilled, too [13].

Note that j always has weak* dense range. Condition (ii), with "or" replaced by "and" easily implies that $ex\, B_{(K_{w^*}(X^*,Y))^*}(= ex\, B_{X^*} \otimes ex\, B_{Y^*}$ by [16], [20]) is weak* sequentially relatively compact. The following, however, seems to be an open problem:

2.7. Problem. Does $K_{w^*}(X^*, Y)$ (or $X \hat{\otimes}_\varepsilon Y$) have a weak* sequentially compact dual unit ball if X and Y have? What about having weak* angelic dual unit balls?

Finally, let us give a list of the results of section 1 and 2. $K_{w^*}(X^*, Y)$ inherits property (n) from X and Y:

(0)	reflexive	no (e.g. $X = Y = l^2$)
(1)	*WCG*	?, yes for $X \hat{\otimes}_\varepsilon Y$
(2)	w.k-a.	yes
(3)	w.c.d.	yes
(4)	class $\mathscr{X}$	yes if X or Y does not contain l^1

(5)	dual ball weak* seq. comp.	?	
(6)	Mazur's condition	?,	yes e.g. if X^* or Y^* has RNP
(7)	Gelfand-Phillips	yes	
(8)	Schur	yes	

These properties are linked as follows:

(0) ⇒ (1) ⇒ (2) ⇒ (3) ⇒ (4) ⇒ (5) ⇒ (7)
(3) ⇒ (6)
(8) and (1) ⇒ separable.

3. M-ideals

In this section we shall investigate a geometrical (rather than topological) property of Banach spaces. According to [1], a closed subspace J of Banach space X is called *M-ideal* if J^0, the polar of J, is an L-summand of X^*, that is, if there is an L^1-direct decomposition

$$X^* = J^0 \oplus_1 V,$$

where V is isometric to J^*. The canonical projection on X^* with range J^0 is called the associated *L-projection*. The reader is referred to [1] and [2] for an exposition of the theory of M-ideals.

We shall prove the following theorem which improves [28, Theorem 2.8], where X was assumed to contain no non-trivial M-ideal.

3.1. Theorem. Suppose X has finitely many M-ideals $J_0 = X, \ldots, J_r = \{0\}$. Then Z is an M-ideal in $X \hat{\otimes}_\varepsilon Y$ iff

$$Z = \bigcap_{i=0}^{r} \left(J_i \hat{\otimes}_\varepsilon Y + X \hat{\otimes}_\varepsilon K_i\right),$$

where K_i is an M-ideal in Y.

Proof. The "if" part immediately follows from [28, Corollary 2.3] and the fact that finite intersections and sums of M-ideals are M-ideals [2, 2.5 and 2.7].

Now, suppose Z is an M-ideal in $X \hat{\otimes}_\varepsilon Y$. Let E denote the L-projection onto Z^0. Throughout the proof we shall make use of the following facts:

(a) $ex\, B_{(X \hat{\otimes}_\varepsilon Y)^*} = ex\, B_{X^*} \otimes ex\, B_{Y^*}$ [16], [20]

(b) $U \oplus_1 V = W$ implies $ex\, B_U \cup ex\, B_V = ex\, B_W$. (Clear!)

(c) For every $q \in ex\, B_{Y^*}$ there is an L-projection P on X^* with weak* closed range such that

$$E(x^* \otimes q) = P(x^*) \otimes q \quad \text{for all} \quad x^* \in X^*.$$

[28, 2.10]

For $0 \leqq i \leqq r$ put

$$C_i = \{q \in ex\, B_{Y^*}: E(x^* \otimes q) = P_i(x^*) \otimes q \quad \text{for all} \quad x^* \in X^*\},$$

where P_i is the L-projection onto J_i^0. By (c) and by assumption on X we have

$$ex\, B_{Y^*} = C_0 \cup \ldots \cup C_r \quad \text{(disjoint union)}.$$

Let as assume $C_0 \neq ex\, B_{Y^*}$, since otherwise $Z = X \hat{\otimes}_\varepsilon Y$, and we are done. Next, apply (c) to again obtain a collection $\{Q_p: p \in ex\, B_{X^*}\}$ of L-projections on Y^* with weak* closed ranges M_p such that

$$E(p \otimes y^*) = p \otimes Q_p(y^*) \quad \text{for} \quad y^* \in Y^*.$$

For $1 \leqq i \leqq r$ put

$$E_i = ex\, B_{J_i^0}.$$

Then (by (b))

$$ex\, B_{X^*} = E_1 \cup \ldots \cup E_r.$$

Also, let $\Delta_p = \{i: p \in E_i\}$ for $p \in ex\, B_{X^*}$, and

$$F_\varphi = \{p: \Delta_p = \varphi\} \quad \text{for} \quad \varphi \subset \{1, \ldots, r\}.$$

We claim

$$(*) \qquad M_p = \text{cl}\, lin^{w^*} \bigcup_{i \in \varphi} C_i \quad \text{for all} \quad p \in F_\varphi.$$

"$\subset$" Let $q \in ex\, B_{M_p}$. Then $q \in \text{ex}\, B_{Y^*}$ by (b).

It follows $E(p \otimes q) = p \otimes Q_p(q) = p \otimes q$ and on the other hand

$$E(p \otimes q) = P_i(p) \otimes q$$

if $q \in C_i$. Hence $P_i(h) = p$ and $p \in E_i$, that is $i \in \Delta_p = \varphi$. Now apply the Krein-Milman theorem.

"$\supset$" Let $i \in \varphi$ and $q \in C_i$. Then

$$\begin{aligned} p \otimes Q_p(q) = E(p \otimes q) &= P_i(p) \otimes q \quad & (q \in C_i) \\ &= p \otimes q & (i \in \Delta_p). \end{aligned}$$

Hence $q \in M_p$.

Finally, put $\Phi = \{\Delta_p: p \in ex\, B_{X^*}\}$ and

$$C_\varphi = \bigcup_{i \in \varphi} C_i \quad \text{for} \quad \varphi \in \Phi.$$

With this notation, $(*)$ implies that

$$K_\varphi: = \{y \in Y: \langle q, y \rangle = 0 \quad \text{for all} \quad q \in C_\varphi\}$$

is an M-ideal in Y for all $\varphi \in \Phi$.

On the other hand, $\Delta_p = \varphi$ means

$$p \in \bigcap_{i\in\varphi} E_i \subset \bigcap_{i\in\varphi} J_i^0 = (\sum_{i\in\varphi} J_i)^0 ,$$

and $\sum_{i\in\varphi} J_i$ is an M-ideal [2, 2.7], so there is a number $i(\varphi) \geqq 1$ such that $i \in \varphi$ iff $E_{i(\varphi)} \subset \subset E_i$, i.e. $\bigcap_{i\in\varphi} E_i = E_{i(\varphi)}$.

Now we are ready to prove

$$(**) \qquad Z = \bigcap_{\varphi\in\Phi} (J_{i(\varphi)} \,{}^\wedge\!\otimes_\varepsilon Y + X \,{}^\wedge\!\otimes_\varepsilon K_\varphi) .$$

We have already pointed out that the space on the right hand side, Z_1 say, is an M-ideal. (Note that Φ is finite.) To prove (**) it is enough to prove $ex\, B_{Z^0} = ex\, B_{Z_1^0}$, that is (by (a) and (b)) $p \otimes q \in Z^0$ iff $p \otimes q \in Z_1^0$ for $p \in ex\, B_{X^*}$, $q \in ex\, B_{Y^*}$. In fact, by construction

$$p \otimes q \in Z_1^0 \quad \text{iff} \quad p \otimes q \in \bigcup_{\varphi\in\Phi} E_{i(\varphi)} \otimes C_\varphi = \mathrm{ex}\, B_{Z^0} .$$

After reordering one obtains the announced representation from (**). This concludes the proof of 3.1.

3.2. Corollary. Under the assumptions of 3.1, Z is an M-ideal iff $Z = \sum_{i=0}^{r} J_i \,{}^\wedge\!\otimes_\varepsilon H_i$ for some M-ideals H_i in Y.

Proof. Again, only the "only if" part needs to be considered. In 3.1. the representation $Z = \bigcap_{i=0}^{r} (J_i \,{}^\wedge\!\otimes_\varepsilon Y + X \,{}^\wedge\!\otimes_\varepsilon K_i)$ was shown, which is equivalent to

$$ex\, B_{Z^0} = \cup_i E_i \otimes F_i$$

with $E_i = ex\, B_{J_i^0}$, $F_i = ex\, B_{K_i^0}$. It follows that $(ex\, B_{X^*} \otimes B_{Y^*}) \setminus ex\, B_{Z^0}$ may be represented as a finite union $\bigcup_j E'_j \otimes F'_j$, where $E'_j(F'_j)$ is the complement of a finite union of E_i's(F_i's):

$$ex\, B_{X^*} \setminus E'_j = \bigcup_{i\in A_j} E_i , \quad ex\, B_{Y^*} \setminus F'_j = \bigcup_{i\in B_j} F_i .$$

Thus,

$$\overline{\mathrm{lin}}^{w^*}\, ex\, B_{X^*} \setminus E'_j = (\bigcap_{i\in A_j} J_i)^0 =: (\hat{J}_j)^0 ,$$

$$\overline{\mathrm{lin}}^{w^*}\, ex\, B_{Y^*} \setminus F'_j = (\bigcap_{i\in B_j} K_i)^0 =: (\hat{H}_j)^0,$$

and $\hat{J}_j$, $\hat{H}_j$ are M-ideals. We conclude

$$Z = \sum \hat{J}_j \,{}^\vee\!\otimes_\varepsilon \hat{H}_j ,$$

since the complement of $ex\, B_{(\hat{J}_j \,{}^\wedge\!\otimes_\varepsilon \hat{H}_j)^0}$ is $E'_j \otimes F'_j$. Again, the announced representation follows.

These results are applicable for example for $X = L(l^p)$, $1 < p < \infty$, here $K(l^p)$ is the only non-trivial M-ideal [12]. Also, a complex C^*-algebra with finitely many closed two-sided ideals may be considered [22].

Let us give a third formulation of 3.1. For a Banach space X, Alfsen and Effros [1, p. 143] introduced the *structure topology* on $ex\, B_{X^*}$ as the topology whose closed sets are exactly those of the form $ex\, B_{X^*} \cap J^0 = ex\, B_{J^0}$, where J runs through the collection of M-ideals in X. It is coarser than the relative weak* topology and non-Hausdorff in general. Define the equivalence relation "$p_1 \sim p_2$ iff p_1 and p_2 are linearly dependent" on $ex\, B_{X^*}$ and the pertaining quotient space $E_X := ex\, B_{X^*}/\sim$. Note that $E_{X\hat{\otimes}_\varepsilon Y} = E_X \times E_Y$, and these topological spaces are homeomorphic if the quotient topology of the weak* topology is considered. Let us write $(E_X, st.)$ to indicate the quotient topology of the structure topology.

3.3. Corollary. Under the assumptions of 3.1, the product $(E_X, st.) \times (E_Y, st)$. is homoeomorphic with $(E_{X\hat{\otimes}_\varepsilon Y}, st.)$.

Indeed, this was shown in the proof of 3.2.

Let us mention some problems. 3.3 suggests the following question.

3.4. Problem. Does the conclusion of 3.3 hold for all Banach spaces X and Y? (Of course, the product topology is always coarser.)

Another problem concerns the possibility of extending our results to $K_{w^*}(X^*, Y)$.

3.5. Problem. Does the conclusion of 3.1 hold for $K_{w^*}(X^*, Y)$? See [28, 2.13] for a partial answer. Also, by the method of [28, 2.11], spaces in which M-ideals are already M-summands (e.g. spaces which are M-ideals in their biduals, [14]) may be considered. The basic obstacle for treating the general case is the following problem:

3.6. Problem. If J is an M-ideal in Y, is $K_{w^*}(X^*, J)$ and M-ideal in $K_{w^*}(X^*, Y)$?

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