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NUMERICAL SIMULATION OF THE TWO-PHASE FLOW THROUGH THE ENERGETICAL EQUIPMENT*

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Abstract

A mathematical model of homogeneous and heterogeneous condensation in the turbomachinery is presented. The mathematical model consists of conservation laws of mass, momentum, energy and conservation laws for the description of the rise and growth of water droplets with the homogeneous and heterogeneous condensation. The relaxation method is introduced for solving the two-dimensional unsteady flow through the turbine cascades. The method is developed for unstructured grids and is applied to the flow in the Laval nozzle.

1. Mathematical model of two-phase wet steam

1.1. Two-phase flow wet steam

M. Šťastný [4], [5] designed the mathematical model for the description of the wet steam flow through an energetical equipment with condensation (phase-change vapour-liquid). The condensation model consists of the homogeneous condensation (spontaneous) and heterogeneous condensation - condensation on chemical impurities emerging in wet steam.

The homogeneous condensation — the condensation cores development and the growth of water droplets are modeled within this phenomena.

The heterogeneous condensation — condensation cores are composed of cluster molecules of salt (NaCl) and the growth of water droplets is modeled.

The mathematical model is based on many theoretical and experimental observations. The model consists of

(a) the description of vapour flow and

(b) a condensation model for the description of dynamical processes during condensation (the rise and growth of water droplets).

Numerical simulation can be used to analyse phenomena, including condensation, ocurring in wet steam flowing through turbine cascades.

The mathematical model of the steam flow with heterohomogenous condensation is decribed by the system of hyperbolic partial differential equations

$$\frac{\partial \mathbf{U}}{\partial t} + \frac{\partial \mathbf{F}(\mathbf{U})}{\partial x} + \frac{\partial \mathbf{G}(\mathbf{U})}{\partial y} = \mathbf{H}(\mathbf{U}), \tag{1}$$

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where $\mathbf{U} = \mathbf{U}(x, y, t)$ is the vector of nine unknown functions (density, velocity, energy, droplets characteristics), $\mathbf{F} = \mathbf{F}(\mathbf{U})$ and $\mathbf{G} = \mathbf{G}(\mathbf{U})$ are the corresponding flow functions, and $\mathbf{H}(\mathbf{U})$ describes environmental interaction. This complex model has been presented in [4].

2. Reconstruction of the gradient on the unstructured grid

In this paragraph, we describe the construction of the gradient of multidimensional function, which is necessary for a numerical method (e.g. relaxation method) on the unstructured grid.

For the description of the reconstruction procedure on an element of the computational grid we use the Taylor expansion of variable [3]

$$\mathbf{U} = \mathbf{U}_m + \nabla \mathbf{U}_m \cdot \mathbf{r}, \tag{2}$$

where $\mathbf{U}_m$ is the cell-averaged value of $\mathbf{U}$ prescribed in the cell center of the element m, $\nabla \mathbf{U}_m$ is the cell-centered gradient, computed as described bellow, $\mathbf{r}$ is the vector outgoing from the cell center to the center of any of three interfaces of the element m.

If the gradient is assumed constant over the cell m, Green's theorem yields

$$\nabla \mathbf{U}_m = \frac{1}{A} \oint_{\delta A} \mathbf{U}\mathbf{n} dS. \tag{3}$$

Gradient plane is uniquely defined by three non-collinear points. We will use the reconstruction technique which computes gradients from two vertices (end points of an interface) and a cell center (on either side of an interface). Consider the two triangles $\triangle 13m, \triangle 1a3$ in Fig. 1. We use equation (3) for elements $\triangle 13m, \triangle 1a3$ and we begin the reconstruction of the gradient.

$$(\mathbf{U}_x)_{13m} = \frac{1}{2A_{13m}} \left[\mathbf{U}_1 y_{m3} + \mathbf{U}_m y_{31} + \mathbf{U}_3 y_{1m}\right], \tag{4}$$

$$(\mathbf{U}_y)_{13m} = \frac{-1}{2A_{13m}} \left[\mathbf{U}_1 x_{m3} + \mathbf{U}_m x_{31} + \mathbf{U}_3 x_{1m}\right], \tag{5}$$

$$(\mathbf{U}_x)_{1a3} = \frac{1}{2A_{1a3}} \left[\mathbf{U}_1 y_{3a} + \mathbf{U}_3 y_{a1} + \mathbf{U}_a y_{13}\right], \tag{6}$$

$$(\mathbf{U}_y)_{1a3} = \frac{-1}{2A_{1a3}} \left[\mathbf{U}_1 x_{3a} + \mathbf{U}_3 x_{a1} + \mathbf{U}_a x_{13}\right], \tag{7}$$

where $y_{m3} = y_3 - y_m, x_{m3} = x_3 - x_m$ etc. are components of the normal vector, defined by the interface of the element. For example, for the vector $\mathbf{m3} = [x_3 - x_m, y_3 - y_m]$ we obtain the normal vector $\mathbf{n_{m3}} = [y_3 - y_m, -(x_3 - x_m)]$. To reconstruct the face gradient (on the alignment of elements $\triangle 13m, \triangle 1a3$) we use the areaweighted average of the gradients for elements $\triangle 13m, \triangle 1a3$

$$(\mathbf{U}_x)_{1a3m} = \frac{A_{13m}(\mathbf{U}_x)_{13m} + A_{1a3}(\mathbf{U}_x)_{1a3}}{A_{13m} + A_{1a3}}, \tag{8}$$

$$(\mathbf{U}_y)_{1a3m} = \frac{A_{13m}(\mathbf{U}_y)_{13m} + A_{1a3}(\mathbf{U}_y)_{1a3}}{A_{13m} + A_{1a3}}. \tag{9}$$

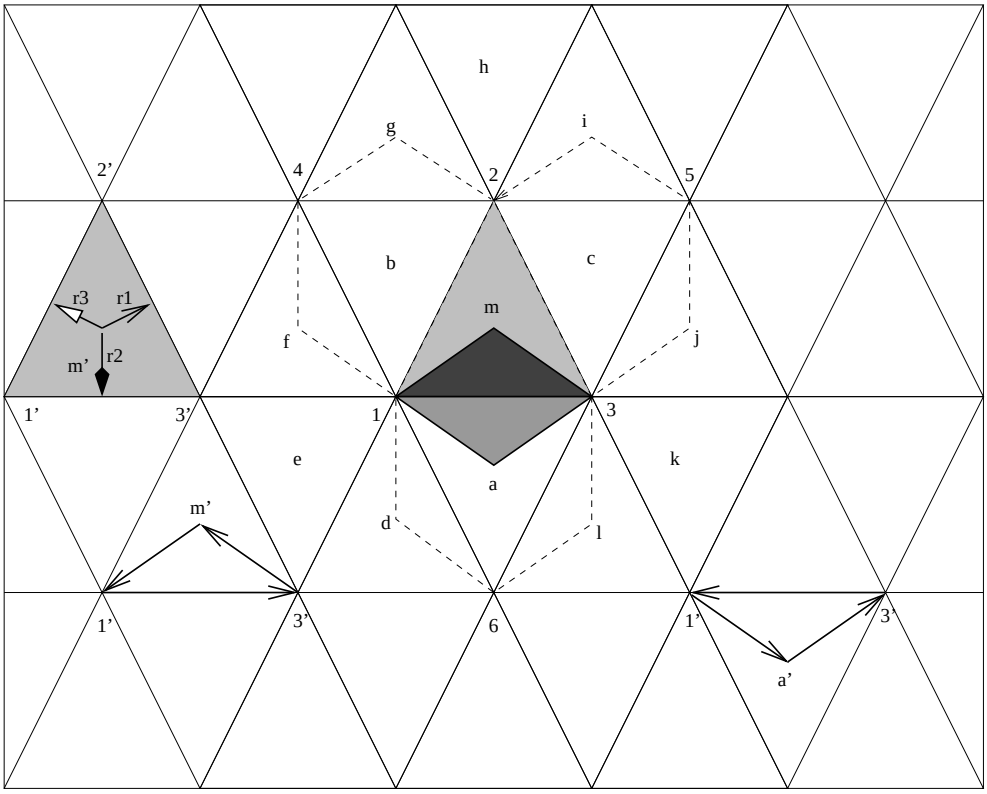

Fig. 1: *Stencil for the reconstruction of the gradient $\nabla \mathbf{U}_m$ and a typical computational cell. The dashed line shows the computational area for function (20).*

Substituing equations (4) and (6) into (8) and similary equations (5) and (7) into (9), we get the face gradient equations in the form

$$(\mathbf{U}_x)_{1a3m} = \frac{1}{2A_{1a3m}} \left[(U_a - U_m) y_{13} + (U_1 - U_3) y_{ma} \right], \tag{10}$$

$$(\mathbf{U}_y)_{1a3m} = \frac{1}{2A_{1a3m}} \left[(U_a - U_m) x_{13} + (U_1 - U_3) x_{ma} \right], \tag{11}$$

where $A_{1a3m} = A_{13m} + A_{1a3}$.

The gradient of U_m on the triangular cell m (triangle $\triangle 123$) is constructed by using the area weighted average of the corresponding face gradients

$$(\mathbf{U}_x)_m = \frac{A_{1a3m}(\mathbf{U}_x)_{1a3m} + A_{3c2m}(\mathbf{U}_x)_{3c2m} + A_{2b1m}(\mathbf{U}_x)_{2b1m}}{A_{1a3m} + A_{3c2m} + A_{2b1m}}, \tag{12}$$

$$(\mathbf{U}_y)_m = \frac{A_{1a3m}(\mathbf{U}_y)_{1a3m} + A_{3c2m}(\mathbf{U}_y)_{3c2m} + A_{2b1m}(\mathbf{U}_y)_{2b1m}}{A_{1a3m} + A_{3c2m} + A_{2b1m}}. \tag{13}$$

3. Relaxation method

The numerical methods of this type are based on the idea [1], [2] to replace the system of equations

$$\frac{\partial \mathbf{U}}{\partial t} + \frac{\partial \mathbf{F}(\mathbf{U})}{\partial x} + \frac{\partial \mathbf{G}(\mathbf{U})}{\partial y} = 0 \tag{14}$$

by the modified system of equations

$$\begin{aligned} \frac{\partial \mathbf{U}}{\partial t} + \frac{\partial \mathbf{V}}{\partial x} + \frac{\partial \mathbf{W}}{\partial y} &= 0, \\ \frac{\partial \mathbf{V}}{\partial t} + \mathbf{A}^2 \frac{\partial \mathbf{U}}{\partial x} &= -\frac{1}{r}\left(\mathbf{V} - \mathbf{F}(\mathbf{U})\right), \\ \frac{\partial \mathbf{W}}{\partial t} + \mathbf{B}^2 \frac{\partial \mathbf{U}}{\partial y} &= -\frac{1}{r}\left(\mathbf{W} - \mathbf{G}(\mathbf{U})\right), \end{aligned} \tag{15}$$

where $r > 0$ is the relaxation parameter and the matrices $\mathbf{A}$ and $\mathbf{B}$ are defined as the diagonal matrices $\mathbf{A} = \mathrm{diag}(a_1, .., a_n)$ and $\mathbf{B} = \mathrm{diag}(b_1, .., b_n)$. The elements of matrices a_i, b_i have to satisfy the condition

$$\frac{|\lambda_i|}{a_i} + \frac{|\mu_i|}{b_i} \leq 1, \quad i = 1, \ldots, n, \tag{16}$$

λ_i and μ_i are the eigenvalues of the Jacobi matrices $\frac{\partial \mathbf{F}(\mathbf{U})}{\partial \mathbf{U}}$ and $\frac{\partial \mathbf{G}(\mathbf{U})}{\partial \mathbf{U}}$.

If $r \to 0$ then the solution of (15) converges to the solution of system (14) and

$$\mathbf{V} \to \mathbf{F}(\mathbf{U}), \quad \mathbf{W} \to \mathbf{G}(\mathbf{U}). \tag{17}$$

Methods of this type on a rectangular grid are described in [2], [1].

Let us describe the usage of the relaxation method on the unstructured grid by applying the gradient reconstruction described in Section 2.

We use an unstructured triangular grid on the computational domain Ω, which have M elements. We define ω_m in the cell center

$$\omega_m = \frac{1}{S_{\Delta_m}} \int_{\Delta_m} \omega(x, y, t) dS, \tag{18}$$

as the integral average over the triangle cell, $m = 1, \ldots, M$. We write the semidiscrete approximation of (15) in the form

$$\begin{aligned} \frac{d\mathbf{U}_m}{dt} + D_x \mathbf{V}_m + D_y \mathbf{W}_m &= 0, \\ \frac{d\mathbf{V}_m}{dt} + \mathbf{A}^2 D_x \mathbf{U}_I &= -\frac{1}{r}\left(\mathbf{V}_m - \mathbf{F}(\mathbf{U})_m\right), \\ \frac{d\mathbf{W}_m}{dt} + \mathbf{B}^2 D_y \mathbf{U}_I &= -\frac{1}{r}\left(\mathbf{W}_m - \mathbf{G}(\mathbf{U})_m\right), \end{aligned} \tag{19}$$

where D_x and D_y are, in our case, differential operators defined by the components of the gradient (12) and (13).

We will use the limited gradients presented in [3]

$$\nabla U_m^l = \omega_a \nabla U_a + \omega_b \nabla U_b + \omega_c \nabla U_c, \tag{20}$$

where ω_i are weights computed as

$$\begin{aligned} \omega_a(g_a, g_b, g_c) &= (g_b g_c + \epsilon^2)/G, \\ \omega_b(g_a, g_b, g_c) &= (g_a g_c + \epsilon^2)/G, \\ \omega_c(g_a, g_b, g_c) &= (g_a g_b + \epsilon^2)/G, \\ G &= g_a^2 + g_b^2 + g_c^2 + 3\epsilon^2, \\ g_a = ||\nabla U_a||_2^2, \quad g_b &= ||\nabla U_b||_2^2, \quad g_c = ||\nabla U_c||_2^2, \end{aligned} \tag{21}$$

where ϵ should be small enough.

The system (19) can be solved by several high-resolution numerical methods. This system is semilinear, i.e., the flux functions are linear. We can easily compute the eigenvalues of this system. We can use this fact to simplify appropriate Riemann solvers that are the important part of every high-resolution scheme. If we use the high-order methods we have to implement a limiting procedure.

4. Numerical results

The numerical results presented in Fig. 2 were obtained by software package written in Matlab. These results were taken from the unsteady model. The initial conditions simulated filling of the empty nozzle. The boundary conditions were chosen as standard ones. We have no experiment based results, so we can not compare our results with experiment, but obtained results fulfilled our expectations.

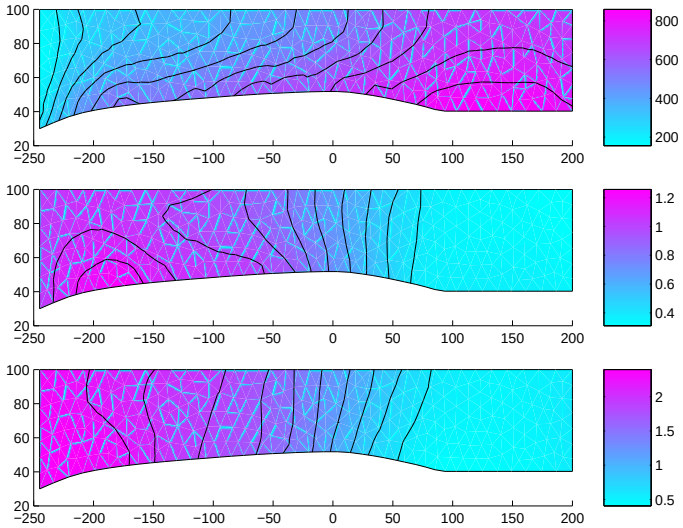

Fig. 2: *Plots of velocity, density and pressure in the nozzle, computed by the first order relaxation scheme (Pressure scale should be multiplied by* 10^5*). Steady state.*

5. Conclusion

We put the main emphasis on the development of a software package for the simulation of steam flow (2D). The software package is written as modular. We expect that modular architecture allows an easy implementation of other numerical methods (realized as modules) and cooperation with a pre- and postprocessor as third-party modules. In future work, we want to implement other types of numerical methods (composite methods, central methods, component-wise methods).

The mathematical model used for the steam flow can be augmented. The augmentation will be realized in two stages. We will use the Navier-Stokes equations for description of the steam flow, at this moment the Euler equations are used. The system will be augmented by a turbulent model at the second stage. We want to use large eddy simulation (LES) approach for the turbulent flow.

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