PANM 14

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In: Jan Chleboun and Petr Přikryl and Karel Segeth and Tomáš Vejchodský (eds.): Programs and Algorithms of Numerical Mathematics, Proceedings of Seminar. Dolní Maxov, June 1-6, 2008. Institute of Mathematics AS CR, Prague, 2008. pp. 103–110.

Persistent URL: http://dml.cz/dmlcz/702863

FINITE ELEMENT MODELING OF WOOD STRUCTURE*

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Abstract

This work is focused on a weak solution of a coupled physical task of the microwave wood drying process with stress-strain effects and moisture/temperature dependency. Due to the well known weak solutions for the individual physical fields, the author concerns with the coupled stress-strain relation coupled with the moisture and temperature distributions. For the scale dependency the subgrid upscaling method was used. The solved region is assumed to be divided into discontinuous subregions according to the investigated scale. This approach suggests sequential type of solution for highly coupled tasks. This way, very huge structures (huge with regard to the geometry and also physics) can be solved in the reasonable time and with reasonable memory consumptions. Main emphasis was put on evaluation of the structural response of the whole complex. Due to the influence of the moisture, temperature, and time, the coupled physical task of the structural response is solved. Suggested approach is of course usable not only for the structural response, but also for the other physical fields, which were taken into account. The weak solution is based on a slight modification of the Ritz-Galerkin method.

Keywords: FEM, multiphysics, microwave wood drying, upscaling, homogenization

1. Introduction

Microwave drying of wood is one of the most difficult problems of Wood Science. The problem is coupled according to the following variables: moisture w, temperature T, velocity of free water within the conductive wood elements $\mathbf{v}$, intensity of electric field $\mathbf{E}$, intensity of magnetic field $\mathbf{B}$, static pressure p and displacement of structural parts $\mathbf{u}$. Parabolic equations arise as models of many partial physical processes which occur during the drying process. The time-dependency affects most of these processes. Generally, diffusive partial differential equations (PDE) represent usually the base constitutive relationships. The electromagnetic field is sufficiently described by the reduced system of Maxwell's equations. To solve the coupled system, we evaluate the following unknowns: $T, w, p, \mathbf{u}, \mathbf{v}, \mathbf{E}, \mathbf{B}, \mathbf{H}, \mathbf{J}, \mathbf{D}$. These quantities are considered as elements of appropriate Hilbert spaces.

The first set of equations in the coupled system consists of the Maxwell's equations

$$\begin{aligned} \nabla \times \mathbf{E} &= \frac{\partial \mathbf{B}}{\partial t}, & \nabla \cdot \mathbf{D} &= \rho_e, \\ \nabla \times \mathbf{H} &= \mathbf{J} + \frac{\partial \mathbf{D}}{\partial t}, & \nabla \cdot \mathbf{B} &= 0, \end{aligned} \tag{1}$$

*This work was supported by grant No. GP106/06/P363 of the Czech Science Foundation.

where t is the time, $\mathbf{B}$ is the magnetic flux density, $\mathbf{D}$ is the electric flux density, $\mathbf{H}$ is the magnetic field intensity, $\mathbf{E}$ is the electric field intensity, $\mathbf{J}$ is the current density, ρ_e is the electric charge density. Due to the anisotropy of wood, we can itemize these variables to $\mathbf{D} = \varepsilon\mathbf{E}$, $\mathbf{B} = \mu\mathbf{H}$, $\mathbf{J} = \sigma\mathbf{E}$, where ε is permittivity, μ is permeability, and σ is the electric conductivity of the material.

Natural requirement for continuous charge is satisfied by the equation of continuity

$$\nabla \cdot \mathbf{J} = -\frac{\partial \rho_e}{\partial t}. \tag{2}$$

However, the electro-dynamical effects are not alone. Also the influence of the moisture and pressure changes in wood should be included. Content of water in the material is obviously separated into free water and water bonded through H-bridges (the chemical bonded water by stronger types of bindings are omitted). Bonded water keeps diffusive character. Interaction of moisture, temperature and static pressure can be described by system of equations (6). Widely disputed is the diffusive character of the static pressure. For this reason, the last equation is often omitted in the following system

$$\begin{aligned}
\rho C\frac{\partial T}{\partial t} - \nabla\mathbf{k_{Tw}}\nabla w - \nabla\mathbf{k_{Tp}}\nabla p - \nabla\mathbf{k_{TT}}\nabla T &= q_{abs} + \mathbf{k_{b_T}}\left(T_{ext} - T\right), \\
\frac{\partial w}{\partial t} - \nabla\mathbf{k_{ww}}\nabla w - \nabla\mathbf{k_{wp}}\nabla p - \nabla\mathbf{k_{wT}}\nabla T &= \mathbf{k_{b_w}}\left(w_{ext} - w\right), \\
\frac{\partial p}{\partial t} - \nabla\mathbf{k_{pw}}\nabla w - \nabla\mathbf{k_{pp}}\nabla p - \nabla\mathbf{k_{pT}}\nabla T &= \mathbf{k_{b_p}}\left(p_{ext} - p\right),
\end{aligned} \tag{3}$$

where ρ is the density, C is the heat capacity, q_{abs} is the density of energy, T_{ext} is the temperature in the surroundings, w is the mass concentration (moisture content), w_{ext} and p_{ext} are moisture content, and static pressure in the surroundings, respectively, $\mathbf{k_{b_w}}, \mathbf{k_{b_T}}, \mathbf{k_{b_p}}$ are convective coefficients and $\mathbf{k_{Tw}}, \mathbf{k_{Tp}}, \mathbf{k_{TT}}, \mathbf{k_{ww}}$, $\mathbf{k_{wp}}, \mathbf{k_{wT}}, \mathbf{k_{pw}}, \mathbf{k_{pp}}$, $\mathbf{k_{pT}}$ are the matrices of diffusion coefficients.

Structural response of the wood structure is described by parabolic equation

$$\begin{aligned}
\rho\frac{\partial \mathbf{u}}{\partial t^2} - \left(\nabla\mathbf{c_{EG}} + \left(w - w_{ext}\right)\nabla\mathbf{c_{K_{bw}}} + \left(T - T_{ext}\right)\nabla\mathbf{c_{K_{bT}}}\right)\nabla\mathbf{u} - \nabla\mathbf{c}_{\lambda_{\mathbf{w,T}}}\nabla\frac{\partial \mathbf{u_{vel}}}{\partial t} \\
+ \mathbf{C_w}\cdot w + \mathbf{C_{w^2}}\cdot w^2 + \mathbf{C_T}\cdot T + \mathbf{C_{T^2}}\cdot T^2 + \mathbf{C_{wT}}\cdot wT + \mathbf{C} = \mathbf{F}.
\end{aligned} \tag{4}$$

Definition of individual coefficients for equation (4) was described in [5], where generally orthotropic elastic properties related to moisture and temperature are defined. This described model is valid for diffusive transport of moisture and temperature. It is not appropriate (due to the physical nature of the phenomenon) for the free water movement. This transport is allocated into inter-cellular spaces and cell lumen. Description of this process can be done with Navier-Stokes equation

$$\frac{\partial \nu}{\partial t} + (\nabla \times \nu) \times \nu + \frac{1}{2} \nabla \nu^2 = -\frac{\nabla p^{fl}}{\rho} - \nabla U + \frac{\eta}{\rho} \nabla^2 \nu, \tag{5}$$

where ∇U is the potential and ν the velocity of the fluid.

Since the weak form of equations (1)–(3) and (5) is well known, see e.g. [2], [1], we focus on equation (4), where we will outline the variational formulation for the mixed type of elements in numerical subgrid upscaling method ([8], [6], [7], and [4]). It should be noted that we will suppose the sequential type of mentioned equations. This assumption leads to simplification of equation (4), where T and w are constant in one time-step.

2. Methods

Assembling of the weak form of equation (4) is realized with regard to the schema of the Ritz-Galerkin method by the following quadratic functional, which should be minimal:

$$\begin{aligned} G(\mathbf{u}) = \left(\rho \frac{\partial^2 \mathbf{u}}{\partial t^2}, \xi\right) - \Big(\Big(\nabla \mathbf{c}_{\mathbf{EG}} + (w - w_{ext})\, \nabla \mathbf{c}_{\mathbf{K_{bw}}} \\ + (T - T_{ext})\, \nabla \mathbf{c}_{\mathbf{K_{bT}}}\Big) \nabla \mathbf{u}, \xi\Big) - \left(\nabla \mathbf{c}_{\lambda_{\mathbf{w},\mathbf{T}}} \nabla \frac{\partial \mathbf{u}}{\partial t}, \xi\right) \\ - 2 \left(\mathbf{F} - \left(\mathbf{C}_w \cdot w + \mathbf{C}_{w^2} \cdot w^2 + \mathbf{C}_T \cdot T + \mathbf{C}_{T^2} \cdot T^2 + \mathbf{C}_{wT} \cdot wT + \mathbf{C}\right), \xi\right) = 0. \end{aligned} \tag{6}$$

This functional is well-defined for all $\xi \in H(\Omega)$ and $(\cdot, \cdot)$ stands for the scalar product on this Hilbert space. By the above mentioned simplifications, we obtain the integral form. Let the functional equation (6) be defined on the vector space V, which is a finite dimensional subspace of $H(\Omega)$. Let us assume the Ω to be partitioned into a finite number of subregions on very fine scale δ_1. Further, we assume that m_1 of these subregions are covered by mesh on this scale (subgrids). Functional from equation (6) is then considered on vector subspaces $V_1^{\delta_1}, V_2^{\delta_1}, \ldots, V_{m_1}^{\delta_1} \subseteq V$, where $V_j^{\delta_j}$ for $j = 1, \ldots, m_1$ are Raviart-Thomas (RT) spaces. Subspaces may not fill the full space V. It means that $V_1^{\delta_1} \cup V_2^{\delta_1} \cup V_3^{\delta_1} \cup \ldots \cup V_{m_1}^{\delta_1} \equiv V^{\delta_1} \subseteq V$. Simultaneously, we declare mentioned vector subspaces with bases

$$\left\{\varphi_{V_j,1}^{\delta_j}, \varphi_{V_j,2}^{\delta_j}, \ldots, \varphi_{V_j,n_1}^{\delta_j}\right\} \subseteq V_j^{\delta_j}, \quad j = 1, 2, \ldots, m_1.$$

Complete basis

$$\varphi^{\delta_1} \equiv \bigcup_{j=1}^{m_1} \left\{\varphi_{V_j,1}^{\delta_1}, \varphi_{V_j,2}^{\delta_1}, \ldots, \varphi_{V_j,n_j}^{\delta_1}\right\} \subset V^{\delta_1}.$$

on vector space V^{δ_1} is derived from the fine mesh of subgrids, where linear basis functions are used. Similarly, let us partition Ω by further linear meshes $\Psi_{\delta_2}, \Psi_{\delta_3}, \ldots, \Psi_{\delta_i}$

for different scales $\delta_1 < \delta_2 < \ldots < \delta_i$, where again regions $m_2, m_3, \ldots, m_i$ cover some parts of Ω on the specific scale. Consequently, similar vector subspaces can be distinguished $V_1^{\delta_k}, V_2^{\delta_k}, \ldots, V_{m_k}^{\delta_k} \subseteq V$, $k = 2, 3, \ldots, i$ with the same requirements:

$$\begin{aligned} V_1^{\delta_2} \cup V_2^{\delta_2} \cup V_3^{\delta_2} \cup \ldots \cup V_{m_2}^{\delta_2} &\equiv V^{\delta_2} \subseteq V, \\ V_1^{\delta_3} \cup V_2^{\delta_3} \cup V_3^{\delta_3} \cup \ldots \cup V_{m_3}^{\delta_3} &\equiv V^{\delta_3} \subseteq V, \\ &\vdots \\ V_1^{\delta_i} \cup V_2^{\delta_i} \cup V_3^{\delta_i} \cup \ldots \cup V_{m_i}^{\delta_i} &\equiv V^{\delta_i} \subseteq V. \end{aligned} \tag{7}$$

In addition, we will tie subspaces by these important rules:

$$V^{\delta_1} \subseteq V^{\delta_2} \subseteq V^{\delta_3} \subseteq \ldots \subseteq V^{\delta_i} \equiv V, \tag{8}$$

where δ_i is maximal scale $V^{\delta_i} \equiv V$ and V is defined on the entire Ω.

All unknown functions can be decomposed to individual scales, e.g.:

$$\mathbf{u} = \mathbf{u}^{\delta_1} + \mathbf{u}^{\delta_2} + \ldots + \mathbf{u}^{\delta_i}, \tag{9}$$

on some Ω_0. This decomposition of unknowns to individual scales affects the solution in the sense of finite elements and the minimization of functional equation (6) does not provide the common appearance of Ritz system.

Let us consider PDE $A\mathbf{u} = f$, $\mathbf{u} \in V$ with a differential operator A and let us follow the common steps in the solution of this task for multi-scale problems.

Functional which will be minimized has the standard form ([3]):

$$G(\mathbf{u}) = (\mathbf{u}, \mathbf{u})_A - 2\,(\mathbf{f}, \mathbf{u})\,. \tag{10}$$

Equation (9) will be substituted into the first part of equation (10):

$$L(\mathbf{u}) = \left(\mathbf{u}^{\delta_1} + \mathbf{u}^{\delta_2} + \ldots + \mathbf{u}^{\delta_i}, \mathbf{u}^{\delta_1} + \mathbf{u}^{\delta_2} + \ldots + \mathbf{u}^{\delta_i}\right)_A. \tag{11}$$

It can be expanded due to the rules of the bilinear form in the following way:

$$L(\mathbf{u}) = \sum_{k=1}^{i} \sum_{j=1}^{i} \left(\mathbf{u}^{\delta_k}, \mathbf{u}^{\delta_j}\right)_A. \tag{12}$$

Our problem is reduced into the task of finding such function $\mathbf{u}$ which minimizes the functional (10). This functional is minimized by a function in the form:

$$\widetilde{\mathbf{u}}_n^{\delta_j} = \sum_{k=1}^{s_j} b_k^{\delta_j} \varphi_k^{\delta_j}, \tag{13}$$

where $s_j = \dim V^{\delta_j}$, $\varphi_k^{\delta_j}$ denote the basis functions in V^{δ_j} and $b_k^{\delta_j}$ represent variables which will be evaluated in the point of the approximate minimum $a_k^{\delta_j}$. As the first step, we estimate the functional in the subgrid on the scale δ_1. The minimizing function is denoted as:

$$\mathbf{u}_n^{\delta_j} = \sum_{k=1}^{s_j} a_k^{\delta_j} \varphi_k^{\delta_j}. \tag{14}$$

To evaluate the coefficients $a_k^{\delta_j}$, we compute the partial derivatives of $L(\mathbf{u}_n)$ with respect to all coefficients on all scales and equal them to zero:

$$\frac{\partial L(\mathbf{u}_n)}{\partial b_l^{\delta_k}}\Big|_{\mathbf{b}_{\delta_1}=\mathbf{a}_{\delta_1},\dots,\mathbf{b}_{\delta_i}=\mathbf{a}_{\delta_i}} \quad \text{for } k = 1,\dots,i;\ l = 1,\dots,s_k, \tag{15}$$

where $\mathbf{a}_{\delta_j} = \left(a_1^{\delta_j},\dots,a_{s_j}^{\delta_j}\right)^T$ and $\mathbf{b}_{\delta_j} = \left(b_1^{\delta_j},\dots,b_{s_j}^{\delta_j}\right)^T$ for $j = 1,\dots,i$. The partial derivatives of $L(\mathbf{u}_n)$ with respect to coefficients $\mathbf{b}_{\delta_k}$ can be computed as follows

$$\frac{\partial \left(\widetilde{\mathbf{u}}_n^{\delta_j}, \widetilde{\mathbf{u}}_n^{\delta_k}\right)_A}{\partial b_1^{\delta_k}\dots\partial b_{s_k}^{\delta_k}} = \mathbf{R}^L_{A_{\delta_j\delta_k}}\mathbf{a}_{\delta_j} \quad \text{for } j \neq k, \tag{16}$$

where the symbol on the left-hand side stands for the vector of partial derivatives with respect to $b_1^{\delta_k},\dots,b_{s_k}^{\delta_k}$ and $\mathbf{R}^L_{A_{\delta_j\delta_k}}$ denotes the modified lower triangular matrix of Ritz system:

$$\mathbf{R}^L_{A_{\delta_j\delta_k}} = \begin{pmatrix} \left(\varphi_1^{\delta_j},\varphi_1^{\delta_k}\right)_A & 0 & \cdots & 0 \\ 2\left(\varphi_1^{\delta_j},\varphi_2^{\delta_k}\right)_A & \left(\varphi_2^{\delta_j},\varphi_2^{\delta_k}\right)_A & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 2\left(\varphi_1^{\delta_j},\varphi_{s_k}^{\delta_k}\right)_A & 2\left(\varphi_2^{\delta_j},\varphi_{s_k}^{\delta_k}\right)_A & \cdots & \left(\varphi_{s_j}^{\delta_j},\varphi_{s_k}^{\delta_k}\right)_A \end{pmatrix}. \tag{17}$$

Similarly, partial derivatives of the first part of equation (10) with respect to coefficients $\mathbf{b}_{\delta_j}$ can be computed as

$$\frac{\partial \left(\widetilde{\mathbf{u}}_n^{\delta_j}, \widetilde{\mathbf{u}}_n^{\delta_k}\right)_A}{\partial b_1^{\delta_j}\dots\partial b_{s_j}^{\delta_j}} = \mathbf{R}^U_{A_{\delta_j\delta_k}}\mathbf{a}_{\delta_k} \quad \text{for } j \neq k, \tag{18}$$

where $\mathbf{R}^U_{A_{\delta_j\delta_k}}$ is modified upper triangular matrix of Ritz system:

$$\mathbf{R}^U_{A_{\delta_j\delta_k}} = \begin{pmatrix} \left(\varphi_1^{\delta_j},\varphi_1^{\delta_k}\right)_A & 2\left(\varphi_1^{\delta_j},\varphi_2^{\delta_k}\right)_A & \cdots & 2\left(\varphi_1^{\delta_j},\varphi_{s_k}^{\delta_k}\right)_A \\ 0 & \left(\varphi_2^{\delta_j},\varphi_2^{\delta_k}\right)_A & \cdots & 2\left(\varphi_2^{\delta_j},\varphi_{s_k}^{\delta_k}\right)_A \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & \left(\varphi_{s_j}^{\delta_j},\varphi_{s_k}^{\delta_k}\right)_A \end{pmatrix}. \tag{19}$$

Finally, partial derivatives of the first part of equation (10) with respect to coefficients $\mathbf{b}_{\delta_j}$ on the same scale δ_j are given by:

$$\frac{\partial\left(\widetilde{\mathbf{u}}_n^{\delta_j}, \widetilde{\mathbf{u}}_n^{\delta_j}\right)_A}{\partial b_1^{\delta_j}\dots\partial b_{s_j}^{\delta_j}} = \mathbf{R}_{A_{\delta_j\delta_j}}\mathbf{a}_{\delta_j}, \tag{20}$$

where $\mathbf{R}_{A_{\delta_j\delta_j}}$ is the well known matrix of the Ritz system:

$$\mathbf{R}_{A_{\delta_j\delta_j}} = 2\begin{pmatrix} \left(\varphi_1^{\delta_j}, \varphi_1^{\delta_j}\right)_A & \left(\varphi_1^{\delta_j}, \varphi_2^{\delta_j}\right)_A & \cdots & \left(\varphi_1^{\delta_j}, \varphi_{s_k}^{\delta_j}\right)_A \\ \left(\varphi_1^{\delta_j}, \varphi_2^{\delta_j}\right)_A & \left(\varphi_2^{\delta_j}, \varphi_2^{\delta_j}\right)_A & \cdots & \left(\varphi_2^{\delta_j}, \varphi_{s_j}^{\delta_j}\right)_A \\ \vdots & \vdots & \ddots & \vdots \\ \left(\varphi_1^{\delta_j}, \varphi_{s_j}^{\delta_j}\right)_A & \left(\varphi_2^{\delta_j}, \varphi_{s_j}^{\delta_j}\right)_A & \cdots & \left(\varphi_{s_j}^{\delta_j}, \varphi_{s_j}^{\delta_j}\right)_A \end{pmatrix}. \tag{21}$$

3. Results and discussion

The main result of this work is the minimization of the quadratic functional with respect to the mentioned subspace division using the common Ritz system and triangular matrices of the Ritz system. Detail reordering of individual equations is beyond the scope of this contribution and also huge and inappropriate for the proceedings. For this reason, just final results are introduced. Using the above mentioned relations, the partial derivatives (15) can be expressed in the following form

$$\begin{aligned}
\frac{\partial L(\mathbf{u}_n)}{\partial b_1^{\delta_1}\dots\partial b_{s_1}^{\delta_1}} &= \mathbf{R}_{A_{\delta_1\delta_1}}\mathbf{a}_{\delta_1} + 2\mathbf{R^U}_{A_{\delta_1\delta_2}}\mathbf{a}_{\delta_2} + \ldots + 2\mathbf{R^U}_{A_{\delta_1\delta_i}}, \\
\frac{\partial L(\mathbf{u}_n)}{\partial b_1^{\delta_2}\dots\partial b_{s_2}^{\delta_2}} &= 2\mathbf{R^L}_{A_{\delta_1\delta_2}}\mathbf{a}_{\delta_1} + \mathbf{R}_{A_{\delta_2\delta_2}}\mathbf{a}_{\delta_2} + 2\mathbf{R^U}_{A_{\delta_2\delta_3}}\mathbf{a}_{\delta_3} + \ldots + 2\mathbf{R^U}_{A_{\delta_2\delta_i}}\mathbf{a}_{\delta_i}, \\
\frac{\partial L(\mathbf{u}_n)}{\partial b_1^{\delta_3}\dots\partial b_{s_3}^{\delta_3}} &= 2\mathbf{R^L}_{A_{\delta_1\delta_3}}\mathbf{a}_{\delta_1} + 2\mathbf{R^L}_{A_{\delta_2\delta_3}}\mathbf{a}_{\delta_2} + \mathbf{R}_{A_{\delta_3\delta_3}}\mathbf{a}_{\delta_3} + 2\mathbf{R^U}_{A_{\delta_3\delta_4}}\mathbf{a}_{\delta_4} + \ldots \\
&\qquad \ldots + 2\mathbf{R^U}_{A_{\delta_3\delta_i}}\mathbf{a}_{\delta_i}, \\
&\vdots \\
\frac{\partial L(\mathbf{u}_n)}{\partial b_1^{\delta_i}\dots\partial b_{s_i}^{\delta_i}} &= 2\mathbf{R^L}_{A_{\delta_1\delta_i}}\mathbf{a}_{\delta_1} + 2\mathbf{R^L}_{A_{\delta_2\delta_i}}\mathbf{a}_{\delta_2} + \ldots + \mathbf{R}_{A_{\delta_i\delta_i}}\mathbf{a}_{\delta_i}.
\end{aligned} \tag{22}$$

This complex system can be rewritten in more readable form (T means vector transposition):

$$S_A(\mathbf{u}_n) = \mathbf{R}_{A_{\delta_j\delta_j}}\mathbf{a}_{\delta_j} + 2\sum_{k=j+1}^{i}\mathbf{R^U}_{A_{\delta_j\delta_k}}\mathbf{a}_{\delta_k} + 2\sum_{k=1}^{j-1}\mathbf{R^L}_{A_{\delta_k\delta_j}}\mathbf{a}_{\delta_k}, \tag{23}$$

where $S_A(\mathbf{u}_n) = \left(\frac{\partial L(\mathbf{u}_n)}{\partial b_1^{\delta_j}}, \frac{\partial L(\mathbf{u}_n)}{\partial b_2^{\delta_j}}, \dots, \frac{\partial L(\mathbf{u}_n)}{\partial b_{s_j}^{\delta_j}} \right)^T$. In this way, we obtained a numerical approximation of the first part of equation (10).

Minimization of functional equation (10) is done by the relation:

$$\frac{\partial L(\mathbf{u}_n)}{\partial b_l^{\delta_k}} \Big|_{\mathbf{b}_{\delta_1}=\mathbf{a}_{\delta_1},\dots,\mathbf{b}_{\delta_i}=\mathbf{a}_{\delta_i}} = 0 \quad \text{for } k = 1, \dots, i;\ l = 1, \dots, s_k. \tag{24}$$

Thus, the approximate solution $\mathbf{u}_n$ can be computed by evaluation of $\mathbf{a}_{\delta_j}$ in the following equation:

$$S_A(\mathbf{u}_n) = \left(\left(f, \varphi_1^{\delta_j}\right), \left(f, \varphi_2^{\delta_j}\right), \dots, \left(f, \varphi_{s_j}^{\delta_j}\right) \right)^T, \tag{25}$$

where $S_A(\mathbf{u}_n)$ is given by (23).

By analogy, the solution of equation (6) with applying of $S_A(\mathbf{u}_n)$ derivation can be rewritten in this form:

$$S_A(\mathbf{u}_n) - S_B(\mathbf{u}_n) - S_C(\mathbf{u}_n) = 2 \left(\left(f_c, \varphi_1^{\delta_j}\right), \left(f_c, \varphi_2^{\delta_j}\right), \dots, \left(f_c, \varphi_{s_j}^{\delta_j}\right) \right)^T, \tag{26}$$

where we use the following differential operators

$$\begin{aligned} S_A(\mathbf{u}) &= \rho \frac{\partial^2}{\partial t^2} \mathbf{u}, \\ S_B(\mathbf{u}) &= \left(\nabla \mathbf{c_{EG}} + (w - w_{ext}) \nabla \mathbf{c_{K_{bw}}} + (T - T_{ext}) \nabla \mathbf{c_{K_{bT}}} \right) \nabla \mathbf{u}, \\ S_C(\mathbf{u}) &= \nabla \mathbf{c}_{\lambda_{\mathbf{w},\mathbf{T}}} \nabla \frac{\partial}{\partial t} \mathbf{u}, \end{aligned}$$

and $f_c = \mathbf{F} - \left(\mathbf{C_w} \cdot w + \mathbf{C_{w^2}} \cdot w^2 + \mathbf{C_T} \cdot T + \mathbf{C_{T^2}} \cdot T^2 + \mathbf{C_{wT}} \cdot w \cdot T + \mathbf{C} \right)$.

If finite elements with linear basis functions are used, then system equation (26) is uniquely solvable. Solution is realized in i consequent steps. In the first step, equation (26) is formed for $j = 1$. Since the results of higher scales are unknown (in Ritz or modified Ritz system), the solution on higher scales in individual nodes is expressed by value of $\mathbf{a}_{\delta_1}$ or other appropriate lower scales. From this step, we obtain suitable extrapolation in some nodes on higher scale(s) which include the region of element on this solved scale. In the next step, we calculate the same equation, but on the following higher scale. At the same time, some nodes on this scale are strictly derived from previous step. This idea is repeated until the highest scale is reached.

4. Conclusions

Advantage of this type of solution is the null requirement of results enumeration on lower scales. Simultaneously, just results on last scale can be enumerated, whereas results on the scale are derived from lower scales. The solution can be simplified by this statement:

If a position of a node for higher scale is in some region of a lower scale mesh, then $\mathbf{a}_{\delta_{\mathbf{j-k}}}$ can be mapped directly to results on higher scale a_{δ_j}, $\left(a_{\delta_{j-k}} \rightarrow a_{\delta_j}\right)$. Let each node of element on some higher scale E^{δ_j} coincides with node in element on lower scale $E^{\delta_{j-k}}$. All contributions of higher scales $\mathbf{a}_{\delta_{\mathbf{j}>\mathbf{1}}}$ to subgrid can be derived from consequent mapping of $\mathbf{a}_{\delta_\mathbf{1}}, \mathbf{a}_{\delta_\mathbf{2}}, \ldots, \mathbf{a}_{\delta_{\mathbf{j-1}}}$ to required $\mathbf{a}_{\delta_\mathbf{j}}$.

5. Summary

The weak solution of coupled stress-strain task with moisture/temperature dependency of material model was obtained in this project. The subgrid upscaling homogenization method for large scale hierarchical structure, which is typical for wood structure, was used. Modified Ritz-Galerkin method for simple solution was derived. The coefficient form of the PDE suitable for nowadays numerical solvers was used ([5]). Suggested weak solution offers unique and relatively accurate solution of large scale problems with dependency on low scale. The solution is very general and slight modification of the approach allows solution of a lot of common tasks in the field of bio-mechanics.

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