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NUMERICAL APPROXIMATION OF DENSITY DEPENDENT DIFFUSION IN AGE-STRUCTURED POPULATION DYNAMICS

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Abstract

We study a numerical method for the diffusion of an age-structured population in a spatial environment. We extend the method proposed in [2] for linear diffusion problem, to the nonlinear case, where the diffusion coefficients depend on the total population. We integrate separately the age and time variables by finite differences and we discretize the space variable by finite elements. We provide stability and convergence results and we illustrate our approach with some numerical result.

1. Introduction

The mathematical problem describing the spatial dispersal of an age-structured population in a region Ω consists in a reaction-diffusion equation for the population density, together with a given initial condition, an integral condition at age $a = 0$, giving the newborns rate, and boundary conditions on $\partial\Omega$ depending on specific features of the population and of the environment. An almost complete review of the results concerning existence, uniqueness and asymptotic behaviour of the solution of age-structured diffusion models can be found in the book by A. Okubo and S. A. Levin ([10], Sec.10.8).

The earliest age-structured models did not include a spatial distribution of the population density (see e.g. [5]). Under the hypothesis of space homogeneity, the problem reduces to a pure first order hyperbolic partial differential equation, which was naturally solved by integration along characteristics in age and time (see for instance [6, 7, 9]). This integration method entails the use of the same discretization step in age and time. However, the presence of different time scales in the dynamics (which is typically the case when space is involved) suggests the use of different steps in the discretization of time and age. This was the approach followed by A. de Roos in [3], and B. Ayati *et al.* in [1], where an approximation space in age is built by discontinuous piecewise polynomials moving along characteristic lines. In [2] a new approach was introduced for the linear diffusion case, where the age and time variables are decoupled and discretized separately by finite differences, while the space variable is discretized by finite elements. The problem is advanced in time by semi-implicit scheme, while a parabolic problem in age and space is solved within the single time step.

In plenty of application of practical interest, the diffusion coefficient depends on the total population itself, and the associated problem is nonlinear (see, e.g. [8]). In this paper we present the extension of the method introduced in [2] to the case of nonlinear diffusion coefficients.

The paper is organized as follows. In Section 2 we describe the nonlinear model we are dealing with. In Sections 3 through 5 we present the finite dimensional approximation, and in Section 6 we outline the algorithmic aspects of the procedure. In Section 7 we state the stability and convergence analysis of the method, and in Section 8 we present some numerical results to illustrate our method.

2. Setting of the problem

We consider an age-structured population diffusing in a bounded spatial domain $\Omega \subset \mathbf{R}^d$, $d = 1, 2, 3$, with boundary $\partial\Omega \in C^2$. We denote by $\rho(t, a, x)$ the density per unit space and age of the population at time t, where $a \in [0, a_\dagger]$ and $x \in \Omega$. The population at time t in a given location $x \in \Omega$, and the total population at time t are thus given by

$$p(t,x) = \int_0^{a_\dagger} \rho(t,a,x)\, da, \qquad\qquad P(t) = \int_\Omega p(t,x)\, dx. \tag{1}$$

We assume the diffusion process to be density- and age-driven, namely the diffusion coefficient in (t, x) depends on the population $p(t, x)$ at the corresponding location in space and time, and on the age of the individuals.

Given a final time $T > 0$, the population density $\rho(t, a, x) \in C(0, T; L^2(0, a_\dagger; H^1(\Omega)))$ satisfies the nonlinear model problem

$$\begin{aligned}
&\rho_t + \rho_a - \mathrm{div}\,(k(p(t,x),a)\,\nabla\rho) = f(t,x) - \mu(a)\,\rho && \text{in } (0,T)\times(0,a_\dagger)\times\Omega\,,\\
&\rho(0,a,x) = \rho_0(a,x) && \text{in } (0,a_\dagger)\times\Omega\,,\\
&\rho(t,0,x) = \int_0^{a_\dagger} \beta(a)\,\rho(t,a,x)\; da && \text{in } (0,T)\times\Omega\,,\\
&k(p(t,x))\,\mathbf{n}\cdot\nabla p = 0 && \text{on } (0,T)\times(0,a_\dagger)\times\partial\Omega\,,
\end{aligned} \tag{2}$$

where $p(t, x)$ is given in (1), the operators $\mathrm{div}(\cdot)$ and $\nabla(\cdot)$ are the standard divergence and gradient operators in Ω, and $\vec{n}$ is the unit vector normal to $\partial\Omega$ pointing outwards.

The coefficients $\mu(a)$ and $\beta(a)$ represent the age-specific mortality and the age-specific fertility, respectively, which are supposed to be non-negative functions of age only. In (2), ρ_0 is the given non-negative initial age distribution, while the integral condition is the so-called renewal condition, providing the newborns rate. Finally, we consider an isolated environment by choosing a zero-flux boundary condition, which reflects the absence of both immigration and emigration, but other boundary conditions can be considered as well (for instance, an homogeneous Dirichlet boundary condition would model an hostile habitat at the boundary of Ω). We refer to [10] for issues concerning existence and uniqueness of a nonnegative solution of (2).

We impose standard conditions on the diffusion coefficient to ensure ellipticity of the associated bilinear form.

$$k \in L^\infty(\mathbb{R}^+ \times (0, a_\dagger)), \quad 0 < k_0 \le k(p, a) \le k_+, \tag{3}$$

and we assume that the age-specific fertility $\beta(\cdot)$ is measurable and essentialy bounded, namely there exists a constant β_+ such that

$$0 \le \beta(a) \le \beta_+. \tag{4}$$

Finally, we assume the age-specific mortality $\mu(\cdot)$ to be a measurable function, satisfying

$$\int_0^{a_\dagger} \mu(\sigma) d\sigma = +\infty, \tag{5}$$

in order to guarantee that the probability for an individual to survive at age a, which is defined as

$$\pi(a) = \exp\left(- \int_0^a \mu(\sigma) d\sigma \right), \tag{6}$$

vanishes at the maximum age $a_\dagger$. The numerical issues arising from the unbounded coefficient $\mu(a)$ can be avoided by performing a standard change of variable.

We let $\rho(t, a, x) = \pi(a) u(t, a, x)$, and we reduce ourselves to the problem of finding $u(t, a, x) \in C(0, T; L^2(0, a_\dagger; H^1(\Omega)))$ such that

$$\begin{aligned}
u_t + u_a - \operatorname{div}\left(k(p(t, x), a)\, \nabla u\right) &= f(t, x) && \text{in } (0, T) \times (0, a_\dagger) \times \Omega\,, \\
p(t, x) &= \int_0^{a_\dagger} \pi(a) u(t, a, x)\, da && \text{in } (0, T) \times \Omega\,, \\
u(0, a, x) &= u_0(a, x) && \text{in } (0, a_\dagger) \times \Omega \\
u(t, 0, x) &= \int_0^{a_\dagger} m(a) u(t, a, x)\ da && \text{in } (0, T) \times \Omega\,, \\
k(a, x)\, \mathbf{n} \cdot \nabla u &= 0 && \text{on } (0, T) \times (0, a_\dagger) \times \partial\Omega\,,
\end{aligned} \tag{7}$$

where now $u_0(a, x) = \frac{\rho_0(a,x)}{\pi(a)}$, while $m(a) = \frac{\beta(a)}{\pi(a)}$ is the so called maternity function. Notice that $m \in L^\infty(0, a)$ as for all $a \in (0, a)$ we have $m(a) \le \beta_+$.

We focus here on the numerical treatment of the problem and we assume throughout the paper existence and uniqueness of smooth, nonnegative solutions [10].

3. Time discretization

Let $t^n = n\Delta t$ $(n = 0, 1, \ldots, N_t)$ be a partition of the interval $(0, T)$ into N_t subintervals (for simplicity we consider an uniform discretization, adaptivity in time being beyond the scope of this paper). We denote with $u^n(a,x)$ and $p^n(x)$ the approximations of $u(t^n,a,x)$ and $p(t^n,x)$, respectively, and we advance in time equation (7) by means of a semi-implicit scheme, where both the initial condition in age and the diffusion coefficient are computed at the previous time step. Moving from t^{n-1} to t^n we solve the following parabolic problem in age and space.

Find $u^n \in L^2(0,a_\dagger;H^1(\Omega))$ such that for all $v \in H^1(\Omega)$

$$\begin{aligned}&\frac{d}{da}\,\langle u^n, v\rangle + A(p^{n-1};a;\,u^n,v) + \frac{1}{\Delta t}(u^n,v) = (f,v) + \frac{1}{\Delta t}(u^{n-1},v)\\ &u^n(0,x) = \int_0^{a_\dagger} m(a)\,u^{n-1}(a,x)\;da, \qquad\qquad p^n(x) = \int_0^{a_\dagger} \pi(a)\,u^n(a,x)\;da,\end{aligned} \tag{8}$$

where $\langle\cdot,\cdot\rangle$ denotes the duality pairing between $H^1(\Omega)$ and $H^{-1}(\Omega)$, and where $A(p^{n-1};a;\cdot,\cdot)$ is the bilinear form given by

$$A(p^{n-1};a;\,w,v) = \int_\Omega k(p^{n-1}(x),a)\nabla w\cdot\nabla v\,dx.$$

By standard coercivity arguments one can prove existence and uniqueness for the solution of (8).

Remark 3.1 *The coercivity and the continuity of the bilinear form $A(p^{n-1};a;\cdot,\cdot) + \frac{1}{\Delta t}(\cdot,\cdot)$ are straightforward. Moreover the fact that the maternity function $m \in L^\infty(0,a_\dagger)$ guarantees that $u^n(0,x) \in L^2(\Omega)$ as long as $u^{n-1} \in L^2([0,a_\dagger]\times\Omega)$.*

4. Space discretization

We discretize in space equation (8) by means of finite elements (see [11] for an introduction to finite element methods). Let then $\Omega = \bigcup_{j=1}^N K_j$, where each $K_j = T_{K_j}(E)$ is an element of the triangulation, E is the reference simplex and T_{K_j} is an invertible affine map. The associated finite element space is then

$$V_h = \left\{\;\varphi_h \in C^0(\Omega)\,|\,\varphi_{h|K_j}\circ T_{K_j} \in \mathbb{P}_k(E)\right\},$$

where $\mathbb{P}_k(E)$ is the space of polynomials of degree at most k on E. A semi-discrete problem in space is then obtained by applying a Galerkin procedure to (8) and choosing a finite element basis for V_h. Letting $\{\varphi_j\}_{j=1,..,N_h}$ be the nodal basis of the finite element space V_h, the semi-discrete solution $u_h^n(a,x)$ is given by

$$u_h^n(a,x) = \sum_{j=1}^{N_h} u_j^n(a)\,\varphi_j(x).$$

By denoting with $\mathbf{u}_h^n(a) = (u_1^n(a), \ldots, u_{N_h}^n(a))^T$, since the finite element basis functions depend only on space, we can rewrite problem (8) as

$$
\begin{aligned}
&M\frac{d\mathbf{u}_h^n}{da} + \mathcal{A}^{(n-1)}(a)\,\mathbf{u}_h^n + \frac{1}{\Delta t}M\mathbf{u}_h^n = \mathbf{f}^n + \frac{1}{\Delta t}M\mathbf{u}_h^{n-1},\\
&\mathbf{u}_h^n(0) = \int_0^{a_\dagger} m(a)\,\mathbf{u}_h^{n-1}(a)\;da, \qquad \mathbf{p}_h^n = \int_0^{a_\dagger} \pi(a)\,\mathbf{u}_h^n(a)\;da,
\end{aligned}
\tag{9}
$$

where M is the mass matrix ($M_{ij} = \int_\Omega \varphi_j\varphi_i\,dx$) and $\mathcal{A}^{(n-1)}$ is the stiffness matrix associated to the bilinear form $A(\mathbf{p}_h^{n-1}; a; \cdot,\cdot)$, ($\left[\mathcal{A}^{(n-1)}(a)\right]_{ij} = A(\mathbf{p}_h^{n-1}; a; \varphi_j, \varphi_i)$).

5. Age discretization

We advance in age the differential problem in (9) by means of the θ-method (see [11]). Let then $a^m = m\Delta a$ ($m = 0, 1, \ldots, N_a$) be a partition of the age interval $[0, a_\dagger]$ into N_a subintervals of uniform amplitude. For $j = 1, .., N_h$, we let $u_j^{n,m}$ denote the approximation of $u_j^n(a^m)$, and the approximation to $u(t^n, a^m, x)$ is then given by

$$
u_h^{n,m}(x) = \sum_{j=1}^{N_h} u_j^{n,m}\,\varphi_j(x).
$$

We denote by $\mathbf{u}_h^{n,m} = (u_1^{n,m}, \ldots, u_{N_h}^{n,m})^T$ the unknown vector at time t^n and age a^m, and we advance from age level a^m to a^{m+1} by the θ-method, which reads, for $0 \le \theta \le 1$,

$$
\begin{aligned}
M\frac{\mathbf{u}_h^{n,m} - \mathbf{u}_h^{n,m-1}}{\Delta a} &+ \theta\left(\mathcal{A}_m^{(n-1)}\,\mathbf{u}_h^{n,m} + \frac{1}{\Delta t}M\mathbf{u}_h^{n,m}\right) + \\
&(1-\theta)\left(\mathcal{A}_{m-1}^{(n-1)}\,\mathbf{u}_h^{n,m-1} + \frac{1}{\Delta t}M\mathbf{u}_h^{n,m-1}\right) = \\
&\qquad \theta\left(\mathbf{f}^{n,m} + \frac{1}{\Delta t}M\mathbf{u}_h^{n-1,m}\right) + (1-\theta)\left(\mathbf{f}^{n,m-1} + \frac{1}{\Delta t}M\mathbf{u}_h^{n-1,m-1}\right),
\end{aligned}
\tag{10}
$$

where $\mathcal{A}_m^{(n-1)} = \mathcal{A}^{(n-1)}(a^m)$. If $\theta = 0$ we have the Forward Euler method (fully explicit in age), if $\theta = 1$ we have the Backward Euler method (fully implicit in age), while $\theta = 1/2$ corresponds to the Crank-Nicholson method [11].

Finally, the values of $\mathbf{u}_h^{n,0}$ and $\mathbf{p}_h^n$ will be computed by replacing the integrals in (9) with suitable quadrature rules. In the numerical result section, we us in both cases a second order Simpson quadrature rule over two adjacent intervals.

6. Stability and convergence

Denoting by $\mathbf{U}_h^n = (\mathbf{u}_h^{n,0}, \mathbf{u}_h^{n,1}, \ldots, \mathbf{u}_h^{n,N_a})$ the approximate solution at time $t = t^n$, we define the discrete $L^1(0, a_\dagger; L^2(\Omega))$ norm as

$$
\|\mathbf{U}_h^n\|_{\mathcal{L}^1(0,a_\dagger;L^2(\Omega))} = \sum_{m=0}^{N_a} \Delta a\,\|\mathbf{u}_h^{n,m}\|_0\,,
$$

where $\|\cdot\|_0$ is the standard $L^2(\Omega)$ norm. Under some mild assumption on the exact solution, the following stability and convergence results for the proposed scheme (with $\theta = 1$) hold.

Proposition 6.1 (Stability) *For any $n = 1, \ldots, N_t$, the following estimate holds:*

$$\|\mathbf{U}_h^n\|_{\mathcal{L}^1(0,a_\dagger;L^2(\Omega))} \le \left(1 + e^{a_\dagger \beta_+^2 T}\right) \|\mathbf{U}_h^0\|_{\mathcal{L}^1(0,a_\dagger;L^2(\Omega))},$$

where β_+ is the one in (4). □

Proposition 6.2 (Convergence) *Let $\mathcal{T}_h$ be a regular family of triangulations on Ω. Assume that the solution $u(t,a,x)$ of the continuous problem is such that, for all $t \in (0,T)$, $\frac{\partial u}{\partial a}(t,\cdot,\cdot)$, $\frac{\partial u}{\partial t}(t,\cdot,\cdot) \in L^1(0,a_\dagger;H^1(\Omega))$, and $\frac{\partial^2 u}{\partial a^2}(t,\cdot,\cdot)$, $\frac{\partial^2 u}{\partial t^2}(t,\cdot,\cdot) \in L^1(0,a_\dagger;L^2(\Omega))$. Then, using linear finite elements, the following estimate holds*

$$\begin{aligned}
\|u(t^n,\cdot,\cdot) - \mathbf{U}_h^n\|_{\mathcal{L}^1(0,a_\dagger;L^2(\Omega))} &\le \left\|\mathbf{U}_h^0 - \Pi_h u_0\right\|_{\mathcal{L}^1(0,a_\dagger;L^2(\Omega))} \\
&+ Ch\, \|u(t^n,\cdot,\cdot)\|_{\mathcal{L}^1(0,a_\dagger;H^1(\Omega))} + Ch \int_0^{t^n} \left\|\frac{\partial u}{\partial t}(t,\cdot,\cdot)\right\|_{\mathcal{L}^1(0,a_\dagger;H^1(\Omega))} dt \\
&+ Ch \sum_{p=0}^{n} \Delta t \left\|\frac{\partial u}{\partial a}(t^p,\cdot,\cdot)\right\|_{L^1(0,a_\dagger;H^1(\Omega))} + C\,\Delta t \int_0^{t^n} \left\|\frac{\partial^2 u}{\partial t^2}(t,\cdot,\cdot)\right\|_{\mathcal{L}^1(0,a_\dagger;L^2(\Omega))} dt \\
&+ C\,\Delta a \sum_{p=0}^{n} \Delta t \left\|\frac{\partial^2 u}{\partial a^2}(t^p,\cdot,\cdot)\right\|_{L^1(0,a_\dagger;L^2(\Omega))},
\end{aligned} \tag{11}$$

where the constant $C > 0$ is independent of h, Δa, and Δt. □

Proofs of the above propositions follow from a generalization of the results in [2], and will be detailed in a forthcoming paper [4].

7. Algorithm

Given $\mathbf{u}_h^{0,m}$ $(m = 1, \ldots, N_a)$, and $\mathbf{p}_h^0$, for $n = 1, \ldots, N_t$:

1. Compute the initial value $\mathbf{u}_h^{n,0}$ from the previous time step via a Simpson quadrature rule over two adjacent age intervals

$$\mathbf{u}_h^{n,0} = \sum_{l=1}^{N_a/2} \frac{\Delta a}{6} \left[m(a^{2(l-1)})\, \mathbf{u}_h^{n-1,2(l-1)} + 4\, m(a^{2l-1})\, \mathbf{u}_h^{n-1,2l-1} + m(a^{2l})\, \mathbf{u}_h^{n-1,2l} \right].$$

2. For $m = 1, \ldots, N_a$

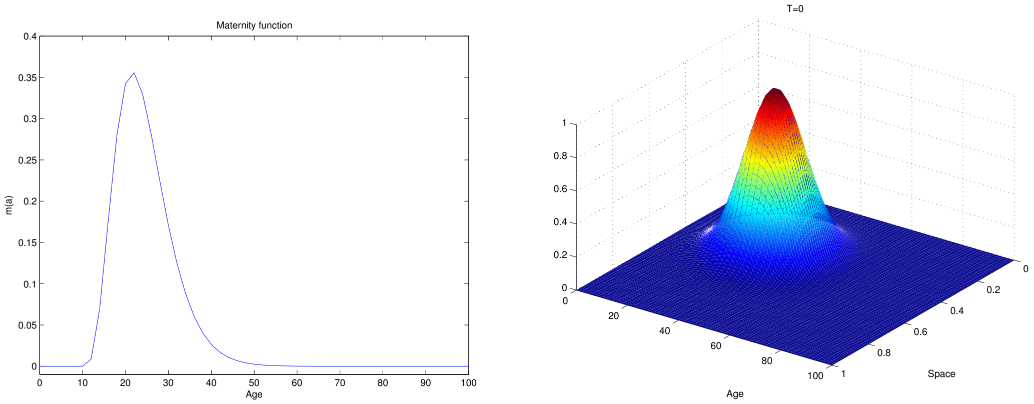

Figure 1: Maternity function (left) and age-space initial profile (right).

(a) Assemble the stiffness matrix $\mathcal{A}_m^{(n)}$ from the population at previous time step

$$\left[\mathcal{A}_m^{(n)}\right]_{ij} = A(\mathbf{p}_h^{n-1} : a^m; \varphi_j, \varphi_i)\,,$$

(b) solve

$$\begin{aligned}&\left[(\Delta t + \theta\Delta a)M + \theta\Delta t\Delta a\,\mathcal{A}_m^{(n)}\right]\mathbf{u}_h^{n,m}\\&= \theta\Delta a\,M\,\mathbf{u}_h^{n-1,m} + \left[(\Delta t - (1-\theta)\Delta a)\,M - (1-\theta)\Delta t\Delta a\,\mathcal{A}_{m-1}^{(n)}\right]\mathbf{u}_h^{n,m-1}\\&+ (1-\theta)\Delta a\,M\,\mathbf{u}_h^{n-1,m-1} + \Delta t\Delta a\Big[\theta\,\mathbf{f}^{n,m} + (1-\theta)\,\mathbf{f}^{n,m-1}\Big]\,.\end{aligned}$$

3. Update the total population $\mathbf{p}_h^n$ via a Simpson quadrature rule over two adjacent age intervals

$$\mathbf{p}_h^n = \sum_{l=1}^{N_a/2} \frac{\Delta a}{6}\left[\pi(a^{2(l-1)})\,\mathbf{u}_h^{n,2(l-1)} + 4\,\pi(a^{2l-1})\,\mathbf{u}_h^{n,2l-1} + m(a^{2l})\,\mathbf{u}_h^{n,2l}\right].$$

8. Numerical results

We present in this section some numerical results to show the effectivity of the method. The spatial domain is $\Omega = (0,1)$, the age interval is $[0, 100]$, and we choose as maximal time $T = 10$. The computational domain is discretized by a uniform mesh in space, time and age, and we choose $\theta = 1$. The numerical simulations are run on a self developed code in Matlab® 7.8.

We consider a non-symmetric initial distribution of population (with respect to both space and age) given by

$$u_0(x,a) = \mathrm{e}^{-\left(\frac{(a-30)^2}{200} + 100(x-0.4)^2\right)},$$

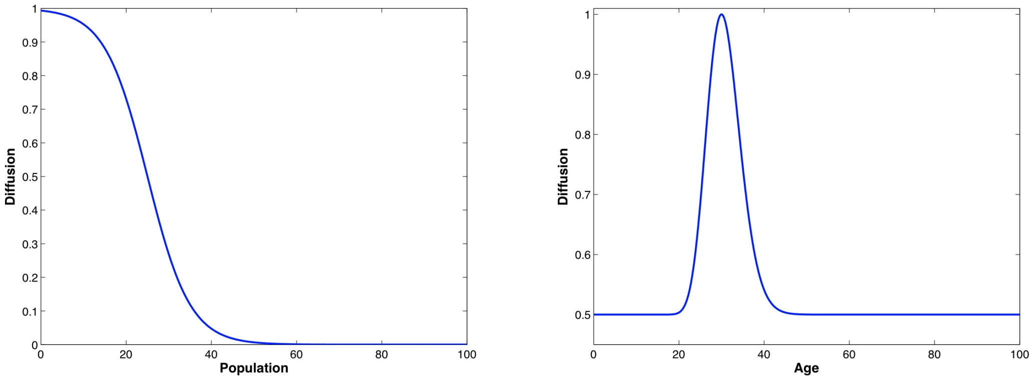

Figure 2: Diffusion coefficients: $k_p(p)$ (left) and $k_a(a)$ (right).

and we choose the mortality and fertility function as

$$\mu(a) = \frac{1}{a_\dagger - a}, \qquad \beta(a) = \begin{cases} 0 & \text{if } a \le a_1 \\ \dfrac{\beta(a-a_1)^{\alpha-1}\mathrm{e}^{-\frac{(a-a_1)}{\vartheta}}}{\vartheta^\alpha \Gamma(\alpha)} & \text{if } a_1 < a < a_2 \\ 0 & \text{if } a \ge a_2, \end{cases}$$

where we set $a_1 = 17$, $a_2 = 70$, $\beta = 7$, $\alpha = 5$, and $\vartheta = 3$. We plot in Figure 1 the resulting maternity function and the initial profile of the problem.

We consider a diffusion coefficient $k(p(t,x),a) = k_p(p) \times k_a(a)$, where we assume $k_p(p)$ to be a monotonic function of the total population $p(t,x)$. The rationale behind this choice is that the population is more keen to move in areas where a lower level of individuals is present, but a different behavior can be easily implemented. We choose in the tests

$$k_p(p) = 1 - \frac{1}{1 + \exp\left(-\left(\frac{p}{5} - 5\right)\right)} \qquad k_a(a) = 0.5 + 0.5 \times \exp\left(-\frac{(a-30)^2}{a}\right),$$

that we plot in Figure 2. With this choice of $k_a(a)$, youngster and old individuals are less mobile.
We investigate numerically the spatial convergence of the method. We consider diffusion coefficients depending on both population and age ($k = k_p \times k_a$), and population only ($k = k_p$): we plot in Figure 3 the corresponding diffusion coefficients in space and age at the initial time $t = 0$. We analyze the relative error $\frac{\|u(t^n,\cdot,\cdot) - U_h^n\|}{\|u(t^n,\cdot,\cdot)\|}$ in the discrete $\mathcal{L}^1(0, a_\dagger; L^2(\Omega))$ norm, with respect to a reference solution computed using a very fine grid in both age and time with $\Delta a = 2\Delta t = 0.1$ and $h = 1/1000$. In Figure 4 we show the work precision in h, for a uniform grid in age and time with $\Delta a = 2\Delta t = 0.2$ for both the case of a density dependent diffusion (left) and density and age dependent diffusion (right). Convergence appears to be robust with respect to the diffusion coefficients. In Figure 5 we plot, for $k = k_p \times k_a$, the age profile at $x = 0.4$ for different times, and the age-space contours at time $T = 5$.

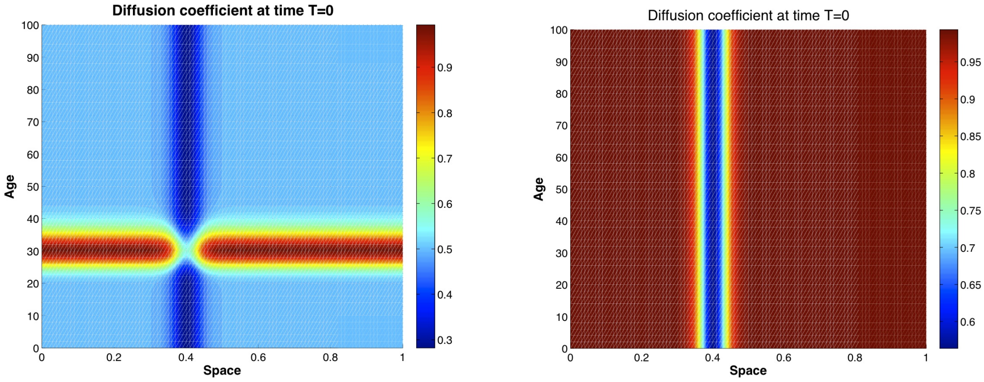

Figure 3: Diffusion coefficients at time $t = 0$. Left: $k = k_p \times k_a$. Right: $k = k_p$.

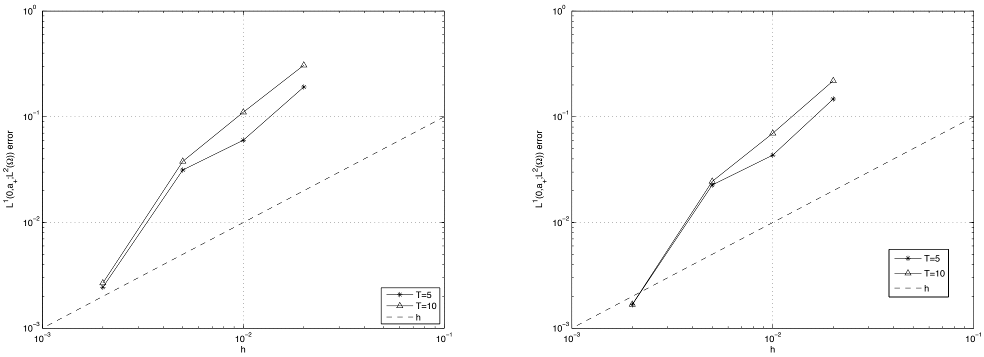

Figure 4: Convergence in $\mathcal{L}^1(0, a_\dagger; L^2(\Omega))$ norm: $k = k_p \times k_a$ (left) and $k = k_p$ (right).

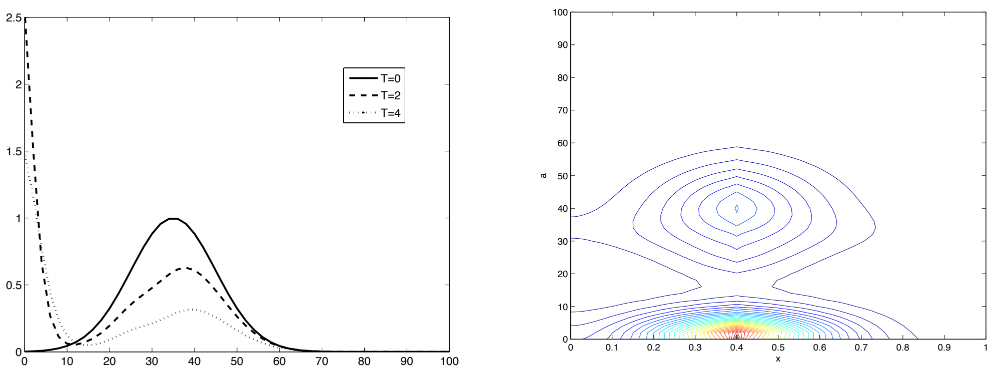

Figure 5: Age profile at $x = 0.4$ (left) and age-space profile at time $T = 5$ (right): $k = k_p \times k_a$.

9. Conclusions

We proposed a Galerkin type method for the numerical approximation of a density dependent diffusion dynamics of an age-structured population. The method is based on a finite elements discretization in space, on a semi-implicit discretization in time, and on the θ-method in age. The separate discretization of time and age, naturally allows for separate adaptivity, which can be necessary when dealing with practical ecological problems. Numerical results showed the effectiveness of the method, that will be analyzed in a more comprehensive way in a forthcoming paper [4].

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