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Algorithms. 36. SNEDECOR. An algorithm for Fisher-Snedecor’s $F$-test without application of critical values


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36. SNEDECOR

AN ALGORITHM FOR FISHER-SNEDECOR'S F-TEST WITHOUT APPLICATION OF CRITICAL VALUES

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The algorithm suggested in this paper computes the probability that Fisher-Snedecor's test statistic will exceed the value $F$ actually observed, i.e.

$$
\alpha_{m,n}(F) = \frac{\left(\frac{m}{n}\right)^{m/2}}{B\left(\frac{m}{2}, \frac{n}{2}\right)} \int_{F}^{\infty} y^{m/2-1} \left(1 + \frac{m}{n} y\right)^{-\frac{(m+n)}{2}} dy,
$$

where $m$, $n$ is the pair of numbers of degrees of freedom. This probability may be hence called the significance degree, similarly as in the case of Student's $t$-statistic treated in our previous paper [1]. Since the latter represents a special case of the present problem (with $m = 1$, $F = t^2$), the features of the algorithm and further remarks made in [1] apply here, too (except the distinction between one-sided and two-sided tests) and will not be repeated.

Analogously as in [1], the relation

$$
\alpha_{m,n}(F) = A_{m,n}(x)
$$

with

$$
x = \left(1 + \frac{m}{n} F\right)^{-1},
$$

$$
A_{m,n}(x) = \frac{1}{B\left(\frac{m}{2}, \frac{n}{2}\right)} \int_{0}^{x} y^{m/2-1}(1 - y)^{n/2-1} dy
$$
holds and the algorithm is based on the following recurrence relations and initial conditions

\[
A_{m,n}(x) = A_{m,n-2}(x) - \frac{\Gamma\left(\frac{m + n}{2} - 1\right)}{\Gamma\left(\frac{m}{2}\right) \Gamma\left(\frac{n}{2}\right)} \chi^{n/2-1}(1 - x)^{m/2-1}
\]

\[\text{for } m > 0, n > 2\],

\[
A_{m,n}(x) = A_{m-2,n}(x) + \frac{\Gamma\left(\frac{m + n}{2} - 1\right)}{\Gamma\left(\frac{m}{2}\right) \Gamma\left(\frac{n}{2}\right)} \chi^{n/2}(1 - x)^{m/2-1}
\]

\[\text{for } m > 2, n > 0\],

\[
A_{1,1}(x) = \frac{2}{\pi} \arcsin \sqrt{x} , \quad A_{2,1}(x) = \sqrt{x} , \quad A_{2,2}(x) = x .
\]

The last relation follows from the definition, the remainder can be proved by differentiation (the relations (5) and (6) being equivalent due to (8)).

If the statistic \( F \) has its usual form

\[
F = \left(\frac{y}{m}\right) / \left(\frac{z}{n}\right)
\]

where \( y, z \) are certain sampling characteristics, then the transform (3), which is used instead of the statistic \( F \), may be evaluated directly from \( y, z \) in the form

\[
x = z/(y + z).
\]

real procedure SNEDECOR(x, m, n); value x; real x; integer m, n;
begin
real a, b, c, d, e, f; integer i;
procedure G;
begin c := c \times x;
    for f := e step 2 until i do
        begin a := a + b; d := b \times c; b := d/f; c := c + 2 \times x end
end G;
procedure H;
begin x := 1 - x; G; b := -d; c := i + 1; e := 3; i := n;
x := 1 - x; G
end H;
procedure P; begin b := sqrt(x); c := 1; H end;
procedure \( Q \); begin \( b := 1; c := n; G; a := a \times (1 - x) \uparrow (n \div 2) \) end;
if \( n > (n \div 2) \times 2 \) then
begin \( i := m; \)
if \( m > (m \div 2) \times 2 \) then
begin
\( a := 0.63661977 \times \arcsin(\sqrt{x}); \)
\( b := 0.63661977 \times \sqrt{(1 - x) \times x}; \)
\( d := b; e := 3; H \)
end
else begin \( a := 0; e := 2; P \) end
end
else begin \( a := 0; e := 2; \)
if \( m > (m \div 2) \times 2 \) then
begin \( i := n; n := m; m := i; x := 1 - x; P; x := 1 - x; \)
\( n := m; m := i; a := 1 - a \)
end
else if \( m > n \) then
begin \( i := n; n := m; Q; n := i; a := 1 - a \) end
else
begin \( i := m; x := 1 - x; Q; x := 1 - x \) end
end;
\( SNEDECOR := a \)
end \( SNEDECOR \)

The result is obtained with the accuracy of at least about 5 decimal places. We give some check values:

\[
\begin{align*}
SNEDECOR (0.3, 1, 1) &= 0.36901 \\
SNEDECOR (0.25, 1, 10) &= 0.00027 \\
SNEDECOR (0.75, 1, 19) &= 0.02099 \\
SNEDECOR (0.5, 4, 10) &= 0.10937 \\
SNEDECOR (0.4, 10, 6) &= 0.58010 \\
SNEDECOR (0.7, 3, 8) &= 0.38890 \\
SNEDECOR (0.6, 4, 9) &= 0.28109 \\
SNEDECOR (0.1, 3, 1) &= 0.39582 \\
SNEDECOR (0.2, 5, 11) &= 0.00143 \\
SNEDECOR (0.3, 7, 3) &= 0.55292 \\
SNEDECOR (0.75, 10, 1) &= 0.99973
\end{align*}
\]
The program has been tested in the symbolic language MOST [3] and implemented in the Biophysical Institute, Faculty of General Medicine, Charles University Prague for the computer ODRA 1013 [4].