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A correction to “Cayley’s problem”, AM 35 (1990) No. 2, 140-146

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Notation as well as numeration of equations follow those of the paper.

In the paper there is an invalid assertion. Solutions of the functional equation

$$\lambda\, g(z) = g\left(\frac{2z^3 + 1}{3z^2}\right) \tag{6}$$

with different values of λ are in question. Solutions are obtained around two fixed points of the Newton iteration function $R(z) = (2z^3 + 1)/3z^2$. The fixed points are $a = \infty$ with its eigenvalue $\lambda = \frac{3}{2}$ and the superattractor $b = 1$, with $\lambda = 2$, the order or the superattractor. The function $g_{3/2}$ is regular at infinity, defined with its series outside the unit circle, and can be continued analitically also inside it, though not independently of the path. The function g_2 however has a logarithmic singularity and has A(1) (immediate basin of attraction) as its natural domain of definition. Therefore $(g_2(z))^\alpha$ and $g_{3/2}(z)$ have different domains and cannot be related multiplicatively as claimed in (14).

However, comparing numerical results on the interval (1, 1·6) we get perfect agreement with the choice of the constant as in (15). What is the explanation?

Instead of the constant M actually we have a Fourier series, where M is just the initial constant term, and the next terms are decreasing rapidly in absolute value, the rate being approximately geometric with the quotient of an order 10^{-6}.

Define the analytic function

$$h(t) = g_{3/2}(f_2(\mathrm{e}^t))\, \mathrm{e}^{-\alpha t} \tag{21}$$

which can easily be proved to be periodic with period ln 2 and can therefore be developed into a Fourier series

$$h(t) = \sum_{k=-\infty}^{\infty} c_k \exp\left(\frac{2k\pi \mathrm{i}}{\ln 2}\right) \tag{22}$$

The constant term can be easily determined as the average over the periodic interval and corresponds to the constant M. The next terms however are so small that numerical errors are of the same order of magnitude as oscillations of $h(t)$.

As $h(t)$ takes real values on the real line, we have $c_{-k} = c_k^*$. Take now a real constant $\beta \in [-\pi/2, \pi/2]$ and apply (22) with $t = r + i\beta$:

$$h(r + i\beta) = \sum_{-\infty}^{\infty} c_k \exp\left(\frac{-2k\pi\beta}{\ln 2}\right) \exp\left(\frac{2k\pi i r}{\ln 2}\right) \tag{23}$$

For positive β, the coefficients with positive indices get a small exponential factor, those with negative indices multiply by a large factor, in the extreme case $\beta = \pi/2$ the factor is a power of $e^{\pi^2/\ln 2} \doteq 1{\cdot}5E + 06$. However a large β is bad numerically. Thus we could only obtain

$$c_0 = {\cdot}71515019\,, \quad c_1 = c_{-1}^* = ({\cdot}8 - 1{\cdot}4)\,E - 08 \tag{24}$$

and the determination of further coefficients remains an interesting problem for numerical analysis.