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# STRONG MAXIMUM PRINCIPLE FOR WEAKLY NONLINEAR PARABOLIC EQUATIONS

J. KADLEC and R. VÝBORNÝ, Praha

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Let $O$ be a region in $E_{n+1}$. Let us assume the functions $a_{ij}(x,t)$ are defined, bounded and measurable on $O$ ($x$ stands for $(x_1,\dots,x_n)$, $t$ is the "time" variable). Let us assume that the quadratic form $\sum_{i,j=1}^{n} a_{ij}(x,t)\,\xi_i\,\xi_j$ is positive definite, i.e., there is a positive constant $\nu$ such that the inequality

$$\sum_{i,j=1}^{n} a_{ij}(x,t)\,\xi_i\,\xi_j \geq \nu \sum_{i=1}^{n} \xi_i^2$$

holds almost everywhere on $O$ and for all real vectors $(\xi_1, \xi_2, \dots, \xi_n)$. Let $u$ be a weak solution of the equation

$$\frac{\partial u}{\partial t} - \sum_{i,j=1}^{n} \frac{\partial}{\partial x_i} a_{ij}(x,t) \frac{\partial}{\partial x_j} u(x,t) + f(u, u_{x_i}) = 0$$

such that $u$ belongs to $W_2^{(1,\frac{1}{2})}(Q)$ for every compact $Q \subset O$. Let us suppose further that the function $f(u, u_{x_i})$ is measurable and that the inequality

$$f(u, u_{x_i}) \geq M\Big(\sum_{i=1}^{n}\Big(\frac{\partial u}{\partial x_i}\Big)^2\Big)^{\frac{1}{2}}$$

is satisfied, where $M$ is a constant. We impose the following continuity hypothesis on the function $u$: For any region $\Omega \subset E_n$ and any numbers $a, b$ $(a < b)$ such that $\bar{\Omega} \times \langle a, b \rangle \subset O$ the limit formula

$$\lim_{t \to t_0} \int_{\Omega} |u(x,t) - u(x,t_0)|^2\, dx = 0$$

is satisfied for all $t_0 \in (a, b)$.

We say that the point $(x_m, t_m)$ can be connected with the point $(x_0, t_0)$ by an admissible polygonal path, if there exists a finite sequence of points $(x_i, t_i)$ ($i = 0,1,2,\dots,m$) such that 1) $t_m < \dots < t_{i+1} < t_i < \dots < t_0$, 2) the line segment connecting the points $(x_{i+1}, t_{i+1})$ and $(x_i, t_i)$ lies in $\mathcal{O}$. Let us denote by $S(x_0, t_0)$ the set of all points which can be connected with $(x_0, t_0)$ by an admissible polygonal path.

The function $u$ is said to have a maximum (in $S(x_0, t_0)$) near the point $(x_0, t_0) \in \mathcal{O}$, provided that for any $n$-dimensional ball $K$ and every $\delta > 0$ such that $(x_0, t_0) \in Q_\delta = K \times \langle t_0 - \delta, t_0 \rangle \subset \mathcal{O}$ the inequality

$$\operatorname*{supess}_{(x,t) \in Q_\delta} u(x,t) \geq \operatorname*{supess}_{(x,t) \in S(x_0,t_0)} u(x,t)$$

holds.

The aim of this paper is to announce the two following statements.

1) The function $u$ is bounded from above on every compact subset of $\mathcal{O}$.

2) If $u$ has a maximum $\mu$ near the point $(x_0, t_0) \in \mathcal{O}$, then $u(x, t) = \mu$ almost everywhere in $S(x_0, t_0)$.