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(Preliminary communication)

Commentationes Mathematicae Universitatis Carolinae, Vol. 6 (1965), No. 1, 19--20

Persistent URL: <http://dml.cz/dmlcz/104990>

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STRONG MAXIMUM PRINCIPLE FOR WEAKLY NONLINEAR PARABOLIC
EQUATIONS

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Preliminary communication.

Let O be a region in E_{n+1} . Let us assume the functions $a_{ij}(x, t)$ are defined, bounded and measurable on O (x stands for (x_1, \dots, x_n) , t is the "time" variable). Let us assume that the quadratic form $\sum_{i,j=1}^n a_{ij}(x, t) \xi_i \xi_j$ is positive definite, i.e., there is a positive constant ν such that the inequality

$$\sum_{i,j=1}^n a_{ij}(x, t) \xi_i \xi_j \geq \nu \sum_{i=1}^n \xi_i^2$$

holds almost everywhere on O and for all real vectors $(\xi_1, \xi_2, \dots, \xi_n)$. Let u be a weak solution of the equation

$$\frac{\partial u}{\partial t} - \sum_{i,j=1}^n \frac{\partial}{\partial x_i} a_{ij}(x, t) \frac{\partial}{\partial x_j} u(x, t) + f(u, u_{x_i}) = 0$$

such that u belongs to $W_2^{(1, \frac{1}{2})}(Q)$ for every compact $Q \subset O$. Let us suppose further that the function $f(u, u_{x_i})$ is measurable and that the inequality

$$f(u, u_{x_i}) \geq M \left(\sum_{i=1}^n \left(\frac{\partial u}{\partial x_i} \right)^2 \right)^{\frac{1}{2}}$$

is satisfied, where M is a constant. We impose the following continuity hypothesis on the function u : For any region

$\Omega \subset E_n$ and any numbers a, b ($a < b$) such that

$\bar{\Omega} \times \langle a, b \rangle \subset O$ the limit formula

$$\lim_{t \rightarrow t_0} \int_{\Omega} |u(x, t) - u(x, t_0)|^2 dx = 0$$

is satisfied for all $t_0 \in (a, b)$.

We say that the point (x_m, t_m) can be connected with the point (x_0, t_0) by an admissible polygonal path, if there exists a finite sequence of points (x_i, t_i) ($i = 0, 1, 2, \dots, m$) such that 1) $t_m < \dots < t_{i+1} < t_i < \dots < t_0$, 2) the line segment connecting the points (x_{i+1}, t_{i+1}) and (x_i, t_i) lies in \mathcal{O} . Let us denote by $S(x_0, t_0)$ the set of all points which can be connected with (x_0, t_0) by an admissible polygonal path.

The function u is said to have a maximum (in $S(x_0, t_0)$) near the point $(x_0, t_0) \in \mathcal{O}$, provided that for any n -dimensional ball K and every $\delta > 0$ such that $(x_0, t_0) \in Q_\delta = K \times (t_0 - \delta, t_0) \subset \mathcal{O}$ the inequality

$$\sup_{(x,t) \in Q_\delta} u(x,t) \geq \sup_{(x,t) \in S(x_0,t_0)} u(x,t)$$

holds.

The aim of this paper is to announce the two following statements.

1) The function u is bounded from above on every compact subset of \mathcal{O} .

2) If u has a maximum μ near the point $(x_0, t_0) \in \mathcal{O}$, then $u(x, t) = \mu$ almost everywhere in $S(x_0, t_0)$.