Jan Krajíček Some theorems on the lattice of local interpretability types

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SOME THEOREMS ON THE LATTICE OF LOCAL INTERPRETABILITY TYPES

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In [2], J. Mycielski introduced the lattice of multidi-mensional local interpretability types of theories (for next shortly type) and he posed some problems (see also a joint ma-muscript [1] with A. Ehrenfeucht). By next three theorems we solved two of them. [T] denotes type of a theory T and \leq deno-tes ordering in the lattice.

A type is meet-irreducible iff it contains a comp-Theorem 1: lete theory.

of the proof: If S is a finitely axiomatizable and essentially undecidable theory and R is its recursi-vely axiomatizable extension then type [R] is not Corollary of the proof: meet-irreducible.

Theorem 2: For each two types t, s such that $s \neq t$ there exists a meet-irreducible type $t \geq t$ such that still

s4t. Corollary: For each type t which is not maximal there exists a meet-irreducible type t ≥ t which is still not maximal.

Theorem 3: Each type contains a theory with a finite language. Also some results about mutual multidimensional interpre-Also some results about mutual multidimensional interpre-tability of various theories of order are proved. Let us defi-ne six theories in the language {=; 3} (= is equality and in the following theories we assume implicitly the axioms of equ-ality, x jy stands for x 3 y × x = y): PO: ∀x; ¬x 3 x + ∀x, y, z; (x 3 y & y 3 z) → x 3 z + ∀x 3 y; x 3 y POD: PO + ∀ x ∀y 5 x 3 t 5 x; y 5 t + ∀x 3 y; y 3 x POS: PO + 3!x ∀y; x 3 t 5 x; y 5 t + ∀x 3 y; y 3 x POS: PO + 3!x ∀y; x 3 z 3 x ∀ ∀ x; y 3 z & (y 3 t → t 3 z) + ∀x ∀y - x 3 z 3 x ∀t 3 x; y 3 z & (y 3 t → t 3 z) LO: PO + ∀x, y; x 3 y ∨ y 3 x; LOD: POD + LO; LOS: POS + LO In the following theorem ≤ denotes 1 adimensional interpreta-bility and ⊥ denotes incomparability with respect to multidi-mensional (1) interpretability. Theorem 4: (i) PO < LO, POS, POD (ii) LO < LOS, LOD (iii) POS ≤ LOS (iv) POS ⊥ POD, LOD (v) LOS ⊥ POD, LOD (vi) |LO| = |LOS| ∧ (LOD) (vii) for each theory T: LO ≤ T → LO ≤ 1T. References: (1) A. Ehrenfeucht, J. Myoielski: Theorems and

References: [1] A. Ehrenfeucht, J. Mycielski: Theorems and problems on the lattice of local interpreta-bility, manuscript [2] J. Mycielski: A lattice of interpretability

types of theories, Journal of Symbolic Logic 42, 1977

A POSSIBLE MODAL REFORMULATION OF COMPREHENSION SCHEME

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In various set theories the Cantor's comprehension is re formulated (Quine's NF) or replaced by a set of axioms (ZF,GB).

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