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On a class of weak Asplund spaces which has some permanence properties

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ANNOUNCEMENTS OF NEW RESULTS

ON ESTIMATING THE DIFFUSION COEFFICIENT

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Consider the diffusion process ξ defined on $(\Omega, \mathcal{F}, P)$ by

$$d\xi_t = a(\xi_t, \vartheta)dt + b(\xi_t, \vartheta)dW_t, \quad \xi_0 = x_0, \quad t \in [0,T],$$

$\vartheta \in \Theta$, where Θ is an open subset of real line, $\{W_t, t \in [0,T]\}$ is a standard Wiener process. Suppose that $a(x,\vartheta), b(x,\vartheta)$ are real-valued functions, continuous on $R \times \Theta$, $b(x,\vartheta) > 0$ for all $(x,\vartheta) \in R \times \Theta$ and such that $a', a'', \dot{a}, \dot{a}', b', b'', b''', \dot{b}, \dot{b}', \dot{b}''$ are continuous on $R \times \Theta$ (here the stroke and the dot denote derivative with respect to x and ϑ respectively). Denote $g(x,\vartheta) = \dot{b}(x,\vartheta)/b(x,\vartheta)$.

The chain $\{X_k\}_{k=0}^n$ of observations of the process ξ_t at discrete sampling points $0=t_0 < t_1 < \ldots < t_n = T$ is the Markov chain which generates on $(\Omega^n, \sigma(X_1, \ldots, X_n))$ the probability measure P_ϑ^n.

Local asymptotic mixed normality (LAMN). The families $\{P_\vartheta^n, \vartheta \in \Theta\}_{n \geq 1}$ satisfy the LAMN condition in some $\vartheta_0 \in \Theta$.

The minimax theorem. For any sequence $\{T_n\}_{n \geq 1}$ of estimators based on X_k, $k=0,1,\ldots,n$, of unknown parameter ϑ_0 holds

$$\lim_{b \to \infty} \underline{\lim}_{n \to \infty} \sup_{|h|<b} E^n_{\vartheta_{n,h}}(l(\sqrt{n}(T_n - \vartheta_{n,h}))) \geq \frac{1}{\sqrt{2\pi}} \int l\left(\frac{z}{\sqrt{w}}\right) e^{-\frac{1}{2}z^2} dz dG(w),$$

where $\vartheta_{n,h} = \vartheta_0 + h/\sqrt{n}$, $l(x)$ is a loss function and $G(w)$ is the distribution function of $\Gamma(\vartheta_0) = \frac{2}{T}\int_0^T g^2(\xi_t, \vartheta)dt$.

The lower bound is obtained only if

$$(T_n - \vartheta_0) \longrightarrow \left[\sum_{k=0}^{n-1} g(X_k, \vartheta_0)(n(\delta W_k)^2 - 1)\right] \cdot \left[2 \sum_{k=0}^{n-1} g^2(X_k, \vartheta_0)\right]^{-1}$$

in $P^n_{\vartheta_0}$-probability as $n \longrightarrow \infty$, where $\delta W_k = W_{k+1} - W_k$.

In particular, if $l(x)=x^2$, then for any $\varepsilon > 0$ and for sufficiently large T

$$\lim_{b \to \infty} \underline{\lim}_{n \to \infty} \sup_{|h|<b} nE^n_{\vartheta_{n,h}}(T_n - \vartheta_{n,h})^2 \gtrsim [\mu(g^2)] - \varepsilon,$$

where μ is the invariant measure of ξ.

ON A CLASS OF WEAK ASPLUND SPACES WHICH HAS SOME PERMANENCE PROPERTIES

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Real Banach spaces, X, Y,... are considered. The set of all linear bounded functionals on X is denoted by X^*.

Let U be the unit ball of X and let $B \subset X^*$ be the polar of U. Let us consider the collection $\mathcal{C} = \{n^{-1}B : n = 1,2,\ldots\}$. Then the following condition C1 is fulfilled.

C1 For each weak* neighbourhood W of the point $0 \in X^*$ there exists $E \in \mathcal{C}$ with the property $B \cap E \subset W$.

If, moreover, X is an Asplund space then, according to [1, Lemma 3], it holds

C2 For each nonempty set $M \subset (1/2)B$ and for each $E \in \mathcal{C}$ there exists a relatively weak* open nonempty subset G of the set M so that $G - G \subset E$.

We now define $\mathcal{K}$ to be the class of all Banach spaces X of which duals X^* have the following property: there exist a weak* compact barrel $B \subset X^*$ and a countable collection $\mathcal{C}$ of weak* closed absolutely convex subsets of X^* so that the conditions C1 and C2 are satisfied.

The main result of this note is expressed by the following assertions (i) - (v).

(i) If $X \in \mathcal{K}$ and $T: X \longrightarrow Y$ is a continuous linear operator with dense range then $Y \in \mathcal{K}$.

(ii) If $Y \in \mathcal{K}$ and $T: X \longrightarrow Y$ is a continuous linear operator having the property $T^* Y^* = X^*$ then $X \in \mathcal{K}$.

(iii) The Cartesian product of two spaces from $\mathcal{K}$ belongs again to $\mathcal{K}$.

(iv) Every Asplund space and every weakly compactly generated Banach space is in $\mathcal{K}$.

(v) Every space from $\mathcal{K}$ is a weak Asplund space.

From (iv),(i),(ii) and (v) it immediately follows

Theorem (Christensen,Kenderov,[2]). Suppose that X is an Asplund space and $T: X \longrightarrow Y$ is a continuous linear operator with dense range. Then every closed linear subspace of Y is a weak Asplund space.

In connection with [3] it is stated that $\mathcal{K}$ is a subclass of the class S.

References

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[3] Ch. Stegall: A class of topological spaces and differentiation of functions on Banach spaces, Vorlesungen aus dem Fachbereich Mathematik der Universität Essen, Heft 10, 1983, 63-77.

WIDTH COMPACTNESS THEOREM AND WELL-QUASI-ORDERING INFINITE GRAPHS

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Robertson and Seymour [1] introduced the following concept of a (tree-)width:

Definition. A tree-decomposition of a graph G is a couple (T,X), where T is a tree and $X=(X_t, t \in V(T))$ such that