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CONSTRUCTION OF THE CLASS FN

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Abstract: In the paper we construct the class of finite natural numbers by a normal formula from a class which is a well-ordering of V. This result shows that in the alternative set theory with Gödel's scheme of existence of classes the class of finite natural numbers exists.

Key words: Alternative set theory, axiom of choice, axiom of extensional coding, finiteness, class of finite natural numbers.

Classification: 03E72, 03H99

In the alternative set theory (AST) we can formulate the axiom of choice in two ways: either as the axiom of well-ordering, or as the axiom of extensional coding. The two formulations are equivalent to each other (see [V],[S1]). However, if we substitute Gödel's scheme of existence of classes for Morse's scheme (i.e. if we restrict the scheme to normal formulas only), then the axiom of well-ordering is stronger. In the theory with the axiom of extensional coding it is impossible to construct the class FN, but in the theory with the axiom of well-ordering we can construct the class FN, as is shown in this paper. This construction also demonstrates that for many issues AST with Gödel's scheme of existence of classes is strong enough.

Theorem. Let R be a well-ordering of the class V. Then we have $(\forall x)(\mathrm{Fin}(x) \equiv \mathrm{Set}(R \cap x^2))$.

Proof. Suppose at first $\mathrm{Fin}(x)$. Then we have $\mathrm{Fin}(x^2)$, this implies $Fin(R\cap x^2)$ and hence $Set(R\cap x^2)$. The converse implication is proved by contradiction. Suppose that $\neg Fin(x)$ and $Set(R\cap x^2)$. Let $r=R\cap x^2$. Every subset of x has an r-maximal element, because r is a linear ordering which is a set (see [V], Ch. I). Let y be the R-first element of x for which $\neg Fin(r''\{y\})$ holds (from the assumption $\neg Fin(x)$ it follows that such an element exists - e.g. the r-maximal element of x satisfies $\neg Fin(r''\{y\})$). Let z be the r-maximal element of $r''\{y\}\setminus\{y\}$. Then we have $Fin(r''\{z\})$, because zRy and $z\neq y$, hence also $r''\{y\} = r''\{z\}\setminus\{y\}$ is a finite set; this is a contradiction, and thus the equivalence is proved.

Now we substitute Fin(x) by an equivalent normal formula $Set(R\cap x^2)$ in the definition of the class FN, and then from Gödel's scheme the existence of the class FN in AST with Gödel's scheme of existence of classes follows.

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