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A NOTE ON EXPONENTIATION IN REGULAR LOCALES

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Abstract. It is known that local compactness characterizes exponentiability in the categories of topological spaces, regular topological spaces and locales (see [1], [5] and [2]). Johnstone [4] showed that locally compact regular locales are exponentiable in the category of regular locales. We will show that the converse is true for spacial locales.

Key words. Locale

We refer to [3] for basic facts concerning locales. Recall that an object A of a category $\mathscr{A}$ is exponentiable if for any $B, C \in \mathscr{A}$ there is a natural bijection between morphisms $A \times C \to B$ and $C \to B^A$.

Proposition: *Let a regular space A be exponentiable in the category of regular locales. Then A is locally compact.*

Proof: Let B, C be regular k-spaces and $q : B \to C$ a quotient map. Following [5] 2.1., it is enough to verify that $1 \times q : A \times B \to A \times C$ is the quotient. By exponentiability, $1 \times_l q : A \times_l B \to A \times_l C$ is the quotient where $\times_l$ denotes the locale product. Consequently, it is sufficient to prove that $A \times X \cong$ $\cong A \times_l X$ for any regular k-space X. Since X is a k-space, it is a quotient $f : Y \to X$ of a regular locally compact space Y. Hence $1 \times_l f : A \times_l Y \to A \times_l X$ is the quotient and thus injective as the frame map. Since $A \times Y \cong A \times_l Y$ (see [3], p. 61), the canonical morphism $A \times X \to A \times_l X$ is injective and thus the isomorphism.

The question whether exponentiable objects in the category of regular locales are locally compact was put in [4]. Johnstone works constructively while the Michael's proof of [5] 2.1. is non-constructive.

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