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# ON THE DOMAIN OF INFLUENCE IN THERMOELASTICITY OF BODIES WITH VOIDS

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Abstract. The domain of influence, proposed by Cowin and Nunziato, is extended to cover the thermoelasticity of bodies with voids. We prove that for a finite time $t > 0$ the displacement field $u_i$, the temperature $\theta$ and the change in volume fraction $\sigma$ generate no disturbance outside a bounded domain $B_t$.

## 1. Introduction

It is remarkable to note that the theory of materials with voids or vacuous pores was first proposed by Nunziato and Cowin [8]. In this theory the authors introduce an additional degree of freedom in order to develop the mechanical behavior of a body in which the skeletal material is elastic and interstices are voids of material. The intended applications of the theory are to geological materials like rocks and soil and to manufactured porous materials. The linear theory of elastic materials with voids was developed by Cowin and Nunziato in [3]. Here the uniqueness and weak stability of solutions are also derived. Iesan in [4] has established the equations of thermoelasticity of materials with voids. An extension of these results to cover the theory of micropolar materials with voids was been made in our studies [6], [7]. In the present paper we first consider the basic equations and conditions of the mixed initial-boundary value problem in the context of thermoelasticity of bodies with voids. Next we define the domain of influence $B_t$ of the data at time $t$ associated with the problem. We adopt the method used in [1] and [5] to establish a domain of influence theorem. The main results assert that in the context of theory considered, the solutions of the mixed initial-boundary value problem vanishes outside $B_t$, for a finite time $t > 0$.

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## 2. Basic equations

An anisotropic elastic material is considered. Assume a such body that occupies a properly regular region $B$ of three-dimensional Euclidian space $R^3$ bounded by a piecewise smooth surface $\partial B$ and we denote the closure of $B$ by $\bar{B}$. We use a fixed system of rectangular Cartesian axes $Ox_i, (i = 1, 2, 3)$ and adopt Cartesian tensor notation. A superposed dot stands for the material time derivate while a comma followed by a subscript denotes partial derivatives with respect to the spatial coordinates.
Einstein summation on repeated indices is also used. Also, the spatial argument and the time argument of a function will be omitted when there is no likehood of confusion. The basic equations from thermoelasticity of bodies with voids are, [4]

$$t_{ij,j} + \varrho f_i = \varrho \ddot{u}_i, \tag{1}$$

$$h_{i,i} + g + \varrho l = \varrho \kappa \ddot{\sigma}, \tag{2}$$

$$\varrho T_0 \dot{\eta} = q_{i,i} + \varrho r. \tag{3}$$

The equation (1) is the motion equation, (2) is the balance of the equilibrated forces and (3) is the energy equations. We complete the above equations with
- the constitutive equations

$$\begin{aligned} t_{ij} &= C_{ijmn} e_{mn} + B_{ij}\sigma + D_{ijk}\sigma_{,k} - \beta_{ij}\theta, \\ h_i &= A_{ij}\sigma_{,j} + D_{mni} e_{mn} + d_i\sigma - a_i\theta, \\ g &= -B_{ij} e_{ij} - \xi\sigma - d_i\sigma_{,i} + m\theta, \\ \eta &= \beta_{ij} e_{ij} + m\sigma + a_i\sigma_{,i} + a\theta, \\ q_i &= k_{ij}\theta_{,j}; \end{aligned} \tag{4}$$

- the kinetic relations

$$2e_{ij} = u_{i,j} + u_{j,i}, \quad \theta = T - T_0, \quad \sigma = \varphi - \varphi_0. \tag{5}$$

In the above equations we have used the following notations: $\varrho$-the constant mass density, $\eta$-the specific entropy, $T_0$-the constant absolute temperature of the body in its reference state, $\kappa$ -the equilibrated inertia, $u_i$- the components of displacement vector, $\varphi$-the volume distribution function which in the reference state is $\varphi_0$, $\sigma$-the change in volume fraction measured from the reference state, $\theta$-the temperature variation measured from the reference temperature $T_0$, $e_{ij}$-the components of the Cauchy strain tensor, $t_{ij}$-the components of the symmetric stress tensor, $h_i$-the components of the equlibrated stress vector, $q_i$-the components of the heat flux vector, $f_i$-the components of the body forces, $r$-the heat supply per unit time, $g$-the intrinsic equilibrated force, $l$-the extrinsic equilibrated body force, $C_{ijmn}, B_{ij}, D_{ijk}, \beta_{ij}, A_{ij}, d_i, a_i, \xi, m, a, k_{ij}$-the characteristic functions of the material, and they obey the symmetry relations

$$\begin{aligned} &C_{ijmn} = C_{mnij} = C_{jimn}, \quad B_{ij} = B_{ji}, \quad A_{ij} = A_{ji}, \\ &D_{ijk} = D_{jik}, \quad \beta_{ij} = \beta_{ji}, \; k_{ij} = k_{ji} \end{aligned} \tag{6}$$

The entropy inequality implies

$$k_{ij}\theta_{,i}\theta_{,j} \geq 0. \tag{7}$$

To the system of field equations (1) – (5) we adjoin the following initial conditions

$$\begin{aligned} u_i(x,0) &= u_i^0(x)\,, \quad \dot{u}_i(x,0) = u_i^1(x)\,, \quad \theta(x,0) = \theta^0(x)\,, \\ \sigma(x,0) &= \sigma^0(x)\,, \quad \dot{\sigma}(x,0) = \sigma^1(x)\,, \quad x \in \bar{B}\,, \end{aligned} \tag{8}$$

and the following prescribed boundary conditions

$$\begin{aligned} &u_i = \bar{u}_i \text{ on } \partial B_1 \times [0,t_0)\,, && t_i = t_{ij}n_j = \bar{t}_i \text{ on } \partial B_1^c \times [0,t_0)\,, \\ &\sigma = \bar{\sigma} \text{ on } \partial B_2 \times [0,t_0)\,, && h = h_i n_i = \bar{h} \text{ on } \partial B_2^c \times [0,t_0)\,, \\ &\theta = \bar{\theta} \text{ on } \partial B_3 \times [0,t_0)\,, && q = q_i n_i = \bar{q} \text{ on } \partial B_3^c \times [0,t_0)\,, \end{aligned} \tag{9}$$

where $\partial B_1, \partial B_2$ and $\partial B_3$ with respective complements $\partial B_1^c, \partial B_2^c$ and $\partial B_3^c$, are subsets of $\partial B$, $n_i$ are the components of the unit outward normal to $\partial B$, $t_0$ is some instant that may be infinite, $u_i^0$, $u_i^1$, $\theta^0$, $\sigma^0$, $\sigma^1$, $\bar{u}_i$, $\bar{t}_i$, $\bar{\sigma}$, $\bar{\theta}$, $\bar{q}$ and $\bar{h}$ are prescribed functions in their domains. Introducing (5) and (4) into equations (1), (2) and (3), we obtain the following system of equations

$$\begin{aligned} \varrho\ddot{u}_i &= (C_{jimn}u_{m,n} + B_{ij}\sigma + D_{ijk}\sigma_{,k} - \beta_{ij}\theta)_{,j} + \varrho f_i\,, \\ \varrho\kappa\ddot{\sigma} &= (D_{mni}u_{m,n} + d_i\sigma + A_{ij}\sigma_{,j} - a_i\theta)_{,i} + \varrho l - \\ &\quad - B_{ij}u_{i,j} - \xi\sigma - d_i\sigma_{,i} + m\theta\,, \\ a\dot{\theta} &= \frac{1}{\varrho T_0}(k_{ij}\theta_{,j})_{,i} + \frac{1}{T_0}r - \beta_{ij}\dot{u}_{i,j} - m\dot{\sigma} - a_i\dot{\sigma}_{,i}\,. \end{aligned} \tag{10}$$

By a solution of the mixed initial boundary value problem of the theory of thermoelasticity of bodies with voids in the cylinder $\Omega_0 = B \times [0,t_0)$ we mean an ordered array $(u_i,\ \theta,\ \sigma)$ which satisfies the system (10) for all $(x,t) \in \Omega_0$, the boundary conditions (9) and the intial conditions (8).

## 3. Main result

We begin this section with the definition of the domain of influence. Next, we establish a domain of influence inequality, which is a counterpart of the inequality established in [5]. Finally, we shall prove a domain influence theorem in the context of thermoelasticity of bodies with voids. In all what follows we shall use the following assumptions on the material properties

i) $\varrho > 0\,,\ \kappa > 0\,,\ T_0 > 0\,,\ a > 0\,;$

ii) $C_{ijmn}x_{ij}x_{mn} + 2B_{ij}x_{ij}z + 2D_{ijk}x_{ij}y_k + 2d_iy_iz + \xi z^2 + A_{ij}y_iy_j \geq$
$\geq \alpha(x_{ij}x_{ij} + y_iy_i + z^2)\,,$ for all $x_{ij} = x_{ji},\ y_i,\ z\,;\ \alpha > 0\,;$

iii) $k_{ij}\eta_i\eta_j \geq \gamma\eta_i\eta_i\,,$ for all $\eta_i\,,\ \gamma > 0\,.$

These assumptions are in agreement with the usual restrictions imposed in the mechanics of continua. The assumption iii) represent a considerable strenghtening of the consequence (7) of the entropy production inequality.
For a sufficiently small $\varepsilon > 0$, let $W_\varepsilon(z)$ be a smooth nondecreasing function, vanishing in $(-\infty, 0]$ and equal to one in $[\varepsilon, \infty)$ and for $0 \le s \le t$,

$$G(x,s) = W_\varepsilon\left(\frac{R-\mathbf{r}}{c} + t - s\right) \tag{11}$$

for some fixed positive $R$ and $t$, where $\mathbf{r} = |x - x_0|$, $x_0$ is an arbitrary fixed point, $c$ is a positive constant to be determined later.
$G(x,s)$ is a smooth function on $B \times [0,t]$, vanishing outside $\Sigma$ where

$$\Sigma = \bigcup_{s\in[0,t]} S[x_0, R + c(t-s)]\,.$$

The sphere $S(x_0, \mathcal{R})$ is defined as

$$S(x_0,\mathcal{R}) = \{x \in R^3 : |x - x_0| < \mathcal{R}\}\,. \tag{12}$$

Let $U(x,s)$ be the function defined as

$$\begin{aligned} U(x,s) = \frac{1}{2}[&\varrho\dot{u}_i\dot{u}_i + \varrho\kappa\dot{\sigma}^2 + a\theta^2 + C_{ijmn}u_{i,j}u_{m,n} + \xi\sigma^2 + \\ &+ A_{ij}\sigma_{,i}\sigma_{,j} + 2B_{ij}\sigma u_{i,j} + 2D_{ijk}\sigma_{,k}u_{i,j} + 2d_i\sigma\sigma_{,i}](x,s)\,. \end{aligned} \tag{13}$$

We also define the function $K(x,s)$

$$K(x,s) = \frac{1}{2}[\varrho\dot{u}_i\dot{u}_i + \varrho\kappa\dot{\sigma}^2 + a\theta^2 + u_{i,j}u_{i,j} + \sigma^2 + \sigma_{,i}\sigma_{,i}](x,s)\,. \tag{14}$$

Taking into account the assumptions i) and ii) from (13) and (14) we deduce

$$K(x,s) \le U(x,s)\,. \tag{15}$$

The next theorem is a necessary step to prove the main result.

**Theorem 1.** *Let* $(u_i, \theta, \sigma)$ *be a solution to the system of equations* (10) *with the initial conditions* (8) *and the boundary conditions* (9). *Then for any* $R > 0$, $t > 0$ *and* $x_0 \in B$, *we have that*

$$\begin{aligned} &\int_{D[x_0,R]} U(x,t)dV + \frac{1}{\varrho T_0}\int_0^t ds \int_{D[x_0,R+c(t-s)]} k_{ij}\theta_{,i}\theta_{,j}dV \le \\ &\le \int_{D[x_0,R+ct]} U(x,0)dV + \int_0^t ds \int_{D[x_0,R+c(t-s)]} \varrho[f_i\dot{u}_i + l\dot{\sigma} + \frac{1}{\varrho^2 T_0}r\theta]dV + \\ &\qquad + \int_0^t ds \int_{\partial D[x_0,R+c(t-s)]} [\bar{t}_i\dot{u}_i + \bar{h}\dot{\sigma} + \frac{1}{\varrho T_0}\bar{q}\theta]dS\,, \end{aligned} \tag{16}$$

*where* $D(x_0,\mathcal{R}) = \{x \in B : |x - x_0| < \mathcal{R}\}$, $\partial D(x_0,\mathcal{R}) = \{x \in \partial B : |x - x_0| < \mathcal{R}\}$.

**Proof.** Multiplying the equation $(10)_1$ by $G\dot{u}_i$, it results

$$(17)\quad \frac{1}{2}G\frac{d}{dt}(\varrho\dot{u}_i\dot{u}_i) = \varrho G f_i\dot{u}_i + (Gt_{ij}\dot{u}_i)_{,j} - G_{,j}t_{ij}\dot{u}_i - \\ - G(C_{ijmn}u_{m,n}\dot{u}_{i,j} + B_{ij}\sigma\dot{u}_{i,j} + D_{ijk}\sigma_{,k}\dot{u}_{i,j} - \beta_{ij}\theta\dot{u}_{i,j})\,.$$

Multiplying the equation $(10)_2$ by $G\dot{\sigma}$, we get

$$(18)\quad \frac{1}{2}G\frac{d}{dt}(\varrho\kappa\dot{\sigma}^2) = \varrho Gl\dot{\sigma} + (Gh_i\dot{\sigma})_{,i} - G_{,i}h_i\dot{\sigma} - \\ - G(A_{ij}\sigma_{,j}\dot{\sigma}_{,i} + D_{mni}u_{m,n}\dot{\sigma}_{,i} + d_i\sigma\dot{\sigma}_{,i} - a_i\theta\dot{\sigma}_{,i}) - \\ - G(B_{ij}u_{i,j}\dot{\sigma} + \xi\sigma\dot{\sigma} + d_i\sigma_{,i}\dot{\sigma} - m\theta\dot{\sigma})\,.$$

At last, multiplying the equation $(10)_3$ by $G\theta$, we are led to

$$(19)\quad \frac{1}{2}G\frac{d}{dt}(a\theta^2) = \frac{1}{T_0}Gr\theta + \frac{1}{\varrho T_0}[(G\theta q_i)_{,i} - G_{,i}\theta q_i] - \\ - \frac{1}{\varrho T_0}Gk_{ij}\theta_{,i}\theta_{,j} - G(\beta_{ij}\theta\dot{u}_{i,j} + m\theta\dot{\sigma} + a_i\theta\dot{\sigma}_{,i})\,.$$

Additing equations (17), (18) and (19) together, it results

$$(20)\quad \frac{1}{2}G\frac{d}{dt}(\varrho\dot{u}_i\dot{u}_i + \varrho\kappa\dot{\sigma}^2 + a\theta^2) = \varrho G f_i\dot{u}_i + \varrho Gl\dot{\sigma} + \frac{1}{T_0}Gr\theta + \\ + G(t_{ij}\dot{u}_i + h_j\dot{\sigma} + \frac{1}{\varrho T_0}\theta q_j)_{,j} - G[C_{ijmn}u_{m,n}\dot{u}_{i,j} + \xi\sigma\dot{\sigma} + A_{ij}\sigma_{,i}\dot{\sigma}_{,j} + \\ + B_{ij}(\dot{u}_{i,j}\sigma + u_{i,j}\dot{\sigma}) + D_{ijk}(u_{i,j}\dot{\sigma}_{,k} + \dot{u}_{i,j}\sigma_{,k}) + d_i(\sigma\dot{\sigma}_{,i} + \dot{\sigma}\sigma_{,i})] - \\ - G_{,j}t_{ij}\dot{u}_i - G_{,i}h_i\dot{\sigma} - \frac{1}{\varrho T_0}G_{,i}q_i\theta - \frac{1}{\varrho T_0}Gk_{ij}\theta_{,i}\theta_{,j}\,.$$

The relation (20) may be restated as follows

$$(21)\quad \frac{1}{2}G\frac{d}{dt}(\varrho\dot{u}_i\dot{u}_i + \varrho\kappa\dot{\sigma}^2 + a\theta^2 + C_{ijmn}u_{m,n}u_{i,j} + \xi\sigma^2 + A_{ij}\sigma_{,i}\sigma_{,j} + \\ + 2B_{ij}u_{i,j}\sigma + 2D_{ijk}u_{i,j}\sigma_{,k} + 2d_i\sigma\sigma_{,i}) = \\ = \varrho G f_i\dot{u}_i + \varrho Gl\dot{\sigma} + \frac{1}{T_0}Gr\theta + G(t_{ij}\dot{u}_i + h_j\dot{\sigma} + \frac{1}{\varrho T_0}\theta q_j)_{,j} - \\ - G_{,j}t_{ij}\dot{u}_i - G_{,i}h_i\dot{\sigma} - G_{,i}\frac{1}{\varrho T_0}\theta q_i - \frac{1}{\varrho T_0}k_{ij}\theta_{,i}\theta_{,j}\,,$$

that is

$$(22)\quad \frac{1}{2}G\dot{U} + \frac{1}{\varrho T_0}k_{ij}\theta_{,i}\theta_{,j} = \varrho G f_i\dot{u}_i + \varrho Gl\dot{\sigma} + \frac{1}{T_0}Gr\theta + \\ + G(t_{ij}\dot{u}_i + h_j\dot{\sigma} + \frac{1}{\varrho T_0}\theta q_j)_{,j} - G_{,j}t_{ij}\dot{u}_i - G_{,i}h_i\dot{\sigma} - G_{,i}\frac{1}{\varrho T_0}\theta q_i\,.$$

Integrating both sides of equations (22) over $B\times[0,t]$ and by using the divergence theorem and the boundary conditions (9), we deduce

$$\frac{1}{2}\int_B GU(x,t)dV+\frac{1}{\varrho T_0}\int_0^t\int_B Gk_{ij}\theta_{,i}\theta_{,j}dVds= \tag{23}$$
$$=\frac{1}{2}\int_B GU(x,0)dV+\int_0^t\int_{\partial B}G(\bar{t}_i\dot{u}_i+\bar{h}\dot{\sigma}+\frac{1}{\varrho T_0}\bar{q}\theta)dVds+$$
$$+\int_0^t\int_B \varrho G(f_i\dot{u}_i+l\dot{\sigma}+\frac{1}{\varrho^2 T_0}r\theta)dVds+\frac{1}{2}\int_0^t\int_B\dot{G}U(x,s)dVds-$$
$$-\int_0^t\int_B(G_{,j}t_{ij}\dot{u}_i+G_{,i}h_i\dot{\sigma}+\frac{1}{\varrho T_0}G_{,i}q_i\theta)dVds\,.$$

Taking into account the definition (11) of the function $G$, we find that

$$|-G_{,j}t_{ij}\dot{u}_i-G_{,i}h_i\dot{\sigma}-\frac{1}{\varrho T_0}G_{,i}q_i\theta|= \tag{24}$$
$$=|\frac{1}{c}W_\varepsilon^{'}\frac{x_j}{\mathbf{r}}t_{ij}\dot{u}_i+\frac{1}{c}W_\varepsilon^{'}\frac{x_i}{\mathbf{r}}h_i\dot{\sigma}+\frac{1}{c\varrho T_0}W_\varepsilon^{'}\frac{x_i}{\mathbf{r}}q_i\theta|=$$
$$=|\frac{1}{c}W_\varepsilon^{'}\frac{1}{\mathbf{r}}[(C_{ijmn}u_{m,n}x_j+B_{ij}\sigma x_j+D_{ijk}\sigma_{,k}x_j-\beta_{ij}\theta x_j)\dot{u}_i+$$
$$+(A_{ij}\sigma_{,j}x_i+D_{mni}u_{m,n}x_i+d_i\sigma x_i-a_i\theta x_i)\dot{\sigma}+\frac{1}{\varrho T_0}k_{ij}\theta_{,j}\theta x_i]|$$

where

$$W_\varepsilon^{'}=\frac{dW_\varepsilon}{d\mathbf{r}}\,.$$

We now make use of arithmetic-geometric mean inequality

$$ab\le\frac{1}{2}(\frac{a^2}{p^2}+b^2p^2) \tag{25}$$

to the last terms of relation (24) and by choosing suitable parameters $p$ we can find $c$ such that

$$|-G_{,j}t_{ij}\dot{u}_i-G_{,i}h_i\dot{\sigma}-\frac{1}{\varrho T_0}G_{,i}q_i\theta|\le W_\varepsilon^{'}K(x,s)\,, \tag{26}$$

and that

$$\int_0^t\int_B\dot{G}U(x,s)dVds-\int_0^t\int_B(G_{,j}t_{ij}\dot{u}_i+G_{,i}h_i\dot{\sigma}+\frac{1}{\varrho T_0}G_{,i}q_i\theta)dVds\le \tag{27}$$
$$\le\int_0^t\int_B W_\varepsilon^{'}(x,s)[K(x,s)-U(x,s)]dVds\le 0\,.$$

By using the inequality (27) in equation (23), it results

$$\begin{aligned}(28)\qquad & \int_B GU(x,t)dV + \frac{1}{\varrho T_0}\int_0^t\int_B Gk_{ij}\theta_{,i}\theta_{,j}dVds \leq \\ & \leq \int_B GU(x,0)dV + + \int_0^t\int_B \varrho G(f_i\dot{u}_i + l\dot{\sigma} + \frac{1}{\varrho^2 T_0}r\theta)dVds + \\ & + \int_0^t\int_{\partial B} G(\bar{t}_i\dot{u}_i + \bar{h}\dot{\sigma} + \frac{1}{\varrho T_0}\bar{q}\theta)dVds\,.\end{aligned}$$

Letting $\varepsilon \to 0$ into relation (28), $G$ tends boundedly to the characteristic function of $\Sigma$ and we get the inequality (16).
Based on the above estimations, we can now prove the main result of our study : the domain of influence theorem.
Let $B(t)$ be the set of points $x \in \bar{B}$ such that:

(1) $x \in B \Rightarrow u_i^0 \neq 0$ or $u_i^1 \neq 0$ or $\sigma^0 \neq 0$ or $\sigma^1 \neq 0$ or $\theta^0 \neq 0$ or $\exists\tau \in [0,t]$ such that $f_i(x,\tau) \neq 0$ or $l(x,\tau) \neq 0$ or $r(x,\tau) \neq 0$;
(2) $x \in \partial B_1 \Rightarrow \exists\tau \in [0,t]$ such that $\bar{u}_i(x,\tau) \neq 0$,
(3) $x \in \partial B_1^c \Rightarrow \exists\tau \in [0,t]$ such that $\bar{t}_i(x,\tau) \neq 0$,
(4) $x \in \partial B_2 \Rightarrow \exists\tau \in [0,t]$ such that $\bar{\sigma}(x,\tau) \neq 0$,
(5) $x \in \partial B_2^c \Rightarrow \exists\tau \in [0,t]$ such that $\bar{h}(x,\tau) \neq 0$,
(6) $x \in \partial B_3 \Rightarrow \exists\tau \in [0,t]$ such that $\bar{\theta}(x,\tau) \neq 0$,
(7) $x \in \partial B_3^c \Rightarrow \exists\tau \in [0,t]$ such that $\bar{q}(x,\tau) \neq 0$.

The domain of influence of the data at instant t is defined as

$$(29)\qquad B_t = \{x_0 \in \bar{B} : B(t) \cap \bar{S}(x_0, ct) \neq \Phi\},$$

where $\Phi$ is the empty set.

**Theorem 2.** *Let* $(u_i, \theta, \sigma)$ *be a solution to the system of equations* (10) *with the initial conditions* (8) *and the boundary conditions* (9). *Then we have*

$$u_i = 0,\;\; \theta = 0,\;\; \sigma = 0, \quad \text{on} \quad \{\bar{B}\setminus B_t\}\times[0,t]\,.$$

**Proof.** For any $x_0 \in \bar{B}\setminus B_t$ and $\tau \in [0,t]$, by using the inequality (16) with $t = \tau$ and $R = c(t-\tau)$, we obtain

$$\begin{aligned}(30)\qquad & \int_{D[x_0,c(t-\tau)]} U(x,\tau)dV + \frac{1}{\varrho T_0}\int_0^\tau\int_{D[x_0,c(t-s)]} k_{ij}\theta_{,i}\theta_{,j}dVds \leq \\ & \leq \int_{D[x_0,ct)]} U(x,0)dV + \int_0^\tau\int_{D[x_0,c(t-s)]}\varrho(f_i\dot{u}_i + l\dot{\sigma} + \frac{1}{\varrho^2 T_0}r\theta)dVds + \\ & + \int_0^\tau\int_{\partial D[x_0,c(t-s)]}\varrho(\bar{t}_i\dot{u}_i + \bar{h}\dot{\sigma} + \frac{1}{\varrho^2 T_0}\bar{q}\theta)dSds\,.\end{aligned}$$

Since $x_0 \in \bar{B} \setminus B_t$, we have $x \in D(x_0, ct) \Rightarrow x \notin B(t)$ and hence

$$\int_{D[x_0,ct]} U(x,0)dV = 0\,. \tag{31}$$

Moreover, since $D[x_0, c(t-s)] \subseteq D(x_0, ct)$, we have

$$\int_0^\tau \int_{D[x_0,c(t-s)]} \varrho(f_i \dot{u}_i + l\dot{\sigma} + \frac{1}{\varrho^2 T_0} r\theta) dV ds = 0\,, \tag{32}$$

$$\int_0^\tau \int_{\partial D[x_0,c(t-s)]} (\bar{t}_i \dot{u}_i + \bar{h}\dot{\sigma} + \frac{1}{\varrho T_0} \bar{q}\theta) dS\, ds = 0\,. \tag{33}$$

Taking into account the assumption iii) and the relations (31), (32) and (33) we obtain

$$\int_{D[x_0,c(t-\tau)]} U(x,\tau)dV \le 0\,, \tag{34}$$

and with aid of inequality (15), we get

$$\int_{D[x_0,c(t-\tau)]} K(x,\tau)dV \le 0\,, \tag{35}$$

From the definition of $K$, it results

$$\dot{u}_i(x_0,\tau) = 0\,, \quad \theta(x_0,\tau) = 0\,, \quad \sigma(x_0,\tau) = 0\,,$$

for any $(x_0, \tau) \in \{\bar{B} \setminus B_t\} \times [0,t]$.
Finally, since $u_i(x_0, 0) = 0$ for any $x_0 \in \bar{B} \setminus B_t$, we deduce

$$u_i(x_0,\tau) = 0, \quad \theta(x_0,\tau) = 0, \quad \sigma(x_0,\tau) = 0\,,$$

for any $(x_0, \tau) \in \{\bar{B} \setminus B_t\} \times [0,t]$ and the proof of Theorem 2 is complete.

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