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DISTURBANCE DECOUPLING FOR NONLINEAR SYSTEMS: A UNIFIED APPROACH[1]

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This note presents an exposition which unifies various (static or dynamic feedback) solutions given in the literature to the disturbance decoupling problem since the early development of modern nonlinear theory up to the end of the 70's. This is possible thanks to some recently introduced generalized transformations depending on a finite number of time derivatives of the input. In this way, some classical controlled invariant distributions can be replaced by a related elementary linear subspace by which a NSC for disturbance decoupling can be derived.

1. INTRODUCTION AND PRELIMINARIES

In this paper we give a general and unified description for the solvability of the Disturbance Decoupling Problem which is solved either by static state feedback (DDP), dynamic state feedback (DDDP) or generalized state feedback (GDDP). Differently from other results ([3, 5, 6]), our description does not rely on any (structure) algorithm. For this purpose a linear algebraic setting is used. Consider the nonlinear system Σ of the form

$$\Sigma = \begin{cases} \dot{x} = f(x) + g(x)\,u \\ y = h(x) \end{cases} \tag{1}$$

where $x \in \mathbb{R}^n$, $u \in \mathbb{R}^m$ and $y \in \mathbb{R}^p$. The entries of $f(x)$, $g(x)$ and $h(x)$ are meromorphic functions. As done in [2], let $\mathcal{K} := \mathcal{K}(x, u, \dot{u}, \dots, u^{(k)}, \dots)$, that is the differential field of meromorphic functions of $x, u, \dot{u}, \dots, u^{(k)}, \dots$, for $k \geq 0$. Over this field we can define a differential vector space, $\mathcal{E} := \operatorname{span}\{dx;\, x \in \mathcal{K}\}$. $\mathcal{E}$ can be decomposed into the sum of two subspaces, $\mathcal{E} = \mathcal{X} \oplus \mathcal{U}$, where

$$\mathcal{X} := \operatorname{span}_{\mathcal{K}}\{dx\}$$
$$\mathcal{U} := \operatorname{span}_{\mathcal{K}}\left\{du^{(k)},\, k \geq 0\right\}.$$

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Define the output differential space

$$\mathcal{Y} := \text{span}_{\mathcal{K}} \left\{ dy^{(k)},\ k \geq 0 \right\}.$$

Thus

$$\mathcal{Y} \subset \mathcal{E} = \mathcal{X} \oplus \mathcal{U}. \tag{2}$$

Recalling the results from the geometric linear control theory ([10]) the key concept to solve the DDP is the so-called supremal (A,B)-invariant subspace contained in the kernel of C, denoted V*. This notion has been generalized to nonlinear system theory by Δ^*, the largest controlled-invariant distribution contained in the kernel of the output map ([7]). In a dual form, it was pointed out that the subspace $\mathcal{X} \cap \mathcal{Y}$ represents the subspace of $\mathcal{X}$ whose observability is not affected by the input and is invariant under regular feedback [11]. For linear systems, remarking that $\mathcal{X}$ contains in a natural way an isomorphic copy of the finite dimensional state space X, we have that V* is isomorphic to $X \cap (\mathcal{X} \cap \mathcal{Y}_k)^{\perp}$, where $\mathcal{Y}_k := \text{span}_{\mathcal{K}}\{dy, d\dot{y}, \ldots, dy^{(k)}\}$, for k large enough. For nonlinear control systems the subspace $\mathcal{X} \cap \mathcal{Y}$, in general, is not closed, in the sense that there does not exist a basis for $\mathcal{X} \cap \mathcal{Y}$ which consists of closed (or locally exact) one-forms only. If we consider generalized transformations, i. e. the state space coordinates transformations, feedbacks and output injections which are parametrized in the input and its derivatives [4], we show that the subspace $\mathcal{X} \cap \mathcal{Y}$ plays a similar role as it does for linear control systems. More precisely $\mathcal{X} \cap \mathcal{Y}$ is shown to describe the maximal observable subsystem with respect to all possible generalized (or quasi static [1]) feedbacks.

Let us recall the definitions (that could be given in different ways) of generalized transformations.

Definition 1.1. Given an isomorphism Φ from $\mathcal{E}$ to $\mathcal{E}$, Φ defines a generalized-state transformation if $\Phi(\mathcal{X})$ is closed and

$$\Phi(\mathcal{X}) \oplus \mathcal{U} = \mathcal{X} \oplus \mathcal{U}. \tag{3}$$

Let $\Phi(\mathcal{X}) = \text{span}_{\mathcal{K}}\{d\xi_i,\ i = 1, 2, \ldots, n\}$. From (3), $\dfrac{\partial(\xi_1, \xi_2, \ldots, \xi_n)}{\partial(x_1, x_2, \ldots, x_n)}$ is nonsingular and $(\xi_1, \xi_2, \ldots, \xi_n)$ defines a generalized-state coordinate system.

Definition 1.2. Given an isomorphism Φ from $\mathcal{E}$ to $\mathcal{E}$, Φ defines a regular generalized state feedback if

- $\Phi(\mathcal{U}) = \text{span}_{\mathcal{K}}\{dv^{(k)},\ k \geq 0\}$ where $v := (v_1, \ldots, v_m)$ and $v_i \in \mathcal{K}$, $i = 1, \ldots, m$,
- and

$$\mathcal{X} \oplus \Phi(\mathcal{U}) = \mathcal{X} \oplus \mathcal{U}. \tag{4}$$

Let $\mathcal{V} = \Phi(\mathcal{U}) = \text{span}_{\mathcal{K}}\{dv^{(k)},\ k \geq 0\}$. The equality (4) implies that $\dfrac{\partial(v_1, \ldots, v_m)}{\partial(u_1, \ldots, u_m)}$ is invertible. Thus, there exist relations as $u_i = u_i(x, v, \dot{v}, \ldots)$ and in the rest of the paper v is considered to be the new input of the closed loop system, *i. e.*, after generalized state feedback $v_1, \ldots, v_m$ and their time derivatives are independent variables.

From the above definition, one may also write

$$\Phi : \Sigma \to \tilde{\Sigma}.$$

For the closed-loop system $\tilde{\Sigma}$ there is a background field $\tilde{K}$, which is the differential field of meromorphic functions of $\xi, v, \dot{v}, \ldots, v^{(k)}, \ldots$, for $k \geq 0$, and a differential vector space $\tilde{\mathcal{E}} := \mathrm{span}_{\tilde{\mathcal{K}}}\{d\eta,\ \eta \in \tilde{K}\}$. The two background fields $\mathcal{K}$ and $\tilde{\mathcal{K}}$ can be identified. As $\xi = \xi(x, u, \dot{u}, \ldots, u^{(r)})$, $v = v(x, u, \dot{u}, \ldots, u^{(s)})$, then every function $\eta \in \tilde{\mathcal{K}}$ can be considered as a function in $\mathcal{K}$, *i.e.* $\eta = \tilde{\eta}(x, u, \dot{u}, \ldots, u^{(q)}) \in \mathcal{K}$, where k, s, r, q are some properly defined integers. Therefore, the transformation Φ sets up a linear map from $\mathcal{E}$ to $\tilde{\mathcal{E}}$ over the same field. Thus, for any $d\eta \in \mathcal{E}$ we can define $d\hat{\eta} := \Phi(d\eta) \in \tilde{\mathcal{E}}$. Especially, let $y_i^{(k)}$ be the kth derivative of the ith output y_i and let $dy_i^{(k)}$ be its differential, then $d\hat{y}_i^{(k)} := \Phi(dy_i^{(k)})$, where $\hat{y}_i^{(k)}$ is the kth derivative of the ith output $\hat{y}_i$ of the closed-loop system $\tilde{\Sigma}$.

The Disturbance Decoupling Problem can now be dealt with in a general and unified way. The definition and solvability conditions of DDDP are given in [3, 5, 6, 8, 9]. For the GDDP it is required to find a generalized-state space change of coordinates and a regular generalized-state feedback to fulfil the disturbance decoupling. More precisely, consider a nonlinear system Σ of the form

$$\Sigma = \begin{cases} \dot{x} = f(x) + g(x)\,u + p(x)\,w \\ y = h(x) \end{cases} \tag{5}$$

which satisfies the general assumptions as system (1). The background field is defined by $\hat{\mathcal{K}} := \mathcal{K}(x, u, \dot{u}, \ldots, u^{(k)}, \ldots, w, \dot{w}, \ldots, w^{(r)}, \ldots)$.

Generalized Disturbance Decoupling Problem Formulation

Find, if possible, an isomorphism Φ from $\mathcal{E}$ to $\mathcal{E}$, which defines a generalized-state transformation and a regular generalized-state feedback such that $\Phi_{|\mathcal{W}}$ is the identity and the differential output space $\mathcal{Y}^*$ of the closed loop system satisfies

$$\mathcal{Y}^* \subset \Phi(\mathcal{X}) \oplus \Phi(\mathcal{U}) = \Phi(\mathcal{X}) \oplus \mathcal{V}.$$

From a practical point of view, the GDDP reduces to find a generalized coordinates system, $\xi_i = \xi_i(x, u, \dot{u}, \ldots, u^{(r_i)})$ $i = 1, 2, \ldots, n$, and new inputs $v_j = v_j(x, u, \dot{u}, \ldots$ $\ldots, u^{(s_i)})$ $j = 1, 2, \ldots, m$ such that $\dfrac{\partial(v_1, v_2, \ldots, v_m)}{\partial(u_1, u_2, \ldots, u_m)}$ is invertible.

Under the new coordinates $\xi = (\xi_1, \xi_2, \ldots, \xi_n)^{\mathrm{T}}$ and the new input $v = (v_1, v_2, \ldots$ $\ldots, v_m)^{\mathrm{T}}$ the system (5) becomes

$$\Sigma_g = \begin{cases} \dot{\xi} = \hat{f}(\xi, v, \dot{v}, \ldots, v^{(r)}, w) \\ y = \hat{h}(\xi) \end{cases} \tag{6}$$

where r is a positive integer and the output y is independent of the disturbance w, *i.e.* the output differential space $\mathcal{Y}^*$ of (6) satisfies

$$\mathcal{Y}^* \subset \Phi(\mathcal{X}) \oplus \Phi(\mathcal{U}) = \Phi(\mathcal{X}) \oplus \mathcal{V}.$$

Remark: (6) has not the most general form after a generalized-state space change of coordinates and a regular generalized-state feedback. More generally, (6) could contain time derivatives of w. In the next section, one verifies that the solvability conditions for DDDP and GDDP are equivalent. More precisely, we show that DDDP, or GDDP, is solvable if and only if

$$\mathcal{X} \cap \mathcal{Y} \subset \operatorname{span}_{\hat{\mathcal{K}}}\{p(x)\}^{\perp}. \tag{7}$$

Condition (7) can be viewed as a natural generalization of the solvability condition of the DDP given in the linear control system theory. In fact, in a dual form, we have $X \cap (\mathcal{X} \cap \mathcal{Y}_k)^{\perp} \approx \mathcal{V}*$, thus, (7) is exactly the same condition appearing in [10].

2. MAIN RESULT

First one shows that condition (7) is a necessary and sufficient condition for the GDDP. After that it will be shown that the solvability conditions for DDDP and GDDP are equivalent.

In order to solve the GDDP it is necessary to find a proper generalized-state space change of coordinates and a regular generalized-state feedback. Let ρ_i, $1 \le i \le p$, denote the orders of the zeros at infinity; then one has

Lemma 2.1.

$$(\mathcal{X} \cap \mathcal{Y}) \oplus \mathcal{U} = \operatorname{span}_{\mathcal{K}}\left\{dy_i^{(k)};\ k < \rho_i,\ 1 \le i \le p\right\} \oplus \mathcal{U}. \tag{8}$$

Theorem 2.2. For system (5) the GDDP is solvable if and only if

$$\mathcal{X} \cap \mathcal{Y} \subset \operatorname{span}_{\hat{\mathcal{K}}}\{p(x)\}^{\perp} \tag{9}$$

where $\hat{\mathcal{K}} := \mathcal{K}(x, u, \dot{u}, \ldots, u^{(k)}, \ldots, w, \dot{w}, \ldots, w^{(r)}, \ldots)$.

Proof. *Sufficiency.* Rewrite system (5) as

$$\begin{cases} \dot{x} = f(x) + g(x)\,u + p(x)\,w = f(x) + \tilde{g}(x)\,\tilde{u} \\ y = h(x) \end{cases} \tag{10}$$

where $\tilde{g}(x) = (g(x)\,p(x))$ and $\tilde{u} = \binom{u}{w}$. $\tilde{u}$ is considered to be the input of system (10).

Without loss of generality, assume $\rho_1 \le \rho_2 \le \cdots \le \rho_q < \infty$, $q \le p$, then since $y_1^{(\rho_1-1)} = y_1^{(\rho_1-1)}(x)$, we have that $dy_1^{(\rho_1-1)} = \dfrac{\partial y_1^{(\rho_1-1)}}{\partial x}\, dx$ belongs, by (9), to $\operatorname{span}_{\hat{\mathcal{K}}}\{p(x)\}^{\perp}$. Hence $y_1^{(\rho_1)} = \dfrac{\partial y_1^{(\rho_1-1)}}{\partial x}\,(f + gu + pw) = \phi_{01}(x) + \phi_{11}(x)\,u$.

Start defining the generalized state feedback by

$$v_1 = \phi_{01}(x) + \phi_{11}(x)\,u \tag{11}$$

where v_1 is a new independent input.

From the definition of ρ_2 ($\rho_1 \leq \rho_2$) it follows that

$$\begin{aligned} y_2^{(k)} &= y_2^{(k)}\left(x,\, y_1^{(\lambda)},\, \lambda \geq \rho_1\right) \quad \text{for any } k < \rho_2, \\ y_2^{(\rho_2)} &= \frac{\partial y_2^{(\rho_2-1)}}{\partial x}(f+gu+pw) + \sum_{\lambda} \frac{\partial y_2^{(\rho_2-1)}}{\partial y_1^{(\lambda)}} y_1^{(\lambda+1)}. \end{aligned}$$

Since

$$dy_2^{(\rho_2-1)} - \sum_{\lambda \geq \rho_1} \frac{\partial y_2^{(\rho_2-1)}}{\partial y_1^{(\lambda)}} dy_1^{(\lambda)} = \frac{\partial y_2^{(\rho_2-1)}}{\partial x} dx \in \mathcal{X} \cap \mathcal{Y},$$

from (9), $y_2^{(\rho_2)} = \phi_{02}\left(x, y_1^{(\lambda)}\right) + \phi_{12}\left(x, y_1^{(\lambda)}\right) u$. The second step for the definition of the generalized state feedback is then

$$v_2 = \phi_{02}\left(x, v_1, \ldots, v_1^{(k)}\right) + \phi_{12}\left(x, v_1, \ldots, v_1^{(k)}\right) u. \tag{12}$$

Repeating the foregoing process, one defines $v_1, \ldots, v_q$ which can be arbitrarily completed to define a regular generalized feedback solution to the GDDP.

Necessity. Assume that (9) is not true, then there exists $\zeta \in \mathcal{X} \cap \mathcal{Y}$ such that $\langle \zeta, p \rangle \neq 0$. By Lemma 2.1, there exists $dy_i^{(k)}$ with $k < \rho_i$, $1 \leq i \leq p$, such that $dy_i^{(k)} = \zeta + a_{i1}\, dv_{i1} + a_{i2}\, dv_{i2} + \cdots + a_{is}\, dv_{is}$ where $dv_{ij} \in \text{span}\{dv_1, d\dot{v}_1, \ldots, dv_q, d\dot{v}_q, \ldots\}$. Thus, applying any generalized transformation Φ the output $\Phi(dy_i^{(k)})$ of the closed loop system equals $\Phi(\zeta) + \Phi\left(a_{i1}\, dv_{i1} + a_{i2}\, dv_{i2} + \ldots + a_{is}\, dv_{is}\right)$. Since $\Phi(a_{i1}\, dv_{i1} + a_{i2}\, dv_{i2} + \ldots + a_{is}\, dv_{is}) \in \Phi(\mathcal{X}) \oplus \mathcal{V}$, and $\Phi(\zeta) \notin \Phi(\mathcal{X}) \oplus \mathcal{V}$, then $\Phi\left(dy_i^{(k)}\right) \notin \Phi(\mathcal{X}) \oplus \mathcal{V}$, which stands in contradiction. □

From the proof of Theorem 2.2 one gets also a structural condition for the solvability of the GDDP, *i. e.* the GDDP is solvable if and only if $\text{span}\{dv_1, \ldots, dv_q\} \subset \mathcal{X} \oplus \mathcal{U}$. By this condition and by the algorithm used in [3] one derives a dynamic state feedback solution to the DDP of (5), consequently one has:

Theorem 2.3. DDDP is solvable if and only if GDDP is solvable.

Condition (9) is an alternative condition to Theorem 2.3 in [5].

Example [6].

$$\begin{aligned} &\dot{x}_1 = x_2\, u_1, && \dot{x}_2 = 5, && \dot{x}_3 = x_2 + x_4 + x_4\, u_1, \\ &\dot{x}_4 = u_2, && \dot{x}_5 = x_1\, u_1 + w, && \\ &y_1 = x_1 && y_2 = x_3. && \end{aligned}$$

One computes $\dot{y}_2 = x_2 + x_4 + x_4 \dfrac{\dot{y}_1}{x_2}$, thus $\mathcal{X} \cap \mathcal{Y} = \text{span}\left\{dx_1,\, dx_3,\, \left(1 - \dfrac{x_4\, \dot{y}_1}{x_2^2}\right) dx_2 + \left(1 + \dfrac{\dot{y}_1}{x_2}\right) dx_4\right\}$ and condition (9) is satisfied although Δ^* is zero.

3. CONCLUSION

A necessary and sufficient condition for the solvability of the GDDP and DDDP has been given. The condition is intrinsic to the system and does not depend on any arbitrary state space extension. Only the data of the original system are involved in (9). The equivalence between GDDP and DDDP should be viewed as the generalization of the equivalence of DDP and DDDP known for linear system.

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