Kybernetika

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Kybernetika, Vol. 18 (1982), No. 6, 545--554

Persistent URL: `http://dml.cz/dmlcz/124850`

KYBERNETIKA — VOLUME *18* (1982), NUMBER 6

FORMALIZED MODELS OF ONTOLOGICAL SYSTEMS

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The article studies questions of real systems formalized models. The notion model of a system is defined on the base of two system relation. There are further specified concepts model$_1$, model$_2$ and model$_3$ and formulated their properties. After introduction of language system connection between notions semantic model of language system and the models of the types 1, 2 and 3 is investigated. A few illustrations of mentioned terms and relations adjoint the paper.

1. BASIC TERMS AND RELATIONS

In this paper, following [5] and [6], I like to study questions of formalized models of real systems.

First I try to specify notion "model of a system". This specification should be in accordance with the common utilization of that in contemporary science like mathematics, semantics, systems theory and physics.

The notion "model" is currently used in different sciences in various sense and meanings. Some authors from logical semantics field use the term „model" and specified that by means of a certain relation between theoretical language sentences class and a sequence of objects from universe of that language. The Tarsky's approach became classical in this regard. He understands by "model of a class L of sentences" any sequence of objects which satisfies every sentence from the class L', where sentences L' are formed from those of L by replacement of all non-logical constants from L by uniquely determined variables.

In further text I take this approach for "semantic model". Suppes takes "model" formed by Tarsky for non-linguistic entity or linguistic entity. In the second case we talk usually about "language model".

Braithwaite forms "model of a theory" as a theory which corresponds with the first as to its deductive structure. This approach is very close to mathematical one, since mathematicians usually assign to a mathematical theory as its model another mathematical theory.

I intend to specify the concept "model of a system" by the use of "relation between two systems". These systems can be both real and language ones. The notion "model" will be related to isomorphy or homomorphy of systems.

In accordance with well-known Carnap's concept I first specify the notion "correlator of two relations at a moment t_i".

D 1. Two member relation $\mathscr{C}$ is a *correlator* of relations R_k and R_l at a moment t_i (symbolically: $\langle \mathscr{C}, R_k^{(j)}, R_l^{(j)}, t_i \rangle \in \mathscr{C}orr$) if coincidentally holds:

a) the relation $\mathscr{C}$ is mutually unique at the moment t_i,

b) every element from relation $R_k^{(j)}$ field belongs to the domain of $\mathscr{C}$ at the moment t_i,

c) every element from relation $R_l^{(j)}$ field belongs to the range of $\mathscr{C}$ at the moment t_i,

d) for each pair of elements a, b holds:

if $\langle a, b \rangle \in \mathscr{C}$ at a moment t_i, then a is element of $R_k^{(j)}$ field at the moment t_i, iff b. is element of $R_l^{(j)}$ field at the moment t_i.

For further use, let me state a few following definitions:

D. 2. Relation $R_k^{(j)}$ is *isomorphic* with respect to relation $R_l^{(j)}$ at a moment t_i (symbolically: $\langle R_k^{(j)}, R_l^{(j)}, t_i \rangle \in \mathscr{I}s$), if there exists two member relation $\mathscr{C}$ such that

$$\langle \mathscr{C}, R_k^{(j)}, R_l^{(j)}, t_i \rangle \in \mathscr{C}orr$$

D. 3. Relation $R_k^{(j)}$ is homomorphic with respect to relation $R_l^{(j)}$ at a moment t_i (symbolically: $\langle R_k^{(j)}, R_l^{(j)}, t_i \rangle \in \mathscr{H}om$), if there exists mapping $\boldsymbol{Z}$ from the relation $R_k^{(j)}$ field on some from its subsets such that the transformation $\boldsymbol{Z}$ uniquely assigns the relation $R_k^{(j)}$ a relation $S_k^{(j)}$, where for every j-tuple of elements from the relation $R_k^{(j)}$ field holds:

if $\langle a_1, \ldots, a_j, t_i \rangle \in R_k^{(j)} \times T$ then $\langle \boldsymbol{Z}(a_1), \ldots, \boldsymbol{Z}(a_j), t_i \rangle \in S_k^{(j)} \times T$ and coincidentally $\langle S_k^{(j)}, R_l'^{(j)}, t_i \rangle \in \mathscr{I}s$.

D. 4. System $\mathscr{S}_1 = \langle \boldsymbol{U}_1, \mathscr{R}_1 \rangle$ is *isomorphic* with respect to system $\mathscr{S}_2 = \langle \boldsymbol{U}_2, \mathscr{R}_2 \rangle$ at a moment t_i (symbolically: $\langle \mathscr{S}_1, \mathscr{S}_2, t_i \rangle \in \mathscr{I}s\mathscr{S}yst$), if coincidentally holds:

a) there exist intervals Δt, $\Delta t'$ so that $t_i \in \Delta t$, $t_i \in \Delta t'$,

$$\langle \mathscr{S}_1, \Delta t \rangle \in \mathscr{S}yst\,, \quad \langle \mathscr{S}_2, \Delta t' \rangle \in \mathscr{S}yst$$

b) for every $R_k^{(j)} \in \mathscr{R}_1$ there is just one $R_l'^{(j)} \in \mathscr{R}_2$ so that

$$\langle R_k^{(j)}, R_l'^{(j)}, t_i \in \mathscr{I}s$$

c) for every $R_l'^{(j)} \in \mathscr{R}_2$ there is just one $R_k^{(j)} \in \mathscr{R}_1$ so that

$$\langle R_l'^{(j)}, R_k^{(j)}, t_i \rangle \in \mathscr{I}s\,.$$

Likewise let us specify homomorphy of two systems:

D. 5. System $\mathscr{S}_1 = \langle \boldsymbol{U}_1, \mathscr{R}_1 \rangle$ is *homomorphic* with respect to system $\mathscr{S}_2 = \langle \boldsymbol{U}_2, \mathscr{R}_2 \rangle$ at a moment t_i (symbolically: $\langle \mathscr{S}_1, \mathscr{S}_2, t_i \rangle \in \mathscr{H}\mathit{omsyst}$), if coincidentally holds:

a) there are intervals Δt, $\Delta t'$ so that $t_i \in \Delta t$, $t_i \in \Delta t'$

$$\langle \mathscr{S}_1, \Delta t \rangle \in \mathscr{S}\mathit{yst}\,, \quad \langle \mathscr{S}_2, \Delta t' \rangle \in \mathscr{S}\mathit{yst}$$

b) for every $R_k^{(j)} \in \mathscr{R}_1$ there exists just one $R_l'^{(j)} \in \mathscr{R}_2$ so that

$$\langle R_k^{(j)}, R_l'^{(j)}, t_i \rangle \in \mathscr{H}\mathit{om}$$

c) for every $R_l'^{(j)} \in \mathscr{R}_2$ there is just one $R_k^{(j)} \in \mathscr{R}_1$ so that

$$\langle R_k^{(j)}, R_l'^{(j)}, t_i \rangle \in \mathscr{H}\mathit{om}$$

Remark. It is obvious that if the mapping $\boldsymbol{Z}$ is "merely" identity, then homomorphy of two relations "degenerates" to their isomorphy. Further, if at a moment t_i system $\mathscr{S}_1$ is homomorphic with respect to system $\mathscr{S}_2$, then there exists subsystem $\mathscr{S}_1'$ of the system $\mathscr{S}_1$ at this moment such that $\mathscr{S}_1'$, $\mathscr{S}_2$ are isomorphic at the moment t_i.
In such a case the mapping $\boldsymbol{Z}$ "reducing" system $\mathscr{S}_1$ on $\mathscr{S}_1'$ is that transformation which forms in $\mathscr{S}_1$ its subsystem $\mathscr{S}_1'$.

Now, when using stated terms, we shall form three others:

D. 6. System $\mathscr{S}_1$ is *model*$_1$ of system $\mathscr{S}_2$ during a period Δt (symbolically: $\langle \mathscr{S}_1, \mathscr{S}_2, \Delta t \rangle \in \mathscr{M}\mathit{od}_1$), if coincidentally holds:

a) there exist such intervals $\Delta t'$, $\Delta t''$ that $\Delta t = \Delta t' \cap \Delta t''$,

$$\langle \mathscr{S}_1, \Delta t' \in \mathscr{S}\mathit{yst}\,, \quad \langle \mathscr{S}_2, \Delta t'' \rangle \in \mathscr{S}\mathit{yst}$$

b) for every $t_i \in \Delta t$ holds: $\langle \mathscr{S}_1, \mathscr{S}_2, t_i \rangle \in \mathscr{I}\mathit{sosyst}$.

Proposed statement of "strong" concept "model$_1$ of system" is adequate to those conceptions which form notion "model" exclusively by isomorphy of system approach.
Weaker concept "model$_2$" related to existence of systems homomorphy can be stated like this:

D. 7. System $\mathscr{S}_1$ is *model*$_2$ of system $\mathscr{S}_2$ within period Δt (symbolically: $\langle \mathscr{S}_1, \mathscr{S}_2, \Delta t \rangle \in \mathscr{M}\mathit{od}_2$), if coincidentally holds:

a) there are intervals $\Delta t'$, $\Delta t''$ that $\Delta t = \Delta t' \cap \Delta t''$,

$$\langle \mathscr{S}_1, \Delta t' \rangle \in \mathscr{S}\mathit{yst}\,, \quad \langle \mathscr{S}_2, \Delta t'' \rangle \in \mathscr{S}\mathit{yst}$$

b) for every $t_i \in \Delta t$ holds: $\langle \mathscr{S}_1, \mathscr{S}_2, t_i \rangle \in \mathscr{H}\mathit{omsyst}$.

It seems to be useful to introduce one more term "model$_3$". Following specification deals with this concept regarding two systems, in each of them can be formed a subsystem such that those would be isomorphic.

D. 8. System $\mathcal{S}_1$ is *model$_3$* of system $\mathcal{S}_2$ in period Δt (symbolically: $\langle \mathcal{S}_1, \mathcal{S}_2, \Delta t \rangle \in \in \mathcal{M}od_3$), if coincidentally holds:

a) there exist such intervals $\Delta t'$, $\Delta t''$ that $\Delta t = \Delta t' \cap \Delta t''$

$$\langle \mathcal{S}_1, \Delta t' \rangle \in \mathcal{S}yst\,, \quad \langle \mathcal{S}_2, \Delta t'' \rangle \in \mathcal{S}yst$$

b) there is a mapping $\boldsymbol{Z}$ defining on $\mathcal{S}_1$ subsystem $\mathcal{S}'_1$, there is a mapping $\boldsymbol{Z}'$ defining on $\mathcal{S}_2$ subsystem $\mathcal{S}'_2$, and for every $t_i \in \Delta t$ holds: $\langle \mathcal{S}'_1, \mathcal{S}'_2, t_i \rangle \in \in \mathcal{I}ssyst$.

This specification well satisfies a way of utilization of concept "model" where one system is being modelled by another one due to their mutual correspondence of only some of their aspects.

From previously formed specification we can conclude some interesting statements: For every system $\mathcal{S}_1$, $\mathcal{S}_2$, $\mathcal{S}_3$ existing within given period holds:

A 1. Relation $\mathcal{M}od_1, \mathcal{M}od_2, \mathcal{M}od_3$ is in every existentional interval of the system reflexive in class of system.*)

A 2. Relation $\mathcal{M}od_1$ and $\mathcal{M}od_3$ is in every interval, where is defined, symmetric in class of systems.

A 3. Relation $\mathcal{M}od_1$ and $\mathcal{M}od_2$ is transitive in class of systems in given intervals Δt_1, Δt_2 in the following sense:

If $[\langle \mathcal{S}_1, \mathcal{S}_2, \Delta t_1 \rangle \in \mathcal{M}od_1$ and coincidentally $\langle \mathcal{S}_2, \mathcal{S}_3, \Delta t_2 \rangle \in \mathcal{M}od_1]$ then [if $\Delta t_1 \leqq \Delta t_2 \Rightarrow \langle \mathcal{S}_1, \mathcal{S}_3, \Delta t_1 \rangle \in \mathcal{M}od_1]$ and coincidentally [if $\Delta t_2 \leqq \Delta t_1 \Rightarrow \langle \mathcal{S}_1, \mathcal{S}_3, \Delta t_2 \rangle \in \mathcal{M}od_1]$.

The same assertion holds for relation $\mathcal{M}od_2$ (obtainable by replacing $\mathcal{M}od_1$ by $\mathcal{M}od_2$).

A 4. If $\langle \mathcal{S}_1, \mathcal{S}_2, \Delta t \rangle \in \mathcal{M}od_1$ then $\langle \mathcal{S}_1, \mathcal{S}_2, \Delta t \rangle \in \mathcal{M}od_2$.

A 5. $\langle \mathcal{S}_1, \mathcal{S}_2, \Delta t \rangle \in \mathcal{M}od_2$ then $\langle \mathcal{S}_1, \mathcal{S}_2, \Delta t \rangle \in \mathcal{M}od_3$,

A 6. For every intervals Δt_1, Δt_2 holds:

a) if $[\langle \mathcal{S}_1, \mathcal{S}_2, \Delta t_1 \rangle \in \mathcal{M}od_1$ and coincidentally $\langle \mathcal{S}_2, \mathcal{S}_3, \Delta t_2 \rangle \in \mathcal{M}od_2]$ then
if $[(\Delta t_1 \leqq \Delta t_2$ then $\langle \mathcal{S}_1, \mathcal{S}_3, \Delta t_1 \rangle \in \mathcal{M}od_2)$ and coincidentally
if $(\Delta t_2 \leqq \Delta t_1$ then $\langle \mathcal{S}_1, \mathcal{S}_3, \Delta t_2 \rangle \in \mathcal{M}od_2)]$

*) Concept of reflexive, symmetric and transitive relation is used here in generalized sense. Relation R is defined to be reflexive in a given class, if for every element a of this class and for every moment t_i holds: the element a is at the moment t_i in a relation R with itself. Likewise for symmetricity and transitivity of relations.

b) if $[\langle \mathcal{S}_1, \mathcal{S}_2, \Delta t_1\rangle \in \mathcal{M}od_2$ and coincidentally $\langle \mathcal{S}_2, \mathcal{S}_3, \Delta t_2\rangle \in \mathcal{M}od_1]$ then
if $[(\Delta t_1 \leqq \Delta t_2$ then $\langle \mathcal{S}_1, \mathcal{S}_3, \Delta t_1\rangle \in \mathcal{M}od_2)$ and coincidentally
if $(\Delta t_2 \leqq \Delta t_1$ then $\langle \mathcal{S}_1, \mathcal{S}_3, \Delta t_2\rangle \in \mathcal{M}od_2)]$.

c) The statement a) with replacement $\mathcal{M}od_2$ by $\mathcal{M}od_3$.

d) The statement b) with replacement $\mathcal{M}od_2$ by $\mathcal{M}od_3$.

e) if $[\langle \mathcal{S}_1, \mathcal{S}_2, \Delta t_1\rangle \in \mathcal{M}od_2$ and coincidentally $\langle \mathcal{S}_2, \mathcal{S}_3, \Delta t_2\rangle \in \mathcal{M}od_3]$ then
if $[\Delta t_1 \leqq \Delta t_2$ then $\langle \mathcal{S}_1, \mathcal{S}_2, \Delta t_1\rangle \in \mathcal{M}od_3]$ and coincidentally
if $[\Delta t_2 \leqq \Delta t_1$ then $\langle \mathcal{S}_1, \mathcal{S}_3, \Delta t_2\rangle \in \mathcal{M}od_3]$.

Relation $\mathcal{M}od_1$ is at every moment reflexive, symmetric and transitive in class of systems and hence is relation of equivalence. For this reason class of systems can be subdivided into subclasses consisting of those systems which are mutually models$_1$. These classes of systems play an important role at systems modelling.

2. LANGUAGE MODELLING OF ONTOLOGICAL SYSTEMS

I shall interpret following reasonings for exact language systems. These systems of the type $\mathcal{S}_L$ are considered to be an ordered pair $\langle U_L, \mathcal{R}_L\rangle$ where

U_L is set of language terms-nonlogical constants of the language L, in which the system is built-up,

$\mathcal{R}_L$ is set of true-sentences of the language L constructed as sequences of elements from U_L, names of moments and terms submitted by logical base of language L (variables, logical connectives and logical operators).

If the language system L is built-up as formalized language system, then among individual elements of the set $\mathcal{R}_L$ (i.e. among true-sentences of the language L) there is a deduction relation. About membership of elements from this relation we talk in metalanguage of the language L.
The notion "language (linquistic) system" can be hence formulated as follows:

D 9. System $\mathcal{S}_L$ is that of language L within a period Δt_i (symbolically: $\langle \mathcal{S}_L, \Delta t\rangle \in L-\mathcal{S}yst$), if coincidentally holds:

a) $$\langle \mathcal{S}_L, \Delta t\rangle \in \mathcal{S}yst$$

b) there exist sets U_L, $\mathcal{R}_L$ such that $\mathcal{S}_L = \langle U_L, \mathcal{R}_L\rangle$ where

U_L is set of nonlogical constants of the language L

$\mathcal{R}_L$ is set of correctly formed sentences sets of the language L valid on the interval Δt,

c) each of these sets of the type ${}^{(S_{j,k})}R_{L,k} \in \mathscr{R}_L$ consists of sentences of the type

$$^{(S_{j,k})}R_{L,k_i} \subset U^{<j>} \times \Delta t^{<\tau>} \times \mathscr{L}^{<l>}$$

where

$1 \leqq j \leqq n,\ 1 \leqq k \leqq i_j,\ 1 \leqq S_{j,k} \leqq S,\ l \geqq 2$

j, n, k, i_j, s, l are natural numerical variables

$S_{j,k}$ is type denomination corresponding with that of highest degree predicate constant in the sentence occurring

j denotes number of places of nonlogical constants from this sentence

k is an ordered number of the given sentence with an upper index j

l is a number of occurrences of symbols from the set L

$\mathscr{L}$ is set of all variables, logical connectives, logical operators of the language L and brackets,

τ is number of places of occurences of symbols from interval Δt.

In proposed interpretation: language L of the system $\mathscr{S}_L$ is that of utmost s-th order, i.e. envolves sentences with predicates of max s-th order. Each of these (correctly formed) sentences is an ordered set consisting of nonlogical constants (individual constants, predicate constants of various orders), further of a name of particular moment (belonging to interval Δt), individual variables and predicate variables of different orders, logical connectives, logical operators and auxiliary symbols (brackets etc.)

For example, let a sentence ${}^{(1)}R^{(2)}_{L,k}$ of language system $\mathscr{S}_L$ have a type:

$$^{(1)}R_k(a_1)\ \bar{t}_i$$

where ${}^{(1)}R_k$, $a_1 \in U_L$, $\bar{t}_i$ is a name of moment t_i in the language L, $\Delta\bar{t}$ is ordered se of names of all moments belonging to Δt(,) $\in \mathscr{L}$

hence

$$^{(1)}R_k \subset U^{<2>} \times \Delta\bar{t} \times \mathscr{L}^{<2>}$$

Let a sentence ${}^{(2)}R^{(3)}_{L,k}$ of the language system $\mathscr{S}_L$ have a form:

$$^{(2)}R_{k,2}({}^{(1)}R_{k,1},\ a,\ \bar{t}_i)$$

("Between a property ${}^{(1)}R_{k,1}$ and element a is at a moment $\bar{t}_i$ a relation ${}^{(2)}R_{k,2}$"), where ${}^{(2)}R_{k,2}$, ${}^{(1)}R_{k,1}$, $a \in U_L$, $\bar{t}_i \in \Delta t$, (,) $\in \mathscr{L}$

hence

$$^{(2)}R^{(3)}_{L,k} \subset U_L^{<3>} \times \Delta t \times \mathscr{L}^{<2>}$$

Let a sentence ${}^{(1)}R^{(s)}_{L,k}$ of the system $\mathscr{S}_L$ be in the form:

$$\forall x\ {}^{(1)}R_{k,1}(x,\ a_1,\ \bar{t}_i) \rightarrow \sim {}^{(1)}R_{k,2}(a_1,\ a_2,\ \bar{t}_i)$$

where

$$^{(1)}R_{k,1},\ {}^{(1)}R_{k,2},\ a_1,\ a_2 \in U_L\ , \quad \bar{t}_i \in \Delta t\ , \quad x, \forall, \rightarrow, \sim, (\ ,\ ) \in \mathscr{L}$$

hence

$$^{(1)}R^{(s)}_{L,k} \subset U_L^{<5>} \times \Delta t^{<2>} \times \mathscr{L}^{<9>}$$

Let us investigate if traditional definition of notion "semantic model of language system" corresponds with some of our three models model$_1$, model$_2$ and model$_3$ presumably considered as a relation between ontological and language systems.

Let us consider first a simple language system, whose sentences are simple statements (without logical connectives), not envolving any quantificated variables, i.e. from the set $\mathscr{L}$ there will be only brackets.

Let there be given a real, ontological system:

$$\mathscr{S}_0 = \langle U_0, \mathscr{R}_0 \rangle \quad \text{on interval } \Delta t$$
$$U_0 = \{a_1, a_2, a_3\}, \qquad \mathscr{R}_0 = \{{}^{(1)}R_1^{(1)}, {}^{(1)}R_1^{(2)}, {}^{(2)}R_1^{(2)}\},$$
$${}^{(1)}R_1^{(1)} = \{\langle a_1, t_i \rangle, \langle a_3, t_j \rangle\}, \quad {}^{(1)}R_1^{(2)} = \{\langle a_1, a_2, t_j \rangle, \langle a_2, a_2, t_i \rangle\}$$
$${}^{(2)}R_1^{(2)} = \{\langle {}^{(1)}R_1^{(1)}, {}^{(1)}R_1^{(2)}, t_i \rangle; \ (t_i, t_j \in \Delta t$$

Let further be given a system of the second order language:

$$\mathscr{S}_L = \{U_L, \mathscr{R}_L\} \text{ also on the interval } \Delta t$$
$$U_L = \{{}^{(1)}R_1^{(1)}, {}^{(1)}R_1^{(2)}, {}^{(2)}R_1^{(2)}, a_1, a_2, a_3\}, \quad \mathscr{R}_L = \{{}^{(1)}R_{L1}^{(1)}, {}^{(1)}R_{L1}^{(2)}, {}^{(2)}R_{L1}^{(2)}\},$$
$${}^{(1)}R_{L1}^{(1)} = \{{}^{(1)}R_{L11}^{(1)}, {}^{(1)}R_{L12}^{(1)}\}, \quad {}^{(1)}R_{L1}^{(2)} = \{{}^{(1)}R_{L11}^{(2)}, {}^{(1)}R_{L12}^{(2)}\}, \quad {}^{(2)}R_{L1}^{(2)} = \{{}^{(2)}R_{L11}^{(2)}\}$$
$${}^{(1)}R_{L11}^{(1)} = {}^{(1)}R_1^{(1)}(a_1, \bar{t}_i), \qquad {}^{(1)}R_{L12}^{(1)} = {}^{(1)}R_1^{(1)}(a_3, \bar{t}_j)$$
$${}^{(1)}R_{L11}^{(2)} = {}^{(1)}R_1^{(2)}(a_1, a_2, \bar{t}_j), \qquad {}^{(1)}R_{L12}^{(2)} = {}^{(1)}R_1^{(2)}(a_2, a_2, \bar{t}_i)$$
$${}^{(2)}R_{L11}^{(2)} = {}^{(2)}R_1^{(2)}({}^{(1)}R_1^{(1)}, {}^{(1)}R_1^{(2)}, \bar{t}_i)$$

Particular sentences from sets of $\mathscr{R}_L$ describe hence only properties and relations of given objects of the system $\mathscr{S}_0$ at given moments.

The language system $\mathscr{S}_L$ (whose sentences are considered to be syntactic structures) is exact description of all relations of system concrete elements. (Hence $\mathscr{S}_L$ is only a descriptive system, whose task is nothing more).

The system $\mathscr{S}_0$ can be understood as semantic model of system $\mathscr{S}_L$ and on the contrary, system $\mathscr{S}_L$ can be considered as language model of the system $\mathscr{S}_0$.
Under given conditions it is possible to define a correlator uniquely assigning individual elements of sets from $\mathscr{R}_0$ particular elements of sets from $\mathscr{R}_L$ and contrary:

to element $\langle a_1, t_i \rangle \in {}^{(1)}R_1^{(1)}$	assigns element ${}^{(1)}R_1^{(1)}(a_1, t_i)$
to element $\langle a_3, t_j \rangle \in {}^{(1)}R_1^{(1)}$	assigns element ${}^{(1)}R_1^{(1)}(a_3, t_j)$
to element $\langle a_1, a_2, t_j \rangle \in {}^{(1)}R_1^{(2)}$	assigns element ${}^{(1)}R_1^{(2)}(a_1, a_2, t_j)$
to element $\langle a_2, a_2, t_i \rangle \in {}^{(1)}R_1^{(2)}$	assigns element ${}^{(1)}R_1^{(2)}(a_2, a_2, t_i)$
to element $\langle {}^{(1)}R_1^{(1)}, {}^{(1)}R_1^{(2)}, t_i \rangle \in {}^{(2)}R_1^{(2)}$	assigns element ${}^{(2)}R_1^{(2)}({}^{(1)}R_1^{(1)}, {}^{(1)}R_1^{(2)}, t_i)$

From $\mathscr{S}_L$ towards $\mathscr{S}_0$ this assignment meets conditions of interpretation, if

a_1	is denotat of	$\bar{a}_1$
a_2		$\bar{a}_2$
a_3		$\bar{a}_3$
${}^{(1)}R_1^{(1)}$		${}^{(1)}\bar{R}_1^{(1)}$

${}^{(1)}R_1^{(2)}$	${}^{(1)}\bar{R}_1^{(2)}$
${}^{(2)}R_1^{(2)}$	${}^{(2)}\bar{R}_1^{(2)}$
t_i	$\bar{t}_i$
t_j	$\bar{t}_j$
Δt	$\Delta\bar{t}$

In the direction from $\mathscr{S}_0$ to $\mathscr{S}_L$ this correspondence can be considered as exact description. In accordance with D 4., the systems $\mathscr{S}_0$ and $\mathscr{S}_L$ are isomorphic at every moment from Δt and mutually serve each other as models_1 on the interval Δt.

Systems homomorphy can be investigated, if a real system $\mathscr{S}_0$ would not be satisfactorily described by system $\mathscr{S}_L$, but only some characteristics of elements from the system $\mathscr{S}_0$ would be modelled by means of the system $\mathscr{S}_L$.

Let us consider, for example, language system $\mathscr{S}'_L = \langle U'_L, \mathscr{R}'_L \rangle$ on Δt:

$$
\begin{aligned}
&\boldsymbol{U}'_L = \{{}^{(1)}R_1^{(1)}, {}^{(1)}R_1^{(2)}, a_1, a_2\} \qquad \mathscr{R}'_L = \{{}^{(1)}R_{L1}^{(1)}, {}^{(1)}R_{L1}^{(2)}\} \\
&{}^{(1)}R_{L1}^{(1)} = \{{}^{(1)}R_{L11}^{(1)}\}, \qquad {}^{(1)}R_{L11}^{(2)} = \{{}^{(1)}R_{L11}^{(2)}, {}^{(1)}R_{L12}^{(2)}\} \\
&{}^{(1)}R_{L11}^{(1)} = {}^{(1)}R_1^{(1)}(a_1, t_i) \\
&{}^{(1)}R_{L11}^{(2)} = {}^{(1)}R_1^{(1)}(a_1, t_i) \\
&{}^{(1)}R_{L11}^{(2)} = {}^{(1)}R_1^{(2)}(a_1, a_2, t_j), \; {}^{(1)}R_{L12}^{(2)} = {}^{(1)}R_1^{(2)}(a_2, a_2, t_i), \quad (t_i, t_j) \in \Delta t
\end{aligned}
$$

Let us define mapping $\boldsymbol{Z}$ associating sets from $\mathscr{R}_0$ of the system $\mathscr{S}_0$:

- set ${}^{(1)}R_1^{(1)}$ with set ${}^{(1)}R_1^{(1)} = \{\langle a_1, t_i \rangle\}$
- set ${}^{(1)}R_1^{(2)}$ again with set ${}^{(1)}R_1^{(2)}$
- set ${}^{(2)}R_1^{(2)}$ the empty set.

Result of this mapping will be subsystem:

$$
\begin{aligned}
&\mathscr{S}'_0 = \langle \boldsymbol{U}'_0, \mathscr{R}'_0 \rangle \quad \text{on interval } \Delta t \\
&\boldsymbol{U}'_0 = \{a_1, a_2\}, \quad \mathscr{R}'_0 = \{{}^{(1)}R_1^{(1)}, {}^{(1)}R_1^{(2)}\} \\
&{}^{(1)}R_1^{(1)} = \{\langle a_1, t_i \rangle\} \\
&{}^{(1)}R_1^{(2)} = \{\langle a_1, a_2, t_j \rangle, \langle a_2, a_2, t_i \rangle\}, \quad (t_i, t_j) \in \Delta t
\end{aligned}
$$

Easily we find a correlator between elements of given sets proving thus isomorphy of $\mathscr{S}'_L$ and $\mathscr{S}'_0$. Hence the system $\mathscr{S}_0$ is a model_2 of the system $\mathscr{S}'_L$ on the interval Δt. Also in this case, interpretation of sentences from $\mathscr{S}'_L$ is correct, if conditions of denotation are met. So I do hope that in both cases, model_1 and model_2, we can talk about semantic models.

What about "model_3"?

Let us consider again a language system:

$$
\begin{aligned}
&\mathscr{S}''_L = \langle \boldsymbol{U}''_L, \mathscr{R}''_L \rangle \quad \text{on interval} \quad \Delta t \\
&\boldsymbol{U}''_L = \{{}^{(1)}R_1^{(1)}, {}^{(1)}R_1^{(2)}, {}^{(1)}R_2^{(2)}, a_1, a_2, a_4\}, \quad \mathscr{R}''_L = {}^{(1)}R_{L_1}^{(1)}, {}^{(1)}R_{L1}^{(2)}, {}^{(1)}R_{L2}^{(1)}\}, \\
&{}^{(1)}R_{L1}^{(1)} = \{{}^{(1)}R_{L11}^{(1)}, {}^{(1)}R_{L12}^{(1)}, {}^{(1)}R_{L13}^{(1)}\},
\end{aligned}
$$

$$
\begin{aligned}
&{}^{(1)}R_{L1}^{(2)} = \{{}^{(1)}R_{L11}^{(2)}, {}^{(1)}R_{L12}^{(2)}\}, \quad {}^{(1)}R_{L2}^{(2)} = \{{}^{(1)}R_{L21}^{(2)}\} \\
&{}^{(1)}R_{L11}^{(1)} = {}^{(1)}R_1^{(1)}(a_1, t_i), \quad {}^{(1)}R_{L12}^{(1)} = {}^{(1)}R_1^{(1)}(a_1, t_j), \quad {}^{(1)}R_{L13}^{(1)} = {}^{(1)}R_1^{(1)}(a_4, t_k) \\
&{}^{(1)}R_{L11}^{(2)} = {}^{(1)}R_1^{(2)}(a_1, a_2, t_j), \quad {}^{(1)}R_{L12}^{(2)} = {}^{(1)}R_1^{(2)}(a_2, a_2, t_i) \\
&{}^{(1)}R_{L21}^{(2)} = {}^{(1)}R_2^{(2)}(a_1, a_4, t_i)
\end{aligned}
$$

Let us define a mapping assigning:

to set ${}^{(1)}R_{L1}^{(1)}$ set $\{{}^{(1)}R_{L11}^{(1)}\}$
to set ${}^{(1)}R_{L1}^{(2)}$ set $\{{}^{(1)}R_{L11}^{(2)}, {}^{(1)}R_{L12}^{(2)}\}$
to set ${}^{(1)}R_{L2}^{(2)}$ empty set.

The result will be a language subsystem:

$$
\begin{aligned}
&\mathscr{S}_L''' = \langle \boldsymbol{U}_L''', \mathscr{R}_L''' \rangle \quad \text{on interval } \Delta t \\
&\boldsymbol{U}_L''' = \{{}^{(1)}R_1^{(1)}, {}^{(1)}R_2^{(2)}, a_1, a_2\}, \quad \mathscr{R}_L''' = \{{}^{(1)}R_{L1}'^{(1)}, {}^{(1)}R_{L1}'^{(2)}\} \\
&{}^{(1)}R_{L1}'^{(1)} = \{{}^{(1)}R_{L11}^{(1)}\} \\
&{}^{(1)}R_{L1}'^{(2)} = \{{}^{(1)}R_{L11}^{(2)}, {}^{(1)}R_{L12}^{(2)}\}, \quad {}^{(1)}R_{L11}^{(1)} = {}^{(1)}R_1^{(1)}(a_1, t_i) \\
&{}^{(1)}R_{L11}^{(2)} = {}^{(1)}R_1^{(2)}(a_1, a_2, t_j), \quad {}^{(1)}R_{L12}^{(2)} = {}^{(1)}R_1^{(2)}(a_2, a_2, t_i), \quad (i_i, t_j) \in \Delta t
\end{aligned}
$$

$\mathscr{S}_L'''$ is obviously identical with $\mathscr{S}'_L$.

The subsystems $\mathscr{S}'_0$ and $\mathscr{S}'_L$ are, as we have already seen, isomorphic and by specification D 8. is the system $\mathscr{S}_0$ model$_3$ of the system $\mathscr{S}''$ on the interval Δt. It is however obvious that $\mathscr{S}_0$ is not semantic model of the language system $\mathscr{S}''_L$, since not all sentences of $\mathscr{S}''_L$ are correctly interpreted by means of objects relations from $\mathscr{S}_0$. $\mathscr{S}'_L$ is "too rich" in terms with respect to the system $\mathscr{S}_0$.

From above mentioned examples it is clear that in case of descriptive language systems having only simple sentences – statements without quantificated variables, it is possible to obtain from notions "model$_1$", "model$_2$" and "model$_3$" concept "semantic model" by following specification: modelling system is ontological one, while modelled system is language one.

On the other hand, this specification seems to be excessive, since mathematicians usually talk about (semantic) model in a relation between two mathematical models. In such cases we require only a language system as modelled one and modelling system can also be a language system. Isomorphy of both systems is then demanded. In this case of modelling we may only omit requirements regarding simplicity of all sentences from language system and absence of quantificated variables. We compare namely two language systems on the base of their mutual correspondence and logical structures.

We may conclude our investigation by finding out that explication of principle notions of systems theory and from that arising unification of terms from logical semantic is currently very actual task enabling mathematicians, logicians and scientists from different spheres to find common ground by approaching their fields as studies of large and complicated dynamic systems.

(Received December 11, 1981.)

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