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OUTPUT TRACKING FOR A FAMILY OF LINEAR PLANTS[1]

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For a given family of linear, continuous-time plants depending on a vector of parameters, the problem of designing a controller ensuring the exact tracking of a specified external reference is considered. Numerical aspects related to the implementation of the control law are addressed in order to improve the practical applicability of the proposed solution.

1. INTRODUCTION

In some classes of control problems the structure of the mathematical model of the plant to be controlled is known but some of its physical parameters may vary in a finite set. This happens, for instance, when different operating conditions of the same plant have to be taken into account or when sudden changes occur as a consequence of component failure [1], [3]. An appropriate controller design should guarantee the achievement of acceptable control requirements for the whole range of possible parameters.

In this paper conditions ensuring an output tracking of a given external reference such that a zero steady state tracking error is attained in a finite time are investigated. This problem is called Robust Output Tracking Problem (briefly ROTP). From a theoretical point of view, a solution for the ROTP has been proposed in [4], under the assumption that each possible configuration of the plant be controllable and detectable. By extending the results stated in [11], [9], a feedback controller in the form of a linear periodic system was proposed in [4]. The controller is obtained by a set of linear, discrete-time time-invariant dead-beat controllers, one for each possible configuration of the plant. The control action consists in applying each time-invariant dead-beat controller for a sufficiently long time interval.

The design of a dead-beat controller requires to compute a feedback gain F for the controllable pair (A, B) of each possible plant such that all the eigenvalues of the closed loop matrix $(A + BF)$ are zero. Since the number of eigenvalues of A exceeds the number of columns of B, this problem is numerically ill-conditioned [12]. Actually, due to possible uncertainties in the coefficients of the matrices and to

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round-off errors, classical algorithms that compute the required feedback gain matrix F with accuracy of ϵ (machine accuracy) can only guarantee closed loop eigenvalues with an accuracy of $\epsilon^{1/\nu}$, where ν is the controllability index of the pair (A, B). This limits the practical applicability of the above control procedure only to those cases in which the dimension of the plant and the number of possible configurations are very small.

To overcome these difficulties, the time-invariant dead-beat controller relative to each possible plant is replaced by a periodic time-varying dead-beat controller. The degree of freedom offered by the periodicity of the feedback gain allows to assign null closed-loop poles with ϵ accuracy [13]. Two different strategies, are suggested for constructing such periodic controllers. Properties and performances of the resulting compensator are investigated in a real situation on a family of linearized models of an underwater vehicle [10].

2. PRELIMINARIES

Let $S(\beta)$ be the linear time-invariant continuous-time system described by

$$\dot{x}(t) = A_C(\beta)\, x(t) + B_C(\beta)\, u(t), \tag{2.1}$$

$$y(t) = C_C(\beta)\, x(t) + D_C(\beta)\, u(t), \tag{2.2}$$

where β is a vector of physical parameters taking value in a known, finite set $\Theta{:=}\{\beta_1, \beta_2, \ldots, \beta_N\}$, $x(t) \in \mathbb{R}^n$ is the state, $u(t) \in \mathbb{R}^p$ is the control input, $y(t) \in \mathbb{R}^q$ is the output to be controlled (which is assumed to be measurable) and $A_C(\beta)$, $B_C(\beta)$, $C_C(\beta)$ and $D_C(\beta)$ are real matrices whose entries depend on β. Assume that $S(\beta)$ is controllable and detectable for all $\beta \in \Theta$. Let $r(t)$ be a reference signal generated as the free output response of the linear time-invariant continuous-time system S_G described by

$$\dot{\xi}(t) = A_G \xi(t), \tag{2.3}$$

$$r(t) = C_G \xi(t), \tag{2.4}$$

where $\xi(t) \in \mathbb{R}^m$ is the state.

The ROTP consists in finding a linear discrete-time controller Σ_C such that, for every initial state of $S(\beta)$, S_G and Σ_C, the output $y(t)$ of $S(\beta)$ track the reference signal $r(t)$ with a zero steady-state error attained in a finite time, independently of the actual value of β. The stabilization problem for system (2.1), (2.2) has been considered in [11], [9], where a discrete-time linear periodic compensator has been proposed. Following the line of [11], a solution of the ROTP has been proposed in [4] based on the hybrid control scheme shown in Figure 1. With reference to Figure 1, ZOH denotes the Zero Order Hold circuit. The hold interval is assumed to be equal to the sampling period T_c of the samplers. The ZOH and the samplers are synchronized at t=0. The sampled plant $\Sigma(\beta)$ is described by

$$x(k+1) = A(\beta)\, x(k) + B(\beta)\, u(k), \tag{2.5}$$

$$y(k) = C(\beta)\, x(k) + D(\beta)\, u(k), \tag{2.6}$$

where $A(\beta) := e^{A_C(\beta)T_c}$, $B(\beta) := \int_0^{T_c} e^{A_C(\beta)(T_c-\tau)} B_C(\beta)\mathrm{d}\tau$, $C(\beta) := C_C(\beta)$ and $D(\beta) := D_C(\beta)$. The values of these matrices for a particular β_i, $i = 1, 2, \ldots, N$, are denoted by $A(\beta_i)$, $B(\beta_i)$, $C(\beta_i)$ and $D(\beta_i)$. The value of the sampling period is chosen in such a way that:

(a) the sampled plant $\Sigma(\beta)$ is reachable and observable,

(b) there is no loss of observability due to sampling in the error system given by the parallel connection of the plant $S(\beta)$ and of the reference signal generator S_G.

Conditions ensuring the fulfilment of the above requirements are well-known [2].

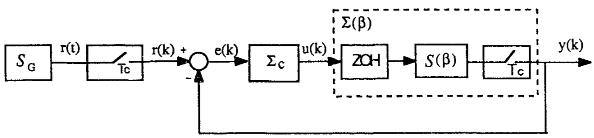

Fig. 1. Control system structure.

The linear periodic discrete-time compensator Σ_C solving the ROTP is given by

$$w(k+1) = P(k)\,w(k) + Q(k)\,e(k), \tag{2.7}$$

$$u(k) = M(k)\,w(k) + R(k)\,e(k), \tag{2.8}$$

where $w(k) \in \mathbb{R}^{\mu}$ is the state, $e(k) := r(k) - y(k)$ is the tracking error, and $P(\cdot)$, $Q(\cdot)$, $M(\cdot)$ and $R(\cdot)$ are periodic matrices of period ω (or, more briefly, ω-periodic) constructed as specified in the following. The compensator Σ_C is designed to insure that for all $\beta \in \Theta$, for any initial state $x(0)$ of system $S(\beta)$, $\xi(0)$ of S_G and $w(0)$ of Σ_C, the reference error $e(k)$ and the control input $u(k)$ of the sampled plant $\Sigma(\beta)$ satisfy

$$e(k) = 0, \qquad u(k) = 0, \qquad \forall\, k \geq k_a, \tag{2.9}$$

where k_a is a finite positive integer.

Taking into account the assumption that there is no loss of observability in the error system, condition (2.9) guarantees an exact tracking for each time instant and not only in correspondence of the sampling instants [4].

For each β_i, $(i = 1, \ldots, N)$, the dead-beat controller Σ_C^i which guarantees the state dead-beat control of system $\Sigma(\beta)$, for $\beta = \beta_i$, described by

$$w_i(k+1) = P_i w(k) + Q_i e(k), \tag{2.10}$$

$$u(k) = M_i w(k) + R_i e(k), \tag{2.11}$$

may be obtained as the series connection of a dead-beat observer Σ_O^i together with a dead-beat feedback F_i.

Using the above N linear time-invariant compensators Σ_C^i, define the ω-periodic matrices of Σ_C in the following way:

$$P(k) := P_i, \quad Q(k) := Q_i, \quad M(k) := M_i, \quad R(k) := R_i \quad \forall\, k \in [k_{i-1}, k_i),\; i = 1, \ldots, N, \tag{2.12}$$

$$P(k+\omega) := P(k), \quad Q(k+\omega) := Q(k), \qquad M(k+\omega) := M(k),$$
$$R(k+\omega) := R(k), \tag{2.13}$$

where $k_j := 2nj$, $j = 0, \ldots, N$, and $\omega := 2nN$. In this way, in each interval $[k_{i-1}, k_i)$, the periodic compensator Σ_C coincides with the dead-beat controller Σ_C^i. Therefore, if the initial state $\xi(0)$ of generator S_G is zero, the state of the closed-loop system goes to zero, as well as $y(k)$ and $e(k)$, in a finite time interval less or equal to ω, for all $\beta \in \Theta$ and arbitrary initial states $x(0)$ of $S(\beta)$ and $w(0)$ of Σ_C. In fact, when $\beta = \beta_i$, in the time interval $[0, k_{i-1})$, the compensator Σ_C does not produce any significative effect on the state of the closed-loop system, while, in the interval $[k_{i-1}, k_i)$ it coincides with Σ_C^i and, therefore it reduces to zero the state of the closed-loop system, as well as $e(k) = r(k) - y(k)$ and $u(k)$. After this time interval, the input $e(k)$ and the state $w(k)$ of the compensator Σ_C are zero, then the control action produced by the compensator Σ_C is zero and the state of the closed-loop system is kept equal to zero.

Observe that the above periodic compensator Σ_C is a solution of the ROTP for the special case of initial state $\xi(0) = 0$ of generator S_G, i. e., reference signal $r(t) = 0$ for all $t \geq 0$. The solvability conditions of ROTP for the general case $\xi(0) \neq 0$ (i. e. $r(t) \neq 0$) are stated in the following theorem [5].

Theorem. The ROTP is solvable with the periodic compensator Σ_C described by (2.7), (2.8), (2.12), and (2.13) if and only if each possible configuration of the continuous-time plant $S(\beta_i)$, $i = 1, \ldots, N$, contains an internal model of the continuous-time external reference generator S_G.

If the condition of the above theorem is not satisfied, a continuous-time precompensator could be connected to the plant $S(\beta)$, so that the series connection of the precompensator and the plant $S(\beta)$ contains a complete internal model of the reference signals for all $\beta \in \Theta$ (see, e. g., [7], [8]).

3. NUMERICAL PROBLEMS – A ROBUST SOLUTION

A key point in implementing the control law described above consists in the construction of a gain matrix F_i which puts all the eigenvalues of the closed loop matrix $A(\beta_i) + B(\beta_i)F_i$ as close as possible to zero, for each value β_i. The crucial point for the effectiveness of the control scheme is that the controller Σ_C^i must bring the regulation error and the internal state of the system to zero in a finite time, when connected with $S(\beta_i)$, so that the successive application of the "wrong" controllers does not modify this situation or, at least, does not destabilize the system.

Given a controllable pair (A, B), with B of full column rank equal to m, it is well known that we can assign to the closed loop matrix $H = A + BF$ only a Jordan form J satisfying the following conditions:

(i) the larger dimension of the blocks in J cannot be smaller than ν (the controllability index of the pair (A, B)),

(ii) J cannot contain more then m blocks.

As a consequence, if the dimension of the state space exceeds the number of columns of B, the closed loop matrix H will have at least one block of dimension equal or greater than ν in its Jordan form. In order to guarantee the dead-beat behavior, all the eigenvalues of H must be zero. In real applications, round-off errors, possible uncertainties in the coefficients of the matrices and the errors introduced by the used algorithms affect the performances of the control scheme. The sensitivity of the eigenvalues under parameter variations mainly depends on the Jordan structure of the matrix, and, in a non diagonalizable matrix, a perturbation of order ϵ in the elements can propagate to a perturbation of order $\epsilon^{1/n}$ in an eigenvalue of multiplicity n. Moreover if more than one block is connected with the same eigenvalue, the presence of a block of larger dimension influences negatively the sensitivity of the eigenvalues of the other blocks [12].

In conclusion, the situation encountered in the considered control scheme is exactly that which is worse: non diagonalizable matrix with multiple eigenvalues. The most reliable algorithms available [6] compute the required feedback gain matrix F within the machine accuracy ϵ, but the accuracy guaranteed on the closed loop eigenvalues is only $\epsilon^{1/\nu}$. This accuracy on the closed-loop eigenvalues makes the tracking error $e(k)$ and the control input $u(k)$ become small, but not necessarily zero, in a finite time interval. This fact severely limits the applicability of the proposed control procedure. To overcome numerical limitations, so enhancing the practical applicability of the proposed control scheme, a time-varying periodic compensator Σ_C^i is introduced for achieving the regulation of the system $\Sigma(\beta_i)$ in a finite time interval. For every $i = 1, \ldots, N$ the compensator Σ_C^i is designed as the connection of a dead-beat observer with a dead-beat periodic state feedback, the period being ω_i. In order to specify the proposed periodic compensator Σ_C^i the following notations are introduced:

$$
\begin{aligned}
A^a(\beta_i) &:= A(\beta_i)^{\omega_i}, \\
B^a(\beta_i) &:= \left[A(\beta_i)^{\omega_i-1}B(\beta_i) \quad A(\beta_i)^{\omega_i-2}B(\beta_i) \quad \cdots \quad B(\beta_i) \right], \\
C^a(\beta_i) &:= \begin{bmatrix} C(\beta_i) \\ C(\beta_i)A(\beta_i) \\ \vdots \\ C(\beta_i)A(\beta_i)^{\omega_i-1} \end{bmatrix}, \\
D^a(\beta_i) &:= \begin{bmatrix} D(\beta_i) & 0 & \cdots & 0 \\ C(\beta_i)B(\beta_i) & D(\beta_i) & \cdots & 0 \\ C(\beta_i)A(\beta_i)B(\beta_i) & C(\beta_i)B(\beta_i) & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ C(\beta_i)A(\beta_i)^{\omega_i-2}B(\beta_i) & C(\beta_i)A(\beta_i)^{\omega_i-3}B(\beta_i) & \cdots & D(\beta_i) \end{bmatrix},
\end{aligned}
$$

where the integer ω_i is the period of Σ_C^i, that it is assumed to be greater or equal to the minimum between the controllability index of the pair $(A(\beta_i), B(\beta_i))$ and the observability index of the pair $(C(\beta_i), A(\beta_i))$. For the construction of the dead-beat periodic state feedback law, two possibilities can be considered: a "dynamic" feedback law and a periodic "non dynamic" feedback law [13]. The two possibilities have been investigated in different applications, and the "dynamic" feedback law has provided better control performances. Therefore, in the following, only the "dynamic feedback law is described.

For a given value $\beta_i \in \Theta$, consider the following ω_i-periodic "dynamic" state feedback law applied to the time-invariant system $\Sigma(\beta_i)$ described by (2.5) and (2.6)

$$u(h\omega_i + j) = F_i(h\omega_i + j)x(h\omega_i), \qquad j = 0, 1, \ldots, \omega_i - 1, \quad \forall\, h \in Z^+, \tag{3.1}$$

where $F_i(\cdot) \in \mathbb{R}^{p \times n}$ is a periodic matrix of period ω_i, namely $F_i(k + \omega_i) = F_i(k)$, for all $k \in Z$, and Z^+ denotes the set of non-negative integers.

Denote by $\Sigma'(\beta_i)$ the ω_i-periodic closed-loop system described by (2.5), (2.6) and (3.1). The state of $\Sigma'(\beta_i)$ satisfies the following equation

$$x((h + 1)\omega_i) = (A^a(\beta_i) + B^a(\beta_i)F_i^a)x(h\omega_i), \tag{3.2}$$

where

$$F_i^a = [\, F_i(0)^T \quad F_i(1)^T \quad \cdots \quad F_i(\omega_i - 1)^T \,]^T . \tag{3.3}$$

In other words, the state $x(k)$ of the ω_i-periodic closed-loop system $\Sigma'(\beta_i)$ in the time instant $k = h\omega_i$ is described by the time-invariant system (3.2). For the chosen period ω_i, the matrix $B^a(\beta_i)$ is full row rank, hence it is possible to find a matrix $F_i^a \in \mathbb{R}^{\omega_i p \times n}$ such that the matrix $A^a(\beta_i) + B^a(\beta_i)F_i^a$ in (3.2) is set equal to zero, avoiding the numerical problems described above. In this way, for any initial state $x(0)$ the state $x(k)$ of $\Sigma'(\beta_i)$ is zero for $k = \omega_i$.

To construct the compensator Σ_C^i when the state $x(k)$ of $\Sigma(\beta_i)$ is not measurable, an observer is needed. For the "dynamic" ω_i-periodic dead-beat feedback (3.1), a dead-beat observer for $x(k)$ is necessary only at time instants $k = h\omega_i$, with $h \in Z^+$. For instance, the following Σ_O^i can be introduced

$$\begin{aligned} \xi((h + 1)\omega_i) \;&=\; (A^a(\beta_i) - G_i^a C^a(\beta_i))\xi(h\omega_i) \\ &+\; G_i^a(y^a(h\omega_i) - D^a(\beta_i)u^a(h\omega_i)) + B^a(\beta_i)u^a(h\omega_i), \end{aligned} \tag{3.4}$$

where

$$G_i^a := [\, G_i(0) \quad G_i(1) \quad \cdots \quad G_i(\omega_i - 1) \,] ,$$

$$y^a(h\omega_i) := [\, y(h\omega_i)^T \quad y(h\omega_i + 1)^T \quad \cdots \quad y(h\omega_i + \omega - i - 1)^T \,]^T ,$$

$$u^a(h\omega_i) := [\, u(h\omega_i)^T \quad u(h\omega_i + 1)^T \quad \cdots \quad u(h\omega_i + \omega_i - 1)^T \,]^T .$$

The inputs $y^a(h\omega_i)$ and $u^a(h\omega_i)$ of the observer Σ_O^i are the staked form over a period ω_i of the output $y(\cdot)$ and of the input $u(\cdot)$ of the system $\Sigma(\beta_i)$, respectively. The estimated error $\varepsilon(k) := \xi(k) - x(k)$ satisfies the following equation

$$\varepsilon((h + 1)\omega_i) = (A^a(\beta_i) - G_i^a C^a(\beta_i))\varepsilon(h\omega_i), \qquad \forall\, h \in Z^+. \tag{3.5}$$

For the chosen period ω_i, the matrix $C^a(\beta_i)$ is full row rank, hence it is possible to find a matrix $G_i^a \in \mathbb{R}^{n \times \omega_i q}$ such that the matrix $A^a(\beta_i) - G_i^a C^a(\beta_i)$ is set equal to zero. In this way, for any initial $\varepsilon(0)$ the estimated error $\varepsilon(k)$ is zero for $k = \omega_i$. Hence, the ω_i-periodic compensator Σ_C^i proposed consists of the series connection of the "dynamic" ω_i-periodic dead-beat feedback described by (3.1) together with the time-invariant dead-beat observer Σ_O^i given by (3.4), with $y^a(h\omega_i)$ replaced by $y^a(h\omega_i) - r^a(h\omega_i)$, where

$$r^a(h\omega_i) := [\, r(h\omega_i)^T \quad r(h\omega_i + 1)^T \quad \cdots \quad r(h\omega_i + \omega_i - 1)^T \,]^T ,$$

that is the stacked form over a period ω_i of the reference signal.

4. NUMERICAL RESULTS

The proposed control scheme has been tested on a family of linearized models of an underwater vehicle whose tasks are the transport of assembling modules for submarine installations and the inspection of underwater structures [10]. In these tasks the vehicle is subjected to very different load configurations, which introduce considerable variations of its mass and inertial parameters. The vehicle is equipped with four thrusters and connected with the surface vessel by a supporting cable controlling the depth, while the vehicle position η and θ and orientation ϕ over planes parallel to the surface are controlled by the thrusters.

The dynamic model of the considered underwater vehicle is non-linear and some physical parameters vary with the operating depth and load configuration. Denoting with $\eta_0(t)$, $\theta_0(t)$ and $\phi_0(t)$ the output trajectory corresponding to the control inputs $T_\eta^0(t)$, $T_\theta^0(t)$ and $M_\phi^0(t)$, the motion equations of the considered underwater vehicle can be linearized around the working point $(\eta_0(\cdot), \theta_0(\cdot), \phi_0(\cdot), T_\eta^0(\cdot), T_\theta^0(\cdot), M_\phi^0(\cdot))$. Assuming for the state $x(\cdot)$, the control input $u(\cdot)$ and the controlled output $y(\cdot)$ the following expressions

$$x(t) := \Big[\eta(t) - \eta_0(t),\ \dot\eta(t) - \dot\eta_0(t),\ \theta(t) - \theta_0(t),\ \dot\theta(t) - \dot\theta_0(t),\ \phi(t) - \phi_0(t),\ \dot\phi(t) - \dot\phi_0(t)\Big]^T$$

$$u(t) := \left[T_\eta(t) - T_\eta^0(t), T_\theta(t) - T_\theta^0(t), M_\phi(t) - M_\phi^0(t)\right]^T$$

$$y(t) := [\eta(t) - \eta_0(t), \theta(t) - \theta_0(t), \phi(t) - \phi_0(t)]^T$$

the state space form of the linearized model can be expressed in the form (2.1), (2.2) with matrices $A_C(\beta)$, $B_C(\beta)$, $C_C(\beta)$ and $D_C(\beta)$ given by

$$A_C(\beta) = \begin{bmatrix} 0 & 1 & 0 & 0 & 0 & 0 \\ a_1(\beta) & a_2(\beta) & 0 & a_3(\beta) & a_4(\beta) & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & a_6(\beta) & a_1(\beta) & a_5(\beta) & a_7(\beta) & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & a_9(\beta) & a_8(\beta) \end{bmatrix},$$

$$B_C(\beta) = \begin{bmatrix} 0 & 0 & 0 \\ b_1(\beta) & 0 & 0 \\ 0 & 0 & 0 \\ 0 & b_2(\beta) & 0 \\ 0 & 0 & 0 \\ 0 & 0 & b_3(\beta) \end{bmatrix},$$

$$C_C(\beta) = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \end{bmatrix}, \quad D_C(\beta) = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}.$$

In such a linearization it is considered an underwater current with intensity of 0.2 m/sec along the η direction and of 0.3 m/sec along the θ direction. The entries $a_i(\beta)$,

Table 1. Numerical values of the coefficients $a_i(\beta)$, $i = 1, \ldots, 9$, and $b_i(\beta)$, $i = 1, 2, 3$ of the linearized model related to three different load configurations C_1, C_2 and C_3 and two different operating depths: 200 meters and 1000 meters.

	C_1 (200m)	C_2 (200m)	C_3 (200m)	C_1 (1000m)	C_2 (1000m)	C_3 (1000m)
a_1	-4.2749e-02	-3.2912e-02	-3.6484e-02	-1.2828e-02	-9.7079e-03	-8.1263e-03
a_2	-8.3429e-02	-9.9248e-02	-1.1031e-01	-8.3429e-02	-9.9248e-02	-1.1031e-01
a_3	-2.9445e-02	-3.5028e-02	-3.8932e-02	-2.9445e-02	-3.5028e-02	-3.8932e-02
a_4	0	0	0	0	0	0
a_5	-1.0796e-01	-2.3761e-01	-1.0944e-01	-1.0796e-01	-2.3761e-01	-1.0944e-01
a_6	-2.9445e-02	-6.4804e-02	-2.9848e-02	-2.9445e-02	-6.4804e-02	-2.9848e-02
a_7	0	0	0	0	0	0
a_8	0	0	0	0	0	0
a_9	9.4368e-04	9.4622e-04	3.2407e-04	9.4368e-04	9.4622e-04	3.2407e-04
b_1	1.1372e-05	7.8926e-05	2.1052e-05	1.1372e-05	7.8926e-05	2.1052e-05
b_2	1.1372e-05	7.8926e-05	2.1052e-05	1.1372e-05	7.8926e-05	2.1052e-05
b_3	7.4074e-06	5.3538e-05	4.4247e-06	7.4074e-06	5.3538e-05	4.4247e-06

$i = 1, \ldots, 9$, and $b_i(\beta)$, $i = 1, 2, 3$ related to six different configurations are reported in Table 1. Three different load configurations have been considered and, for each load configuration, two different operating depths have been examined. The vector parameters β may assume six different values, $\Theta = \{\beta_1, \beta_2, \ldots, \beta_6\}$. All the tests are performed with constant reference signals. Therefore, the reference generator S_G described by (2.3) and (2.4) is characterized by $A_G = 0$ and $C_G = I_3$.

This control problem could be also solved by continuously monitoring depth and load at each time instant, defining six different time-invariant controllers (not necessarily dead-beat), each for each possible configuration of the plant, and then coordinating the switching among these six controllers. Application of the periodic control schemes here proposed has a twofold motivation. First, this offers the possibility of testing the performance of the proposed solutions on an actual problem, secondly, the periodic controller allows to define a completely automatic control structure without any need of knowing the actual operating conditions and introducing a supervisor coordinating the switching among the time-invariant controllers.

The linearized model of the underwater vehicle does not contain an internal model of the continuous-time external reference generator for all $\beta \in \Theta$. Hence, for the applicability of the proposed control scheme, a continuous-time precompensator S_C has been introduced, so that the series connection of the precompensator and the linearized model contains a complete internal model of reference signals for all $\beta \in \Theta$.

It is assumed a sampling period of 0.5 sec. Two different realizations of the compensator Σ_C are analyzed:

(i) time-invariant dead-beat controller Σ_C^i, $i = 1, \ldots, N$, described by (2.10) and (2.11);

(ii) ω_i-periodic dead-beat controller Σ_C^i, $i = 1, \ldots, N$, realized by the ω_i-periodic dynamic feedback (3.1) together with the time-invariant dead-beat observer given by (3.4).

Numerical experiments were carried out by assuming the configuration C_3 and the value $r(t) = [\,10 \quad -20 \quad 1\,]^T$ for the reference signal. Simulation results are reported in Figures 2, 3, 4 and 5. The results related to Σ_C^i of the form (i) are shown in Figures 2 and 3, those relative to Σ_C^i of the form (ii) are reported in Figures 4 and 5. The sequences of integers reported in the mentioned figures, denote the time intervals in which the controllers Σ_C^i, $i = 1, \ldots, 6$, are operating. The compensator settling the tracking error to zero is Σ_C^3. Note that, the component $y_3(t)$ of the output is regulated to the desired value in the time interval 6. In fact, the linearized model related to this output variable is independent of the depth. Hence, for this part of the linear model, configuration 3 coincides with configuration 6.

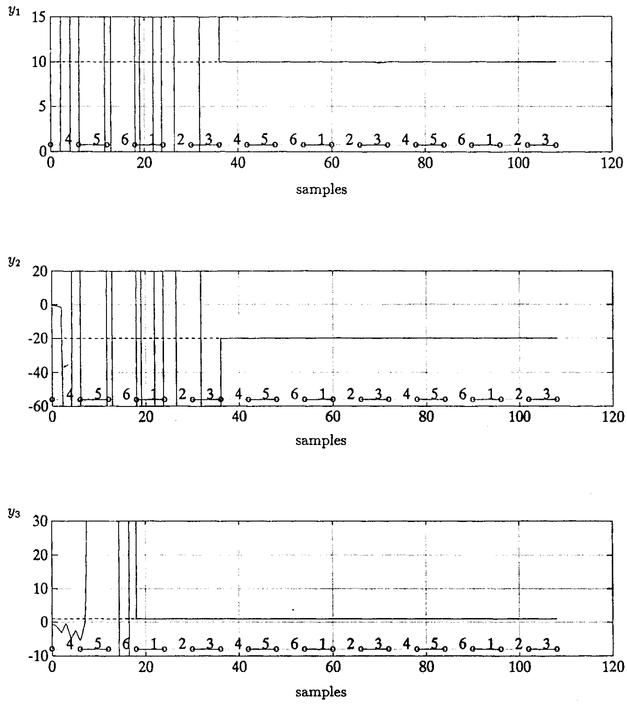

Fig. 2. Simulation results with Σ_C of the form (i): y_1, y_2, y_3 are the outputs of the linearized model.

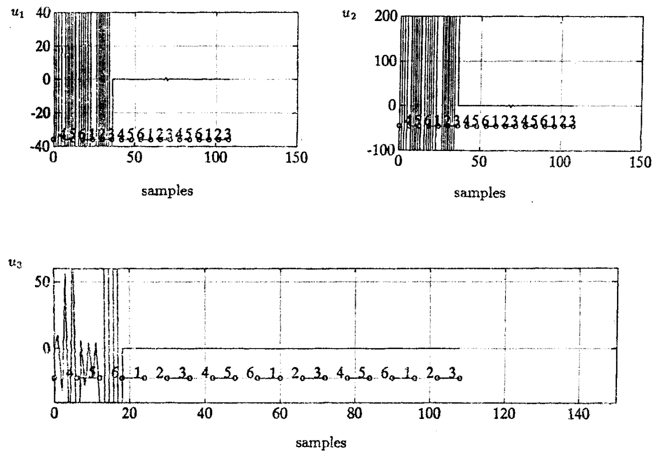

Fig. 3. Simulation results with Σ_C of the form (i): u_1, u_2, u_3 are the outputs of compensator Σ_C .

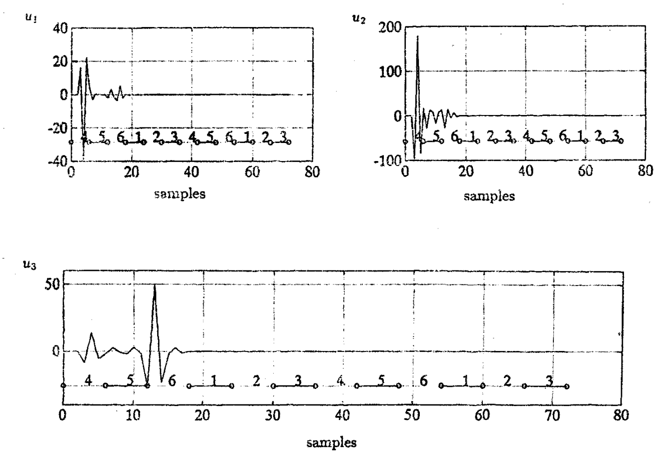

Fig. 5. Simulation results with Σ_C of the form (ii): u_1, u_2, u_3 are the outputs of compensator Σ_C .

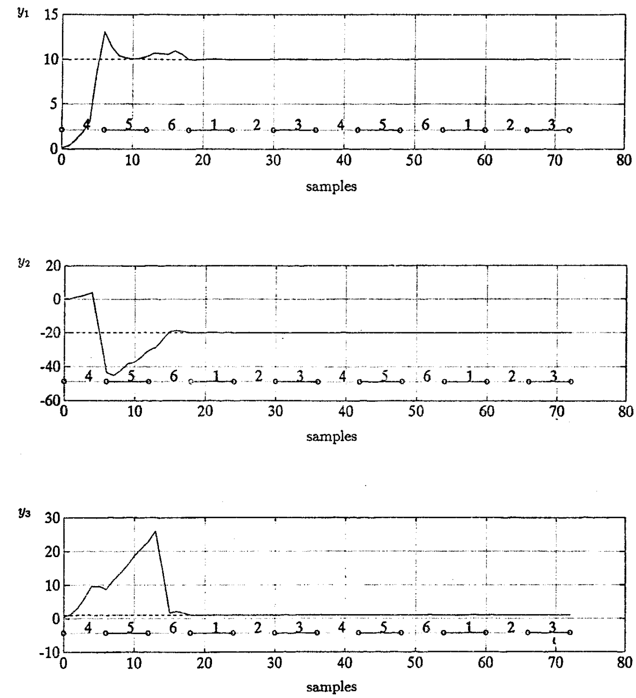

Fig. 4. Simulation results with Σ_C of the form (il): y_1, y_2, y_3 are the outputs of the linearized model.

Diagrams evidence that both the proposed controller realizations provided satisfactory results in terms of steady-state precision. A little numerical difference, which is not completely visible in the used scale, exists in favour of realization (ii). Diagrams also show an evident difference of control performance in terms of transient behaviour. Realization (i) provided unacceptable transient error and control effort samples. Their order of magnitude resulted to be 10^7 and 10^8, respectively. Vice versa, a perfectly acceptable transient behaviour has been produced by realization (ii). This can be explained in terms of a greater robustness of realization (ii) with respect to random perturbation of plant parameters. Figure 6 shows closed-loop eigenvalue positions for random perturbations of parameters on matrices $A_C(\beta)$ and $B_C(\beta)$ for $\beta = \beta_3$. Realization (ii) provides closed-loop eigenvalues with a smaller dispersion around the origin; note also that for random perturbations of 20% it preserves asymptotic stability, while this property is lost with realization (i). Analogous results have been obtained for the other configurations of the plant. This robustness property makes the controllers Σ_C^i, $i = 1, \ldots, 6$, $i \neq 3$, obtained by realization (ii), less "destabilizing" then those of realization (i). As a consequence realization (ii) is able to produce control actions yielding a better transient behaviour.

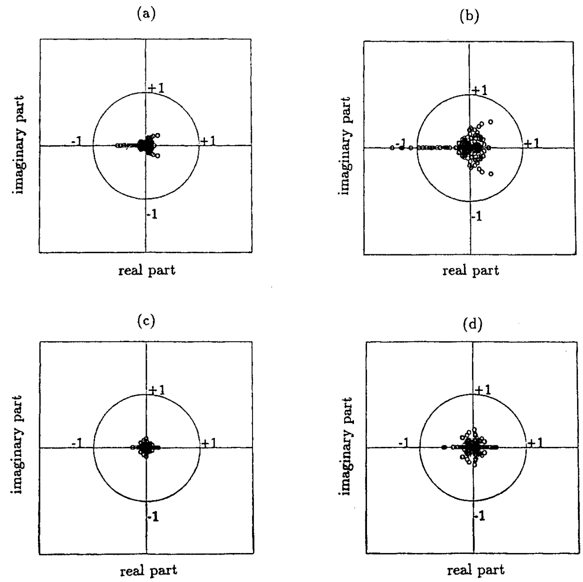

Fig. 6. Distribution of closed-loop eigenvalues corresponding of fifty random parameter perturbations on matrices $A_C(\beta)$ and $B_C(\beta)$ for $\beta = \beta_3$: (a) realization (i) with perturbation of 10 %; (b) realization (i) with perturbation of 20 %; (c) realization (ii) with perturbation of 10 %; (d) realization (ii) with perturbation of 20 %.

5. CONCLUDING REMARKS

Some numerical issues related to the Robust Output Tracking Problem for a finite set of linear plants have been investigated. The proposed solution is based on a periodic controller given by the periodic switching among the dead-beat compensators that can be designed for all the possible configurations of the plant. To improve the numerical robustness of the proposed solution, the dead-beat compensators are realized with a priodic dynamic feedback law. This solution is able to place to zero the closed-loop eigenvalues within the machine accuracy. Simulations results confirmed the feasibility of the approach.

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REFERENCES

[1] J. Ackermann: Multi-Model Approaches to Robust Control System Design. Technical Report, DFVLR Institut für Dynamic der Flugsysteme, Oberpfaffenhofen, Germany 1985.

[2] C.T. Chen: Linear System Theory and Design. Holt, Rinehart and Winston, New York 1984.

[3] E. J. Chow and A. S. Willsky: Analytical redundancy and the design of robust failure detection systems. IEEE Automat. Control *29* (1984), 603–613.

[4] G. Conte, L. Jetto, S. Longhi and A. M. Perdon: Simultaneous tracking for a family of linear plants, In: Proceedings of the 31st IEEE Conference on Decision and Control, Tucson, Arizona 1992, pp. 2884–2887.

[5] G. Conte, L. Jetto, S. Longhi and A. M. Perdon: Exact output control for a family of linear plants with parameter uncertainties. In: Proceedings of the 12th IFAC World Congress, Sydney, Australia 1993, vol. 10, pp. 65–68.

[6] P. Van Dooren: Dead-beat control: a special inverse eigenvalue problem. *BIT 24* (1984), 681–699.

[7] G. F. Franklin and A. Emami-Naeini: Design of ripple-free multivariable robust servomechanism. IEEE Trans. Automat. Control *AC-31* (1986), 7, 661–664.

[8] O. M. Grasselli, S. Longhi and A. Tornambè: Robust ripple-free tracking and regulation for sampled-data systems under uncertainties of physical parameters. In: Proceedings of the Second European Control Conference, Groningen, The Netherlands 1993, pp. 432–437.

[9] P.P. Khargonekar, K. Poolla and A. Tannenbaum: Robust control of linear time-invariant plants using periodic compensation. IEEE Trans. Automat. Control *AC-30* (1985), 1088–1096.

[10] S. Longhi and A. Rossolini: Adaptive control for an underwater vehicle: simulation studies and implementation details. In: Proceedings of the IFAC Workshop on Expert Systems and Signal Processing in Marine Automation, Lyngby, Denmark 1989, pp. 271–280.

[11] A. W. Olbrot: Robust stabilization of uncertain systems by periodic feedback. Internat. J. Control *45* (1987), 747–758.

[12] J. L. Wilkinson: The Algebraic Eigenvalue Problem. Oxford Science Publications, Oxford 1988.

[13] J. L. Willems, V. Kučera and P. Brunovsky: On the assignment of invariant factors by time-varying feedback strategies. System Control Lett. *5* (1984), 75–80.

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