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RINGS OF MAPS: SEQUENTIAL CONVERGENCE AND COMPLETION

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Abstract. The ring $B(R)$ of all real-valued measurable functions, carrying the pointwise convergence, is a sequential ring completion of the subring $C(R)$ of all continuous functions and, similarly, the ring $\mathbb{B}$ of all Borel measurable subsets of R is a sequential ring completion of the subring $\mathbb{B}_0$ of all finite unions of half-open intervals; the two completions are not categorical. We study $\mathscr{L}_0^*$-rings of maps and develop a completion theory covering the two examples. In particular, the σ-fields of sets form an epireflective subcategory of the category of fields of sets and, for each field of sets $\mathbb{A}$, the generated σ-field $\sigma(\mathbb{A})$ yields its epireflection. Via zero-rings the theory can be applied to completions of special commutative $\mathscr{L}_0^*$-groups.

Keywords: Rings of sets, completion of sequential convergence rings, $Z(2)$-generation, $Z(2)$-completion, σ-rings of maps, epireflection, fields of events, foundation of probability

MSC 2000: Primary 54A20, 54B30; Secondary 54H13, 60A99

0. Introduction

We continue our investigations of $\mathscr{L}_0^*$-rings of functions started in [BKF]. Recall that an $\mathscr{L}_0^*$-ring is a ring (even though $C(R)$ and $\mathbb{B}_0$ possess the unit element, in general we do not assume its existence) carrying a sequential convergence having unique limits and satisfying the Urysohn axiom and such that if sequences $\langle x_n\rangle$ and $\langle y_n\rangle$ converge then their difference $\langle x_n - y_n\rangle$ and product $\langle x_n y_n\rangle$ converge to the difference and product of their limits, respectively. Background information on the completion of $\mathscr{L}_0^*$-rings can be found in [FKO] and [FKT]. Notice that the field Q of rational numbers can carry an $\mathscr{L}_0^*$-field convergence having no completion ([FZE]) and the usual convergence on Q admits $\exp\exp\omega$ nonhomeomorphic $\mathscr{L}_0^*$-field completions ([FCB]).

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In [BKF] it was shown that the categorical $\mathscr{L}_0^*$-ring completion of $C(R)$ exists and is different from $B(R)$. For the undefined categorical notions the reader can consult [HES].

In the present paper we investigate completions of $\mathscr{L}_0^*$-rings of maps into a complete $\mathscr{L}_0^*$-ring. Section 1 is devoted to the $\mathscr{L}_0^*$-ring $\mathbb{B}_0$ of half-open intervals, i.e., the field of sets consisting of all finite unions of intervals of the form $[a,b)$, $(-\infty,b)$, $(-\infty,+\infty)$, $[a,+\infty)$, where $a,b \in R$, carrying the usual sequential convergence. In Section 2 we describe the categorical background of the completions of $C(R)$ and $\mathbb{B}_0$. In Section 3 we present two additional results: on rings of sets and the application of the completion of $\mathscr{L}_0^*$-rings to commutative $\mathscr{L}_0^*$-groups via zero-rings.

1. The $\mathscr{L}_0^*$-ring $\mathbb{B}_0$

It is known that every ring of sets can be visualized as a ring of characteristic functions. Each set is identified with its characteristic function, the symmetric difference as addition becomes the pointwise addition of characteristic functions modulo 2, the intersection as product becomes the usual pointwise product, and the usual convergence of sets ($A = \lim A_n$ means $A = \limsup A_n = \liminf A_n$) becomes the pointwise convergence of the corresponding characteristic functions; in this way $\mathbb{B}_0$ and $\mathbb{B}$ become $\mathscr{L}_0^*$-subrings of $Z(2)^R = \{0,1\}^R$. Denote by $\mathbb{B}_1$ the first pointwise sequential closure of $\mathbb{B}_0$ in $Z(2)^R$ (the limits of sequences in $\mathbb{B}_0$) and, for $\alpha \in \omega_1$, denote by $\mathbb{B}_\alpha$ the α-th closure of $\mathbb{B}_0$. Then each $\mathbb{B}_\alpha$ is a ring of sets and $\mathbb{B} = \mathbb{B}_{\omega_1}$, see [NOV], [LAC].

In [BKF] it is proved that the pointwise sequential convergence on the $\mathscr{L}_0^*$-ring $B_1(R)$ of the first Baire class functions fails to be strict. We shall prove that the pointwise convergence on $\mathbb{B}_1$ fails to be strict, too. Recall the definition of strictness (cf. Definition 1.2 in [FSZ]).

Let $(X, \mathbb{L})$ be an $\mathscr{L}_0^*$-ring ($\mathbb{L}$ denotes the sequential convergence as a subset of $X^{\mathbb{N}} \times X$) and let X_0 be a subring of X. The first sequential closure X_1 of X_0 is a ring. Denote by $\mathbb{L}_0$ and $\mathbb{L}_1$ the restrictions of $\mathbb{L}$ to X_0 and X_1, respectively. We say that $\mathbb{L}_1$ is *strict* (more exactly it should be: strict with respect to X_0) if the following holds:

(s) Let $\langle x_n \rangle$ be a sequence ranging in $X_1 \setminus X_0$ which converges under $\mathbb{L}_1$ to $x \in X_1$. Then there exist a subsequence $\langle x'_n \rangle$ of $\langle x_n \rangle$ and sequences $\langle y_n^{(k)} \rangle$ ranging in X_0, $k \in \mathbb{N}$, such that the sequence $\langle x_n^{(k)} \rangle$ converges under $\mathbb{L}_1$ to x'_k and each diagonal sequence $\langle y_{d(n)}^{(n)} \rangle$, $d\colon \mathbb{N} \to \mathbb{N}$, converges under $\mathbb{L}_1$ to x.

Proposition 1.1. *The pointwise convergence in $\mathbb{B}_1$ fails to be strict.*

P r o o f. Let $p_1, p_2, p_3, \dots$ denote the increasing sequence of all prime numbers. For each $n \in \mathbb{N}$, let $A_n = \{k/p_n;\ k = 1, 2, \dots, p_n - 1\}$. Then $A_n \in \mathbb{B}_1 \setminus \mathbb{B}_0$ and the sequence $\langle A_n \rangle$ pointwise converges to $\emptyset$. For each $k \in \mathbb{N}$, let $\langle B_n^k \rangle$ be a sequence in $\mathbb{B}_0$ which pointwise converges to A_k. We show that there exists a mapping u of $\mathbb{N}$ into $\mathbb{N}$ such that for each mapping v of $\mathbb{N}$ into $\mathbb{N}$, $v(k) > u(k)$ for each $k \in \mathbb{N}$, and for each strictly increasing mapping s of $\mathbb{N}$ into $\mathbb{N}$ the subsequence $\langle B_{v(s(n))}^{(s(n))} \rangle$ of the diagonal sequence $\langle B_{v(n)}^{(n)} \rangle$ does not converge to $\emptyset$. Clearly, then the pointwise convergence on $\mathbb{B}_1$ fails to be strict.

So, since all sets A_k are finite, for each $k \in \mathbb{N}$ choose $u(k) \in \mathbb{N}$ such that $A_k \subset B_n^{(k)}$ for each $n > u(k)$. Let v be a mapping of $\mathbb{N}$ into $\mathbb{N}$ such that $v(k) > u(k)$ for each $k \in \mathbb{N}$ and let s be a strictly increasing mapping of $\mathbb{N}$ into $\mathbb{N}$. There exists an interval $I_1 = [a_1, b_1)$, $a_1 < b_1$, such that $p_{s(1)} \in I_1 \subset B_{v(s(1))}^{(s(1))}$. Put $t(1) = 1$. By induction, define a strictly increasing mapping t of $\mathbb{N}$ into $\mathbb{N}$ and a sequence of half-open intervals $I_n = [a_n, b_n), a_n < b_n$, such that $I_n \subset B_{v(s(t(n)))}^{(s(t(n)))}$ and $I_{n+1} \subset I_n$. However, since the distance between two neighboring points of A_n tends to 0, and each B_n^k is a finite union of half-open intervals, the construction of $\langle I_n \rangle$ is trivial and we omit its description. Because the intersection $\bigcap_{n=1}^{\infty} I_n$ is nonempty, the sequence $\langle B_{v(s(n))}^{(s(n))} \rangle$ cannot pointwise converge to $\emptyset$. This completes the proof. □

Observe that we have proved a rather strong negation of (s) for $\mathbb{B}_1$.

It is known that a commutative $\mathscr{L}_0^*$-group can have many nonequivalent $\mathscr{L}_0^*$-group completions and its Novák completion [NOV] yields its categorical $\mathscr{L}_0^*$-group completion ([FKO]). On the one hand, the convergence in the Novák $\mathscr{L}_0^*$-group completion is strict ([FSZ]), on the other hand, we show that the convergence in the Novák $\mathscr{L}_0^*$-group completion of $\mathbb{B}_0$ fails to be an $\mathscr{L}_0^*$-ring convergence.

Example 1.2. Let $\mathbb{L}$ denote the usual pointwise convergence on $\mathbb{B}_0$. Then the Novák $\mathscr{L}_0^*$-group completion of $\mathbb{B}_0$ has $\mathbb{B}_1$ as its underlying group and is equipped with an $\mathscr{L}_0^*$-group convergence $\mathbb{L}_1^*$ defined as follows: $\langle B_n \rangle$ converges to B under $\mathbb{L}_1^*$ iff for each subsequence $\langle B'_n \rangle$ of $\langle B_n \rangle$ there exists its subsequence $\langle B''_n \rangle$ such that $B''_n \triangle B = A_n \triangle A$, $n \in \mathbb{N}$, where $\langle A_n \rangle$ is a sequence in $\mathbb{B}$ pointwise converging to $A \in \mathbb{B}_1$. The sequence $\langle [n, n+1) \rangle$ converges to $\emptyset$ and $N \in \mathbb{B}_1$, but their product $\langle N \cap [n, n+1) \rangle = \langle \{n\} \rangle$ fails to converge under $\mathbb{L}_1^*$. Indeed, otherwise there would exist a sequence $\langle M_n \rangle$ in $\mathbb{B}_0$ converging pointwise to $M \in \mathbb{B}_1$ such that $\{n\} = M_n \triangle M$ for infinitely many $n \in \mathbb{N}$, which is impossible. Hence $\mathbb{L}_1^*$ fails to be an $\mathscr{L}_0^*$-ring convergence.

Interesting results concerning strictness can be found in [PAU].

It is known that an $\mathscr{L}_0^*$-field need not have any $\mathscr{L}_0^*$-ring completion ([FZE]). In [BKF] it was shown that if an $\mathscr{L}_0^*$-ring $(X, \mathbb{L})$ has a completion, then it has the categorical one: the epireflection in the subcategory of all complete $\mathscr{L}_0^*$-rings; denote it by $\varrho(X,\mathbb{L}) = (\widehat{X},\widehat{\mathbb{L}})$. Further, by Theorem 3.3 in [BKF], $(X,\mathbb{L})$ is ω-dense in $(\widehat{X},\widehat{\mathbb{L}})$. That means that at most ω iterations of the sequential closure of X with respect to $\widehat{\mathbb{L}}$ are sufficient to get $\widehat{X}$. But $C(R)$ fails to be ω-dense in $B(R)$ and hence the latter $\mathscr{L}_0^*$-ring fails to be the categorical $\mathscr{L}_0^*$-ring completion of $C(R)$ (Corollary 3.4 in [BKF]). By the same argument we get the following

Corollary 1.3. *$\mathbb{B}$ carrying the pointwise convergence fails to be the categorical completion of $\mathbb{B}_0$.*

Then the question arises whether there is any categorical construction of an $\mathscr{L}_0^*$-ring completion such that $B(R)$ and $\mathbb{B}$ are the corresponding completions of $C(R)$ and $\mathbb{B}_0$, respectively. We show that the answer is affirmative.

2. H-completions

Let $(H,\mathbb{K})$ be a complete $\mathscr{L}_0^*$-ring. For each $\mathscr{L}_0^*$-ring $(X,\mathbb{L})$ denote by $\mathrm{Hom}(X,H)$ the set of all sequentially continuous ring homomorphisms of $(X,\mathbb{L})$ into $(H,\mathbb{K})$.

Recall the notion of an initial structure. Let X be a ring, $\{(X_a,\mathbb{L}_a);\ a\in A\}$ a set of $\mathscr{L}_0^*$-rings and $\mathscr{F}=\{f_a\colon X\to X_a;\ a\in A\}$ a set of ring homomorphisms. If $\mathbb{L}$ is an $\mathscr{L}_0^*$-ring convergence and it is the coarsest of all $\mathscr{L}_0^*$-ring convergences $\mathbb{L}'$ on X (we do not assume that uniqueness of limits and the Urysohn axiom) such that all $f_a\colon (X,\mathbb{L}')\to(X_a,\mathbb{L}_a)$ are sequentially continuous, then $\mathbb{L}$ is called the initial $\mathscr{L}_0^*$-ring convergence with respect to $\mathscr{F}$. For example, the product convergence is the initial structure with respect to the projections.

Definition 2.1. We say that $(X,\mathbb{L})$ is *H-generated* if $\mathbb{L}$ is the initial $\mathscr{L}_0^*$-ring convergence with respect to $\mathrm{Hom}(X,H)$.

Note: Since we assume uniqueness of the limits, if $(X,\mathbb{L})$ is H-generated, then every pair of distinct points of X is separated by some $h\in\mathrm{Hom}(X,H)$; in fact, the initial structure with respect to $\mathscr{F}$ exists whenever $\mathscr{F}$ separates points of X; also, if $\langle x_n\rangle$ does not converge to x under $\mathbb{L}$, then there exists $h\in\mathrm{Hom}(X,H)$ such that $\langle h(x_n)\rangle$ does not converge to $h(x)$ under $\mathbb{K}$; hence if H is the real line (or sequentially regular), then each H-generated $\mathscr{L}_0^*$-ring is sequentially regular (see [FMR]).

Example 2.2. For each $r\in R$ and each $f\in R^R$ define $\mathrm{ev}_r(f)=f(r)$. Since we identify $A\subset R$ with its characteristic function, we have $\mathrm{ev}_r(A)=1$ if $r\in A$ and $\mathrm{ev}_r(A)=0$ otherwise. It is easy to see that ev_r is, in fact, a sequentially continuous ring homomorphism of R^R, equipped with pointwise convergence, into R and also a sequentially continuous ring homomorphism of $Z(2)^R$ into $Z(2)$, where $Z(2)$ is the usual discrete ring $\{0,1\}$ with addition modulo 2. In the same way we define evaluations $\mathrm{ev}_x(f)$ for each set M, $x \in M$ and $f \in R^M$, or $f \in Z(2)^M$.

The next assertion was proved in [FPA] via a straightforward calculation.

Lemma 2.3. *Let h be a sequentially continuous ring homomorphism of $\mathbb{B}_0$ into $Z(2)$. Then there exists $r \in R$ such that $h = \mathrm{ev}_r$.*

Lemma 2.4. *Let h be a sequentially continuous ring homomorphism of $C(R)$ into R. Then there exists $r \in R$ such that $h = \mathrm{ev}_r$.*

P r o o f. Since h is a linear functional on $C(R)$, it follows from Theorem 1 in [ITH] that h is a finite linear combination of evaluations at points of R. Using the fact that h is a ring homomorphism, we easily infer that h is an evaluation at some $r \in R$. □

Corollary 2.5. (i) *$C(R)$ is R-generated.*
(ii) *$\mathbb{B}_0$ is $Z(2)$-generated.*

We omit the straightforward proofs of the next two propositions.

Proposition 2.6. (i) *Each $\mathscr{L}_0^*$-subring of an H-generated $\mathscr{L}_0^*$-ring is H-generated.* (ii) *If each $(X_a, \mathbb{L}_a), a \in A$, is an H-generated $\mathscr{L}_0^*$-ring, then their product is H-generated, too.*

Proposition 2.7. *Let $(X, \mathbb{L})$ be an $\mathscr{L}_0^*$-ring. Then the following are equivalent:*

(i) *$(X, \mathbb{L})$ is H-generated.*
(ii) *The canonical map φ of $(X, \mathbb{L})$ into the power $H^{\mathrm{Hom}(X,H)}$, defined by $\varphi(x) = (h(x); h \in \mathrm{Hom}(X,H))$, is a homeomorphic isomorphism of $(X, \mathbb{L})$ onto an $\mathscr{L}_0^*$-subring of $H^{\mathrm{Hom}(X,H)}$.*
(iii) *There exists a homeomorphic isomorphism of $(X, \mathbb{L})$ onto an $\mathscr{L}_0^*$-subring of a power H^M of $(H, \mathbb{K})$.*

Definition 2.8. Let $(X, \mathbb{L})$ be an $\mathscr{L}_0^*$-ring and let $(Y, \mathbb{L} \restriction Y)$ be its $\mathscr{L}_0^*$-subring. If each sequentially continuous ring homomorphism of $(Y, \mathbb{L} \restriction Y)$ into $(H, \mathbb{K})$ can be extended to a sequentially continuous homomorphism of $(X, \mathbb{L})$ into $(H, \mathbb{K})$, then $(Y, \mathbb{L} \restriction Y)$ is said to be *H-embedded* in $(X, \mathbb{L})$.

Definition 2.9. Let $(X, \mathbb{L})$ be an H-generated $\mathscr{L}_0^*$-ring. If X is sequentially closed in each H-generated $\mathscr{L}_0^*$-ring $(X', \mathbb{L}')$ in which it is H-embedded, then $(X, \mathbb{L})$ is said to be *H-complete*.

Definition 2.10. Let $(X, \mathbb{L})$ be an H-generated $\mathscr{L}_0^*$-ring. A sequence $\langle x_n \rangle$ is said to be *H-fundamental* if for each $h \in \operatorname{Hom}(X, H)$ the sequence $\langle h(x_n) \rangle$ converges in H under $\mathbb{K}$.

Proposition 2.11. *Let $(X, \mathbb{L})$ be an H-generated $\mathscr{L}_0^*$-ring. Then the following are equivalent:*

(i) *$(X, \mathbb{L})$ is H-complete.*
(ii) *Every H-fundamental sequence in $(X, \mathbb{L})$ converges.*
(iii) *The image of $(X, \mathbb{L})$ under the canonical map φ is a sequentially closed subring of the $\mathscr{L}_0^*$-ring $H^{\operatorname{Hom}(X,H)}$.*
(iv) *There exists a homeomorphic isomorphism of $(X, \mathbb{L})$ onto a sequentially closed $\mathscr{L}_0^*$-subring of some power H^M of $(H, \mathbb{K})$.*

Let $\mathscr{A}$ be the category of all $\mathscr{L}_0^*$-rings with sequentially continuous ring homomorphisms as morphisms. Let $\mathscr{H}(H)$ and $\mathscr{H}_c(H)$ be the full and isomorphism closed subcategories of all H-generated and H-complete $\mathscr{L}_0^*$-rings, respectively.

Definition 2.12. Let $(X, \mathbb{L})$ be an H-generated $\mathscr{L}_0^*$-ring. We say that an H-generated $\mathscr{L}_0^*$-ring $(X', \mathbb{L}')$ is an *H-completion* of $(X, \mathbb{L})$ provided that

(i) $(X, \mathbb{L})$ is an H-embedded $\mathscr{L}_0^*$-subring of $(X', \mathbb{L}')$ and X' is the smallest sequentially closed subset of X' containing X;
(ii) $(X', \mathbb{L}')$ is H-complete.

Proposition 2.13. *Each H-generated $\mathscr{L}_0^*$-ring $(X, \mathbb{L})$ has an H-completion and is uniquely determined up to a homeomorphic isomorphism pointwise fixed on X.*

Proof. According to Proposition 2.7, the canonical map φ of $(X, \mathbb{L})$ into $H^{\operatorname{Hom}(X,H)}$ is a homeomorphic isomorphism of $(X, \mathbb{L})$ onto the corresponding $\mathscr{L}_0^*$-subring of $H^{\operatorname{Hom}(X,H)}$; for the sake of simplicity we identify $(X, \mathbb{L})$ with its φ-image. Denote by $\eta(X, \mathbb{L}) = (X_H, \mathbb{L}_H)$ the smallest sequentially closed $\mathscr{L}_0^*$-subring of $H^{\operatorname{Hom}(X,H)}$ containing $(X, \mathbb{L})$. It follows from the categorical properties of $H^{\operatorname{Hom}(X,H)}$ that $(X, \mathbb{L})$ is H-embedded in $(X_H, \mathbb{L}_H)$. By Proposition 2.11, $(X_H, \mathbb{L}_H)$ is H-complete and hence an H-completion. By a standard categorical argument it follows that $(X_H, \mathbb{L}_H)$ is uniquely determined up to a homeomorphic isomorphism pointwise fixed on X.

(Remember, we work with unique sequential limits and if two sequentially continuous maps agree on a topologically dense subset, then they are identical.) □

Corollary 2.14. *$\mathscr{H}_c(H)$ is epireflective in $\mathscr{H}(H)$ and η yields the epireflector.*

Corollary 2.15. *$(X_H, \mathbb{L}_H)$ is a maximal H-generated $\mathscr{L}_0^*$-ring containing $(X, \mathbb{L})$ as a topologically dense H-embedded $\mathscr{L}_0^*$-ring.*

Corollary 2.16. (i) *$B(R)$ is the epireflection of $C(R)$ into R-complete $\mathscr{L}_0^*$-rings.* (ii) *$\mathbb{B}$ is the epireflection of $\mathbb{B}_0$ into $Z(2)$-complete $\mathscr{L}_0^*$-rings.*

3. Additional results

Assume that $\mathbb{A}$ is a ring of subsets of a set M, i.e. $\mathbb{A}$ is closed with respect to the symmetric difference and the intersection. Unlike in the case of $\mathbb{B}_0$, it can happen that not every sequentially continuous ring homomorphism of $\mathbb{A}$ into $Z(2)$ is an evaluation ev_x at some $x \in M$. Despite this fact, we show that the generated σ-ring $\sigma(\mathbb{A})$ is the $Z(2)$-completion of $\mathbb{A}$. The following construction has been suggested in [FPA].

Proposition 3.1. *An $\mathscr{L}_0^*$-ring $(X, \mathbb{L})$ is $Z(2)$-generated iff it is a ring of sets.*

P r o o f. 1. Let $(X, \mathbb{L})$ be $Z(2)$-generated. Put $M = \mathrm{Hom}(X, Z(2))$. Then the canonical map φ sends $x \in X$ into a subset $A(x) = \{h \in \mathrm{Hom}(X, Z(2));\ h(x) = 1\}$. Clearly, this yields a homeomorphic isomorphism of $(X, \mathbb{L})$ onto the corresponding ring of subsets of M.

2. If $\mathbb{A}$ is a ring of subsets of a set M, then $A_n \to A$ in $\mathbb{A}$ iff $\mathrm{ev}_x(A_n) \to \mathrm{ev}_x(A)$ for each $x \in M$. Obviously, the canonical map φ of $\mathbb{A}$ into $Z(2)^{\mathrm{Hom}(\mathbb{A}, Z(2))}$ defined by $\varphi(a) = \{h(a);\ h \in \mathrm{Hom}(\mathbb{A}, Z(2))\}$ is a homeomorphic isomorphism. □

Proposition 3.2. *A ring of sets is $Z(2)$-complete iff it is a σ-ring.*

P r o o f. The assertion follows from Proposition 2.11. □

Corollary 3.3. *Let $\mathbb{A}$ be a ring of sets. Then $\mathbb{A} \in \mathscr{H}(Z(2))$ and the generated σ-ring $\sigma(\mathbb{A})$ is its $Z(2)$-completion, i.e. the epireflection of $\mathbb{A}$ into $\mathscr{H}_c(Z(2))$.*

Concerning rings of sets and the extension of sequentially continuous maps we call the reader's attention to the pioneering [NOE].

We end with the following observation. Let $(X, \mathbb{L})$ be a commutative $\mathscr{L}_0^*$-group. Then X can be viewed as a zero-ring and $(X, \mathbb{L})$ as an $\mathscr{L}_0^*$-ring. The construction of an H-completion therefore also yields a categorical completion theory of $\mathscr{L}_0^*$-groups for suitable commutative $\mathscr{L}_0^*$-groups $(H, \mathbb{K})$. In particular, it might be interesting to investigate H-completions when H is the torus (cf. [FCA]).

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