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g-METRIZABLE SPACES AND THE IMAGES OF SEMI-METRIC SPACES

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Abstract. In this paper, we prove that a space X is a g-metrizable space if and only if X is a weak-open, π and σ-image of a semi-metric space, if and only if X is a strong sequence-covering, quotient, π and $mssc$-image of a semi-metric space, where "semi-metric" can not be replaced by "metric".

Keywords: g-metrizable spaces, sn-metrizable spaces, weak-open mappings, strong sequence-covering mappings, quotient mappings, π-mappings, σ-mappings, $mssc$-mappings

MSC 2000: 54C10, 54D55, 54E25, 54E35, 54E40

1. INTRODUCTION

g-metrizable spaces as a generalization of metric spaces have many important properties [17]. To characterize g-metrizable spaces as certain images of metric spaces is an interesting question in the theory of generalized metric spaces, and many "nice" characterizations of g-metrizable spaces have been obtained ([6], [8], [7], [13], [18], [19]).

Theorem 1.1. *The following are equivalent for a space X.*
(1) *X is a g-metrizable space.*
(2) *X is a quotient, π, σ-image of a metric space* [6].
(3) *X is a compact-covering, quotient, π, σ-image of a metric space* [13].
(4) *X is a 1-sequence-covering, quotient, σ-image of a metric space* [8].

Recently, the following results were given.

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Proposition 1.2. *The following are equivalent for a space X.*

(1) *X is a g-metrizable space.*

(2) *X is a weak-open, π, σ-image of a metric space* [10].

(3) *X is a strong sequence-covering, quotient, π, mssc-image of a metric space* [9].

Unfortunately, the proposition is not true. In this paper, we give an example to show that there exists a g-metrizable space which is not a weak-open, π, σ-image of a metric space and is not a strong sequence-covering, quotient, π, *mssc*-image of a metric space. As a further investigation on g-metrizable spaces the following is the main theorem of this paper.

Theorem 1.3. *The following are equivalent for a space X.*

(1) *X is a g-metrizable space.*

(2) *X is a weak-open, π, σ-image of a semi-metric space.*

(3) *X is a strong sequence-covering, quotient, π, mssc-image of a semi-metric space.*

Throughout this paper, all spaces are assumed to be regular and T_1, all mappings are continuous and onto.

2. Definitions and remarks

Definition 2.1 [4]**.** Let X be a space.

(1) $P \subset X$ is called a sequential neighborhood of x in X, if each sequence $\{x_n\}$ converging to x is eventually in P.

(2) A subset U of X is called sequentially open if U is a sequential neighborhood of each of its points.

(3) X is called a sequential space if each sequential open subset of X is open.

Definition 2.2 [14]**.** Let $\mathscr{P} = \bigcup\{\mathscr{P}_x\colon\ x \in X\}$ be a cover of a space X with each $x \in \bigcap \mathscr{P}_x$.

(1) $\mathscr{P}$ is called a network of X, if for each $x \in U$ with U open in X, there exists $P \in \mathscr{P}_x$ such that $P \subset U$, where $\mathscr{P}_x$ is called a network at x in X.

(2) $\mathscr{P}$ is a cs^*-network of X, if each sequence S converging to a point $x \in U$ with U open in X, is frequently in $P \subset U$ for some $P \in \mathscr{P}_x$.

Definition 2.3. Let $\mathscr{P} = \bigcup\{\mathscr{P}_x\colon\ x \in X\}$, where $\mathscr{P}_x$ is a network at x in X, and satisfies the following condition $(*)$ for each $x \in X$.

$(*)$ If $P_1, P_2 \in \mathscr{P}_x$, then there exists $P \in \mathscr{P}_x$ such that $P \subset P_1 \cap P_2$.

(1) $\mathscr{P}$ is called a weak base of X [1], if whenever $G \subset X$ and for each $x \in G$ there exists $P \in \mathscr{P}_x$ such that $P \subset G$, then G is open in X, where $\mathscr{P}_x$ is called a weak neighborhood base at x in X.

(2) $\mathscr{P}$ is called an sn-network of X [12], if each element of $\mathscr{P}_x$ is a sequential neighborhood of x for each $x \in X$, where $\mathscr{P}_x$ is called an sn-network at x in X.

Definition 2.4.

(1) A space X is called g-metrizable [17] (resp. sn-metrizable [5]), if X has a σ-locally finite weak base (resp. sn-network).

(2) A space X is called g-first countable [1] (resp. sn-first countable [5]), if X has a weak base (resp. an sn-network) $\mathscr{P} = \bigcup\{\mathscr{P}_x \colon x \in X\}$ such that $\mathscr{P}_x$ is countable for each $x \in X$.

Notation 2.5. *Let d be a non-negative real valued function defined on $X \times X$ such that $d(x, y) = 0$ if and only if $x = y$, and $d(x, y) = d(y, x)$ for all $x, y \in X$. d is called a d-function on X. For each $x \in X, n \in \mathbb{N}$, put $S_n(x) = \{y \in X \colon d(x, y) < 1/n\}$.*

Definition 2.6. Let d be a d-function on a space X. A space (X, d) is called an sn-symmetric space (resp. a symmetric space, a semi-metric space), if d satisfies the following condition (A) (resp. (B), (C)), where d is called an sn-symmetric (resp. a symmetric, a semi-metric) on X.

(A) $\{S_n(x)\}$ is an sn-network at x in X for each $x \in X$.

(B) $\{S_n(x)\}$ is a weak neighborhood base at x in X for each $x \in X$.

(C) $\{S_n(x)\}$ is a neighborhood base at x in X for each $x \in X$.

Remark 2.7. Each weak base of a space is an sn-network, and each sn-network of a sequential space is a weak base [12]. Thus

(1) g-metrizable spaces $\Longleftrightarrow$ Sequential and sn-metrizable spaces.

(2) Symmetric spaces $\Longleftrightarrow$ Sequential and sn-symmetric spaces.

(3) g-first countable spaces $\Longleftrightarrow$ Sequential and sn-first countable.

(4) Semi-metric spaces $\Longleftrightarrow$ First countable and sn-symmetric spaces.

Definition 2.8 ([15], [18]). Let (X, d) be an sn-symmetric (resp. symmetric, semi-metric, metric) space. A mapping $f \colon X \to Y$ is called a π-mapping with respect to d, if for each $y \in U$ with U open in Y, $d(f^{-1}(y), X - f^{-1}(U)) > 0$.

Definition 2.9. Let $f \colon X \to Y$ be a mapping.

(1) f is called a 1-sequence-covering mapping [12], if for each $y \in Y$ there exists $x \in f^{-1}(y)$ such that whenever $\{y_n\}$ is a sequence converging to y in Y, there exists a sequence $\{x_n\}$ converging to x in X with each $x_n \in f^{-1}(y_n)$.

(2) f is called a strong sequence-covering mapping [9], if whenever $\{y_n\}$ is a convergent sequence in Y, there exists a convergent sequence $\{x_n\}$ in X with each $f(x_n) = y_n$.

(3) f is called a sequentially quotient mapping [2], if whenever S is a convergent sequence in Y, there exists a convergent sequence L in X such that $f(L)$ is a subsequence of S.
(4) f is called a weak-open mapping [20] if there exists a weak base $\bigcup\{\mathscr{P}_y\colon\ y \in Y\}$ of Y such that for each $y \in Y$, there exists $x \in f^{-1}(y)$, such that whenever U is a neighborhood of x in X, then $P \subset f(U)$ for some $P \in \mathscr{P}_y$.
(5) f is called a σ-mapping [13], if there exists a base $\mathscr{B}$ of X such that $f(\mathscr{B})$ is σ-locally-finite in Y.
(6) f is called an $mssc$-mapping [13], if X is a subspace of the product space $\prod\limits_{n\in\mathbb{N}} X_n$ in which each X_n is metrizable, and for each $y \in Y$, there exists a sequence $\{V_n\}$ of open neighborhoods of y in Y such that each $\overline{p_n(f^{-1}(V_n))}$ is a compact subset of X_n, where $p_n\colon \prod\limits_{i\in\mathbb{N}} X_i \to X_n$ is the projection.

Remark 2.10.
(1) "Strong sequence-covering mappings" in Definition 2.9(2) were called "sequence-covering mappings" in [7], [12], [16], [18], [19], [20].
(2) Quotient mappings from sequential spaces are sequentially quotient [2].
(3) Sequentially quotient mappings onto sequential spaces are quotient [2].
(4) Weak-open mappings from first countable spaces are equivalent to 1-sequence-covering, quotient mappings [20].
(5) $mssc$-mappings are σ-mappings [13].

3. The main results

The following example shows that Proposition 1.2 is not true.

Example 3.1. There exists a g-metrizable space which is not a strong sequence-covering, π-image of a metric space.

Proof. Let C_n be a convergent sequence containing its limit point p_n for each $n \in \mathbb{N}$, where $C_n \cap C_m = \emptyset$ if $n \neq m$. Let $\mathbb{Q} = \{q_n\colon\ n \in \mathbb{N}\}$ be the set of all rational numbers of the real line $\mathbb{R}$. Put $M = (\bigoplus\{C_n\colon\ n \in \mathbb{N}\}) \oplus \mathbb{R}$, and let X be the quotient space obtained from M by identifying each p_n in C_n with q_n in $\mathbb{R}$. Then

(1) X is a quotient, compact image of a separable metric space M from [18, Example 2.14(3)]. So X has a countable weak base from [12, Corollary 4.7], thus X is g-metrizable, hence X is symmetric.

Recall that a symmetric space (Y, d) is a Cauchy space if for each convergent sequence $\{y_n\}$ in Y and each $\varepsilon > 0$, there exists $k \in \mathbb{N}$ such that $d(y_n, y_m) < \varepsilon$ for all $n, m > k$. Y. Tanaka[18] proved that a space is a Cauchy space if and only if it is a strong sequence-covering, quotient, π-image of a metric space.

(2) X is not a Cauchy space from [11, Example 3.1.13(2)], so X is not a strong sequence-covering, quotient, π-image of a metric space by Tanaka's result. X is not a strong sequence-covering, π-image of a metric space from Remark 2.10(3).

The mistake in the papers [9, 10] is the following lemma: Suppose (X, d) is a metric space and $f\colon X \to Y$ is a quotient mapping. Then Y is a symmetric space if and only if f is a π-mapping with respect to d. The example 16 in [13] shows that there exists a metric space (M, d) and a quotient mapping $f\colon M \to X$ such that X is a symmetric space, but f is not a π-mapping with respect to d. □

The following Lemma is due to the proof of [12, Theorem 4.4].

Lemma 3.2. *Let $f\colon X \to Y$ be a mapping. If $\{B_n\}$ is a decreasing network at some x in X, and each $f(B_n)$ is a sequential neighborhood of $f(x)$ in Y, then whenever $\{y_n\}$ is a sequence converging to $f(x)$ in Y, there is a sequence $\{x_n\}$ converging to x in X with each $x_n \in f^{-1}(y_n)$.*

Proof. Let $\{y_n\}$ be a sequence converging to $y = f(x)$ in Y. For each $k \in \mathbb{N}$, there exists $n_k \in \mathbb{N}$ such that $y_n \in f(B_k)$ for each $n > n_k$. Thus $f^{-1}(y_n) \cap B_k \neq \emptyset$ for each $n > n_k$. Without loss of generality, we can assume $1 < n_k < n_{k+1}$ for each $k \in \mathbb{N}$. For each $n \in \mathbb{N}$, pick

$$x_n \in \begin{cases} f^{-1}(y_n), & n < n_1, \\ f^{-1}(y_n) \cap B_k, & n_k \leqslant n < n_{k+1}. \end{cases}$$

Then each $x_n \in f^{-1}(y_n)$. We show that $\{x_n\}$ converges to x as follows. Let U be a neighborhood of x. There exists $k \in \mathbb{N}$ such that $x \in B_k \subset U$. For each $n > n_k$, there exists $k' \geqslant k$ such that $n_{k'} \leqslant n < n_{k'+1}$, so $x_n \in B_{k'} \subset B_k \subset U$. This proves that $\{x_n\}$ converges to x. □

Lemma 3.3. *Let $f\colon M \to X$ be a mapping with sn-symmetric d on M.*

(1) *If X is an sn-symmetric space, then f is a π-mapping with respect to some sn-symmetric on M.*

(2) *If f is a sequentially quotient, π-mapping, then X is an sn-symmetric space.*

Proof. (1) Let (X, d') be an sn-symmetric space. Put $\delta(a, b) = d(a, b) + d'(f(a), f(b))$ for $a, b \in M$. It is clear that δ is a d-function on M. Let $a \in M, x \in X$ and $n \in \mathbb{N}$; we denote $\{b \in M\colon \delta(a, b) < 1/n\}$, $\{b \in M\colon d(a, b) < 1/n\}$ and $\{y \in X\colon d'(x, y) < 1/n\}$ by $S_n(a)$, $S_n^1(a)$ and $S_n^2(x)$ respectively.

Claim 1. $\{S_n(a)\}$ is a network at a in M for each $a \in M$.

Let $a \in U$ with U open in M. Since d is an sn-symmetric on M, there exists $n \in \mathbb{N}$ such that $S_n^1(a) \subset U$. Since $d(a,b) \leqslant \delta(a,b)$ for each $b \in M$, $S_n(a) \subset S_n^1(a) \subset U$. Hence $\{S_n(a)\}$ is a network at a in M.

Claim 2. $S_n(a)$ is a sequential neighborhood of a for each $a \in M, n \in \mathbb{N}$.

Let $\{a_k\}$ be a sequence converging to a in M. Then $\{f(a_k)\}$ converges to $f(a)$ in X. There exist $k_0 \in \mathbb{N}$ such that $d(a, a_k) < 1/2n$ and $d'(f(a), f(a_k)) < 1/2n$ for all $k > k_0$. Then $\delta(a, a_k) = d(a, a_k) + d'(f(a), f(a_k)) < 1/n$ for each $k > k_0$. That is $a_k \in S_n(a)$ for all $k > k_0$. So $\{a_k\}$ is eventually in $S_n(a)$, and $S_n(a)$ is a sequential neighborhood of a in M.

By Claim 1 and Claim 2, δ is an sn-symmetric on M.

Claim 3. f is a π-mapping with respect to δ.

Let $x \in U$ with U open in X. There exists $n \in \mathbb{N}$ such that $S_n^2(x) \subset U$. If $a \in f^{-1}(x), b \in M - f^{-1}(U)$, then $f(b) \notin U$, and $d'(x, f(b)) \geqslant 1/n$, thus $\delta(a,b) \geqslant d'(f(a), f(b)) = d'(x, f(b)) \geqslant 1/n$. So $\delta(f^{-1}(x), M - f^{-1}(U)) \geqslant 1/n$.

(2) Let f be a sequentially quotient, π-mapping. Put $d'(x,y) = d(f^{-1}(x), f^{-1}(y))$ for each $x, y \in X$. It is clear that d' is a d-function on X. Let $a \in M, x \in X$ and $n \in \mathbb{N}$; we denote $\{b \in M\colon\ d(a,b) < 1/n\}$ and $\{y \in X\colon\ d'(x,y) < 1/n\}$ by $S_n(a)$ and $S'_n(x)$ respectively.

Claim 1. $\{S'_n(x)\}$ is a network at x in X for each $x \in X$.

Let U be an open neighborhood of x in X. There exists $n \in \mathbb{N}$ such that $d(f^{-1}(x), M - f^{-1}(U)) \geqslant 1/n$. If $y \notin U$, then $f^{-1}(y) \subset M - f^{-1}(U)$, hence $d'(x,y) = d(f^{-1}(x), f^{-1}(y)) \geqslant d(f^{-1}(x), M - f^{-1}(U)) \geqslant 1/n$, so $y \notin S'_n(x)$. This proves that $S'_n(x) \subset U$.

Claim 2. $S'_m(x)$ is a sequential neighborhood of x for each $x \in X, m \in \mathbb{N}$.

Let $\{x_n\}$ be a sequence converging to x. Since f is sequentially quotient, there exists a sequence $\{a_k\}$ converging to $a \in f^{-1}(x)$ such that each $f(a_k) = x_{n_k}$. There exists $k_0 \in \mathbb{N}$ such that $d(a, a_k) < 1/m$ for all $k \geqslant k_0$. So $d'(x, x_{n_k}) = d(f^{-1}(x), f^{-1}(x_{n_k})) \leqslant d(a, a_k) < 1/m$ for all $k \geqslant k_0$, that is, $x_{n_k} \in S'_m(x)$ for all $k \geqslant k_0$. Thus $\{x_n\}$ is frequently in $S'_m(x)$. It is easy to check that $S'_m(x)$ is a sequential neighborhood of x.

By Claim 1 and Claim 2, d' is an sn-symmetric on X. □

Corollary 3.4. *Each sn-metrizable space is an sn-symmetric space.*

Proof. Let X be an sn-metrizable space. Then X is a sequentially quotient, π, σ-image of a metric space from [6, Theorem 3.4]. Thus (X, d) is an sn-symmetric space by Lemma 3.3(2). □

Theorem 3.5. *The following are equivalent for a space X.*

(1) *X is an sn-metrizable space.*

(2) *X is a 1-sequence-covering, π, $mssc$-image of a semi-metric space.*

(3) *X is a sequentially quotient, π, σ-image of an sn-symmetric space.*

Proof. Since each $mssc$-mapping is a σ-mapping by Remark 2.10(5), we only need to prove that (1) $\Longrightarrow$ (2) and (3) $\Longrightarrow$ (1).

(1) $\Longrightarrow$ (2). Suppose that X has a σ-locally-finite sn-network $\mathscr{P}=\bigcup\{\mathscr{P}_x\colon x\in X\}=\bigcup\{\mathscr{P}_n\colon n\in\mathbb{N}\}$, where each $\mathscr{P}_x$ is an sn-network at x in X and each $\mathscr{P}_n=\{P_\beta\colon \beta\in A_n\}$ is a locally finite family of subsets of X. Without loss of generality, we can suppose that each $\mathscr{P}_n$ is closed under finite intersections and $X\in\mathscr{P}_n\subset\mathscr{P}_{n+1}$. Each A_n is endowed the discrete topology. Put

$$M=\{b=(\beta_n)\in\prod_{n\in\mathbb{N}}A_n\colon \{P_{\beta_n}\} \text{ is a network at some point } x_b \text{ in } X\}.$$

Then M is a metric space, and $f\colon M\to X$ defined by $f(b)=x_b$ is a mapping.

Claim 1. f is a 1-sequence-covering mapping.

Let $x\in X$. For each $n\in\mathbb{N}$, there exists $\beta_n\in A_n$ such that $P_{\beta_n}=\bigcap\{P\in\mathscr{P}_n\ P\in\mathscr{P}_x\}\in\mathscr{P}_x$. Thus $\{P_{\beta_n}\}$ is a network at x in X. Put $b=(\beta_n)$, then $b\in f^{-1}(x)$. Let $B_n=\{(\gamma_k)\in M\colon \gamma_k=\beta_k\ for\ k\leqslant n\}$ for each $n\in\mathbb{N}$. We prove that $f(B_n)=\bigcap\limits_{k\leqslant n}P_{\beta_k}\in\mathscr{P}_x$ for each $n\in\mathbb{N}$ as follows.

In fact, let $c=(\gamma_k)\in B_n$. Then $f(c)\in\bigcap\limits_{k\in\mathbb{N}}P_{\gamma_k}\subset\bigcap\limits_{k\leqslant n}P_{\beta_k}$, so $f(B_n)\subset\bigcap\limits_{k\leqslant n}P_{\beta_k}$. On the other hand, let $y\in\bigcap\limits_{k\leqslant n}P_{\beta_k}$. Then there exists $c'=(\gamma'_k)\in M$ such that $f(c')=y$. For each $k\in\mathbb{N}$, put $\gamma_k=\beta_k$ if $k\leqslant n$, and $\gamma_k=\gamma'_{k-n}$ if $k>n$. Then $\{P_{\gamma_k}\}$ is a network at y in X. Let $c=(\gamma_k)$, then $c\in B_n$ and $f(c)=y$, so $y\in f(B_n)$. Thus $\bigcap\limits_{k\leqslant n}P_{\beta_k}\subset f(B_n)$.

It is obvious that $\{B_n\}$ is a decreasing neighborhood base at b in M. Thus f is a 1-sequence-covering mapping by Lemma 3.2.

Claim 2. f is an $mssc$-mapping.

For each $x\in X, n\in\mathbb{N}$, there exists an open neighborhood V_n of x in X such that V_n only meets with finite by many elements in $\mathscr{P}_n$ because $\mathscr{P}_n$ is locally finite in X. Let $\Lambda_n=\{\beta\in A_n\colon P_\beta\cap V_n\neq\emptyset\}$, then Λ_n is finite in A_n and $\overline{p_n(f^{-1}(V_n))}\subset\Lambda_n$ is compact. Hence f is an $mssc$-mapping.

Claim 3. f is a π-mapping with respect to some semi-metric on M.

X is an sn-symmetric space from Corollary 3.4. Thus f is a π-mapping with respect to some semi-metric on M from Lemma 3.3(1) and Remark 2.7(4).

(3) $\Longrightarrow$ (1). Let M be an sn-symmetric space, and $f\colon M \to X$ a sequentially quotient, π, σ-mapping. Then X is an sn-symmetric space from Lemma 3.4(2). Thus X is sn-first countable. Since a space is sn-metrizable if and only if it is an sn-first countable space with a σ-locally finite cs^*-network [6], to complete the proof it suffices to prove that X has a σ-locally finite cs^*-network. Since f is a σ-mapping, there exists a base $\mathscr{B}$ of M such that $f(\mathscr{B})$ is a σ-locally-finite family in X. Let S be a sequence converging to $x \in U$ with U open in X. There exists a sequence L converging to some $a \in f^{-1}(x)$ such that $f(L)$ is a subsequence of S. Thus there exists $B \in \mathscr{B}$ such that $a \in B \subset f^{-1}(U)$. So L is eventually in B, hence $f(L)$ is eventually in $f(B) \subset U$. Thus S is frequently in $f(B) \in f(\mathscr{B})$. So $f(\mathscr{B})$ is a cs^*-network of X. $\square$

We have the following main theorem of this paper by Remarks 2.7, 2.10 and Theorem 3.5.

Theorem 3.6. *The following are equivalent for a space X.*

(1) *X is a g-metrizable space.*

(2) *X is a weak-open, $\pi, mssc$-image of a semi-metric space.*

(3) *X is a weak-open, π, σ-image of a semi-metric space.*

(4) *X is a strong sequence-covering, quotient, $\pi, mssc$-image of a semi-metric space.*

(5) *X is a strong sequence-covering, quotient, π, σ-image of a semi-metric space.*

Remark 3.7. By Example 3.1, "semi-metric" in Theorem 3.6 can not be replaced by "metric".

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