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A CLASS OF DIFFERENTIAL EQUATIONS SIMILAR TO LINEAR EQUATIONS

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In the paper it is shown that certain properties, especially those connected with some differential inequalities (monotonicity, disconjugacy, etc.) of a linear differential (fort short d.) equation

$$x^{(n)}+\sum_{k=1}^{n} P_k(t)x^{(n-k)}=Q(t)$$

can be extended to the class of nonlinear d. equations of the form

$$x^{(n)}+\sum_{k=1}^{n} p_k(t, x, x', \ldots, x^{(n-1)})x^{(n-k)}=q\ (t, x, x', \ldots, x^{(n-1)})$$

or to a special case of that class. In this way a Hartman—Wintner's result has been generalized. This also extends a theorem of Anichini—Schuur. The main tool in the proof is the application of the Fan and Glicksberg fixed point theorem in which a compactness condition plays an important role. Further the existence of a solution to a nonlinear boundary value problem is proved, which generalizes a result of Kannan—Locker.

1. First we introduce some notions. Let $I=[a, b)$, $-\infty<a<b\leqq\infty$, $J=(-\infty, \infty)$. Let $C^{n-1}(I)$ be the vector space of all real functions (in what follows only real functions will be considered) which have $n-1$ continuous derivatives on I. The topology on $C^{n-1}(I)$ is introduced by the countable family of seminorms

$$p_m(x)=\max_{0\leqq i\leqq n-1}\ \max_{t\in[a, a+m]} |x^{(i)}(t)|$$

(if $b=\infty$) and in the case $b<\infty$ by

$$p_m(x)=\max_{0\leqq i\leqq n-1}\ \max_{t\in[a, b-\frac{1}{m}]} |x^{(i)}(t)|$$

for all m such that $a<b-\dfrac{1}{m}$. In this topology $C^{n-1}(I)$ is a Fréchet space and the convergence $x_p\to x$ in this space means the locally uniform convergence in I of $x_p^{(i)}$ to $x^{(i)}$ up to the order $n-1$. In a similar way the Fréchet spaces $C^0(I)$, $C^{n-1}(J)$, $C^0(J)$ are defined.

Lemma 1 (The Fan and Glicksberg fixed point theorem, see [6], [7], [1, p. 249]). *If S is a closed, convex, nonempty subset of a Fréchet space X and if T satisfies:* i) *for each $u \in S$, $T(u)$ is nonempty, compact, convex subset of X;* ii) *T is a closed mapping;* iii) *$T(S)$ is contained in a compact subset of S, then there is a $u \in S$ such that $u \in T(u)$.*

Lemma 2. *Let $P_{k,m} \in C^0(I)$, $Q_m \in C^0(I)$, $k=1, \ldots, n$, $m=1,2,\ldots$, be bounded in the topology of $C^0(I)$, i.e. on each compact subinterval of I the sequences $\{P_{k,m}\}_{m=1}^{\infty}$, $\{Q_m\}_{m=1}^{\infty}$ $(k=1,\ldots,n)$ are uniformly bounded. Then the following statement holds:*

If $\{x_m\}_{m=1}^{\infty}$ is a sequence of solutions of the d. equations

$$(1_m) \qquad x^{(n)} + \sum_{k=1}^{n} P_{k,m}(t)x^{(n-k)} = Q_m(t)$$

which is bounded in the $C^0(I)$ topology, then it is relatively compact in the topology of $C^{n-1}(I)$.

Proof. The case $n=1$ is clear. Suppose, therefore, $n>1$. Let $[c,d]$ be a compact subinterval of I. Denote by $\|\cdot\|_0$ the sup-norm on this interval. By the assumptions of the lemma there exists an $\alpha>0$ such that

$$(2) \qquad \|P_{k,m}\|_0 \leqq \alpha, \quad \|x_m\|_0 \leqq \alpha \quad \text{and} \quad \left\| x_m^{(n)} + \sum_{k=1}^{n} P_{k,m} x_m^{(n-k)} \right\|_0 \leqq \alpha$$

$$(k=1,\ldots,n, \quad m=1,2,\ldots).$$

Without loss of generality we can assume that $\alpha \geqq 1$, $n!\ \alpha \geqq (d-c)^n$. Put $\|x_m^{(n)}\|_0 = \beta_m$. By [10, p. 1260; 3, p. 140], there exist constants $a_{n,k}>0$, $k=1,\ldots,n-1$, such that

$$(3) \qquad \begin{aligned} \|x_m^{(k)}\|_0 &\leqq a_{n,k}\alpha^{(n-k)/n}\left[\max\left(\beta_m, \frac{n!}{(d-c)^n}\alpha\right)\right]^{k/n} \leqq \\ &\leqq a_{n,k}\alpha^{(n-1)/n}\left[\max\left(\beta_m, \frac{n!}{(d-c)^n}\alpha\right)\right]^{(n-1)/n}, \quad k=1,\ldots,n-1. \end{aligned}$$

Two cases should be distinguished.

1. $\beta_m \leqq \dfrac{n!\,\alpha}{(d-c)^n}$,

2. $\dfrac{n!\,\alpha}{(d-c)^n} < \beta_m$.

In the latter case, by (2), (3),

$$\beta_m \leqq \alpha + \sum_{k=1}^{n-1} a_{nn\,n-k}\alpha^{(2n-1)/n}\beta_m^{(n-1)/n}$$

and, hence,

$$\beta_m \leqq \alpha^n \left[1+\sum_{k=1}^{n-1} a_{n,n-k}\alpha^{(n-1)/n}\right]^n .$$

Let

$$\beta = \max\left(\frac{n!}{(d-c)^n}\,\alpha,\ \alpha^n\left[1+\sum_{k=1}^{n-1} a_{n,n-k}\alpha^{(n-1)/n}\right]^n\right).$$

Then $\|x_m^{(n)}\|_0 \leqq \beta$ and, again by (3), $\|x_m^{(k)}\|_0 \leqq a_{n,k}\alpha^{(n-1)/n}\beta^{(n-1)/n}$, $(m=1, 2, \ldots,$ $k=1, \ldots, n-1)$.

Hence, by the Ascoli lemma, any uniformly bounded sequence $\{x_m\}$ in $[c, d]$ contains a subsequence $\{x_{m(p)}\}$ which is uniformly convergent on $[c, d]$ with its derivatives up to the order $n-1$. I can be covered by a sequence of compact subintervals, and, by a diagonalization process, a subsequence $\{x_{m(r)}\}$ can be extracted such that $\{x_{m(r)}^{(i)}\}$, $i=0, 1, \ldots, n-1$ converges uniformly on any compact subinterval of I. This means that the sequence $\{x_m\}$ is relatively compact in $C^{n-1}(I)$.

With respect to Corollary 4.1 ([8, p. 73]), the last lemma yields

Corollary. *If the sequences $\{P_{k,m}\}$ and $\{Q_m\}$ are locally uniformly convergent to the functions P_k and Q, respectively, on I for $k=1, \ldots, n$, and $\{x_m\}$ is a sequence of solutions of (1_m) which are uniformly bounded on each compact subinterval of I, then there exists a subsequence $\{x_{m(r)}\}$ and a solution x of*

$$x^{(n)}+\sum_{k=1}^{n} P_k(t)x^{(n-k)} = Q(t) \quad (t \in I)$$

such that $\{x_{m(r)}^{(i)}\}$ uniformly converges to $x^{(i)}$ on each compact subinterval of I for $i=0, 1, \ldots, n-1$.

Remark. Lemma 2 and its Corollary remain valid when instead of I the open interval J is considered.

The next lemma describes a property of linear d. equations.

Lemma 3 (Hartman—Wintner, [9, p. 204]). *Let m, $0<m\leqq n$ be fixed. Let $P_k \in C^0(I)$, $k=1, \ldots, n$ and $P_k(t)\geqq 0$ for $k=m+1, \ldots, n$ if $m<n$, and for all $t\in I$. Let the m-th order d. equation*

$$(4) \qquad (L_m(x)\equiv)x^{(m)}+\sum_{k=1}^{m}(-1)^{k+1}P_k(t)x^{(m-k)}=0$$

possess a set of solutions $u_1, \ldots, u_m$ satisfying $W_k(u_1, \ldots, u_k)(t) = \det(u_i^{(j-1)}(t))>0$, $i, j=1, \ldots, k$ for $k=1, \ldots, m$, $t\in I$. Then

$$(L_n(x)\equiv)\, x^{(n)}+\sum_{k=1}^{n}(-1)^{k+1}P_k(t)x^{(n-k)}=0$$

has a solution x satisfying

$$x(t)>0 \quad \textit{and} \quad (-1)^k x^{(k)}(t) \geqq 0 \quad \textit{for} \quad k=0, 1, \dots, n-m.$$

Corollary. *If $P_n(t) \equiv 0$ is not true in any subinterval of I and $0<m<n$, then the mentioned solution x shows the property*

$$(-1)^k x^{(k)}(t)>0 \quad (k=0, \dots, n-m-1, t \in I)$$

and $(-1)^{n-m} x^{(n-m)}$ has less than $\frac{m+1}{2}$ $\left(\frac{m}{2}+1\right)$ different zeros on I when m is odd (m is even).

Proof. When x is the considered solution, the function $y=x^{(n-m)}$ satisfies the nonhomogeneous d. equation

$$L_m(y)=\sum_{k=m+1}^{n} (-1)^k P_k(t) x^{(n-k)}(t), \quad t \in I. \tag{5}$$

Denote the right-hand side of (5) as h. Then h does not vanish identically on any subinterval of I and its sign is equal to $(-1)^n$. Further all zeros of y are of multiplicity at least 2. If m is odd and y has $\frac{m+1}{2}$ different zeros $t_1<t_2<\dots<t_j$, $j=\frac{m+1}{2}$, then the Green function G corresponding to the problem

$$L_m(y)=0, \quad y(t_k)=y'(t_k)=0, \quad k=1, \dots, \frac{m+1}{2}$$

is, on the basis of a result of Levin [11, pp. 80—81], nonnegative. y can be written in the form $y(t)=\int_{t_1}^{t_j} G(t, s)h(s)\, \mathrm{d}s$, $t \in [t_1, t_j]$, which is a contradiction since the signs on the two sides of this equality are mutually different.

When m is even and y has $\frac{m}{2}+1$ different zeros, then we consider the Green function G_1 of the problem

$$L_m(y)=0, \quad y(t_k)=y'(t_k)=0, \quad k=1, \dots, \frac{m}{2}$$
$$y(t_l)=0, \quad l=\frac{m}{2}+1$$

Since $G_1 \leqq 0$ and $y(t)=\int_{t_1}^{t_l} G_1(t, s)h(s)\, \mathrm{d}s$, we again have a contradiction. Using the fact that $x^{(n-m)}$ is of a constant sign and has only finitely many zeros, we get the statement of the corollary.

Remarks. 1. Since the lemma and its corollary are based on Theorem 2.1, [8, p. 592], which is true also on an open interval, in this lemma and its corollary the interval I can be replaced by J both in the assumptions and in the statements.

2. If $m=1$, then (4) clearly satisfies the assumption of Lemma 3. For $m>1$ a sufficient condition for the existence of a *Markov system* of solutions $u_1, \ldots, u_m$ of (4) (i.e. with Wronskians $W_k(u_1, \ldots, u_k)>0$, $k=1, \ldots, m$, on I), is the existence of $m-1$ functions $y_1, \ldots, y_{m-1} \in C^m(I)$ which form a *Descartes system* on I (i.e. the Wronskians $W_k(y_{i_1}, \ldots, y_{i_k})$ $(1 \leqq i_1 < \ldots < i_k \leqq m-1, k=1, \ldots, m-1)$ are positive on I), and satisfy the inequalities $(-1)^{m-k} L_m(y_k)(t) \geqq 0$ $(k=1, \ldots, m-1, t \in I)$ ([4, p. 123]). Another sufficient condition on a compact or on an open interval j is that the equation (4) should be disconjugate on j ([4, pp. 94, 116]).

Lemma 3 and its Corollary will be generalized to the nonlinear d. equation

$$x^{(n)} + \sum_{k=1}^{n} (-1)^{k+1} p_k(t, x) x^{(n-k)} = 0. \tag{6}$$

Theorem 1. *Let* $1 \leqq m \leqq n$, $p_k \in C^0(I \times R)$, $k=1, \ldots, n$, *and if* $m<n$, *let* $p_k(t, x) \geqq 0$ *on* $I \times R$, $k=m+1, \ldots, n$. *If* $1<m$, *let there exist* $m-1$ *functions* $u_l \in C^m(I)$, $l=1, \ldots, m-1$, *which form a Descartes system on I and satisfy*

$$(-1)^{m-l} \left[u_l^{(m)}(t) + \sum_{k=1}^{m} (-1)^{k+1} p_k(t, x) u_l^{(m-k)}(t) \right] \geqq 0 \quad (t \in I)$$

for each point $x \in R$, $l=1, \ldots, m-1$.

Then for any $c>0$ (6) *possesses a solution* x *on I such that*

$$\begin{gathered} x(a)=c, \quad x(t)>0, \quad (-1)^k x^{(k)}(t) \geqq 0 \\ \text{for} \quad k=0, 1, \ldots, n-m, \quad t \in I. \end{gathered} \tag{7}$$

If in the case $m<n$ $p_n(t, x)>0$ *on* $I \times R$, *then* x *satisfies*

$$(-1)^k x^{(k)}(t)>0 \quad (k=0, \ldots, n-m-1, t \in I)$$

and $x^{(n-m)}$ *has less than* $\frac{m+1}{2}$ $\left(\frac{m}{2}+1\right)$ *different zeros on I, when m is odd (m is even).*

Proof. 1. The case $m<n$. Consider the Fréchet space $C^{n-1}(I)$ topologized as above. Let $S=\{x \in C^{n-1}(I): x(a)=c, (-1)^k \cdot x^{(k)}(t) \geqq 0, t \in I, k=0, 1, \ldots, n-m\}$. S is a closed, convex and nonempty subset of $C^{n-1}(I)$. For $u \in S$ let $T(u)=\{x \in S$: x is a solution of the d. equation

$$x^{(n)} + \sum_{k=1}^{n} (-1)^{k+1} p_k(t, u(t)) x^{(n-k)} = 0 \tag{8_u}$$

which satisfies (7)$\}$. By Lemma 3 and linearity of (8_u), $T(u) \neq \emptyset$ and $T(u)$ is convex. Since $T(u) \subset S$ and S is bounded in the topology of $C^0(I)$, by Lemma 2, $T(u)$ is relatively compact in $C^{n-1}(I)$. But $T(u)$ is also closed, and hence, compact in the C^{n-1} topology. Thus the so defined mapping T: $S \rightarrow 2^S$ satisfies the requirement i) of Lemma 1.

If $u_p \in S$, $u_p \to u_0$ and $x_p \in T(u_p)$, $x_p \to x_0$, the convergence being considered in $C^{n-1}(I)$, then the functions $p_k(\cdot, u_p(\cdot))$ converge locally uniformly on I to $p_k(\cdot, u_0(\cdot))$, $k=1, \ldots, n$, and by Corollary 4.1, [8, p. 73], $x_p \to y_0$, where y_0 is the solution of (8_{u_0}) satisfying the same initial condition as x_0. Therefore $x_0 = y_0$ and $x_0 \in T(u_0)$. Thus T is a closed mapping.

As S is bounded in the topology of $C^0(I)$ and $T(S) \subset S$, Lemma 2 guarantees that $T(S)$ is relatively compact in $C^{n-1}(I)$, hence its closure $\overline{T(S)} \subset S$ is compact. Thus all assumptions of Lemma 1 are satisfied. By this lemma there exists an $x \in S$ such that $x \in T(x)$. x is then the searched solution. When $p_n(t, x)$ is everywhere positive, the Corollary to Lemma 3 implies the last statement of the theorem.

2. If $m=n$, the definition of S must be changed. The other steps of the proof remain the same. Consider the functions u_l, $l=1, \ldots, n-1$. Since $c_l u_l$, $c_l > 0$, $l=1, \ldots, n-1$, also form a Descartes system on I, we can assume that all $u_l(a)=c$, $l=1, \ldots, n-1$. Let $S=\{x \in C^{n-1}(I)\colon x(a)=c,\ 0 \leqq x(t) \leqq u_1(t)\ (t \in I)\}$. Then S is a closed, convex and nonempty subset of $C^{n-1}(I)$. By Theorem 18 [4, p. 128], there is a fundamental system $U_1, \ldots, U_n$ of (8_u) which forms a Markov system and is such that

$$\frac{U_1'}{U_1} \leqq \frac{u_1'}{u_1} \leqq \frac{U_2'}{U_2} \leqq \ldots \leqq \frac{u_{n-1}'}{u_{n-1}} \leqq \frac{U_n'}{U_n}$$

on I. Then by Theorem 12 [4, p. 110] there exists a principal solution U of (8_u) which is positive and $W(U, U_1) \geqq 0$ on I. Therefore $\frac{U'}{U} \leqq \frac{U_1'}{U_1}$. The principal solution is uniquely determined by the condition $U(a)=c$. Using the inequalities above we come to the conclusion that the set $T(u)=\{x \in S\colon x$ is the principal solution of (8_u) which satisfies $x(a)=c\}$ consists of exactly one element.

Remark. Theorem 1 is a generalization of Theorem 3 in [1, p. 253].

2. In the second part of the paper a theorem of Kannan and Locker dealing with a nonlinear boundary value problem in [12, p. 3] is strengthened. Here the function f need not be bounded and the coefficients a_i do not belong to $C^\infty([a, b])$ as we shall see.

Let $-\infty < a < b < \infty$ and denote $K=[a, b]$. Consider the real Hilbert space $L^2(K)$ with the norm $\|\cdot\|$, and let $C^{n-1}(K)$ ($C^0(K)$) be provided with the norm $\|\cdot\|_{n-1}$ ($\|\cdot\|_0$) defined by

$$\|x\|_{n-1} = \sum_{i=0}^{n-1} \max_{t \in K} |x^{(i)}(t)| \quad (x \in C^{n-1}(K))$$

$$\|x\|_0 = \max_{t \in K} |x(t)| \quad (x \in C^0(K)).$$

In accordance with the definition of $\|\cdot\|_{n-1}$ the norm $|\cdot|$ in R^n will be taken as

$$|p| = \sum_{i=1}^{n} |p_i| \quad (p = (p_1, \ldots, p_n) \in R^n).$$

Let L be an n-th order formal operator given by

$$L(x) = \sum_{i=0}^{n} a_i(t) x^{(i)}, \tag{9}$$

where $a_i \in C^i(K)$, $i = 0, 1, \ldots, n$ and $a_n(t) \neq 0$ on K. Let

$$B_i(x) = \sum_{j=1}^{n} \alpha_{ij} x^{(j-1)}(a) + \sum_{j=1}^{n} \beta_{ij} x^{(j-1)}(b) \quad (i = 1, \ldots, n)$$

be a set of n linearly independent boundary conditions where α_{ij}, β_{ij} $(i, j = 1, \ldots, n)$ are real numbers.

In [5, p. 463] (Lemma 16, Chapter XIII.2) the following properties of the space $H^n(K)$, the subspace of $L^2(K)$ consisting of all functions $x \in C^{n-1}(K)$ with $x^{(n)} \in L^2(K)$ have been derived. The second statement gives a useful compactness condition.

Lemma 4 (See also [12, p. 3]). 1. *The space $H^n(K)$ is a Banach space under the norm $\|x\|_{n-1} + \|x^{(n)}\|$.*

2. *If $a_i \in C^i(K)$ $(i = 0, 1, \ldots, n)$ and $a_n(t) \neq 0$ in K, then there exists a constant M_1 depending only on L, K and n such that for each $x \in H^n(K)$*

$$\|x\|_{n-1} + \|x^{(n)}\| \leqq M_1 [\|x\| + \|L(x)\|]. \tag{10}$$

In fact, the lemma has been proved under the assumption that $a_i \in C^\infty(K)$, but the proof is still valid under a weaker assumption $a_i \in C^i(K)$, $i = 0, 1, \ldots, n$.

Suppose the problem

$$\pi\colon L(x) = \lambda x, \quad B_i(x) = 0 \quad (i = 1, \ldots, n)$$

is self-adjoint ([2, p. 189]). Then there exists an orthonormal basis for $L^2(K)$ made up of eigenfunctions Φ_i, $i = 1, 2, \ldots$ of π and let λ_i, $i = 1, 2, \ldots$ be the corresponding eigenvalues of π. We have that $|\lambda_i| \to \infty$ as $i \to \infty$.

The following theorem generalizes Theorem 1 in [12, p. 3].

Theorem 2. *Let the problem π be self-adjoint. Let $h\colon K \times R^n \to R$ and $f\colon K \times R^n \to R$ be two continuous functions such that*

a) *there exist real numbers p, q with $p \leqq h(t, x) \leqq q$, $t \in K$, $x \in R^n$,*

b) $\lambda_i \notin [p, q]$, $i = 1, 2, \ldots$,

c) $\liminf_{r \to \infty} \frac{Q_r}{r} = 0$, *where* $Q_r = \max_{t \in K,\, |x| \leqq r} |f(t, x)|$ $(0 < r < \infty)$.

Then the nonlinear boundary value problem

$$L(x)-h(t,x,x',\dots,x^{(n-1)})x=f(t,x,x',\dots,x^{(n-1)})$$
$$B_i(x)=0 \quad (i=1,\dots,n)$$

has at least one solution.

Proof. The proof is a modification of the proof of Theorem 1 in [10]. Instead of $H^{n-1}(K)$ its subspace $C^{n-1}(K)$ is used. First, for any function $w\in C^{n-1}(K)$ the linear boundary value problem

$$(11)\quad L(x)-h[t,w(t),w'(t),\dots,w^{(n-1)}(t)]x=f[t,w(t),w'(t),\dots,w^{(n-1)}(t)]$$
$$B_i(x)=0 \quad (i=1,\dots,n)$$

is considered. Since $C^{n-1}(K)\subset H^{n-1}(K)$, there exists by what has been proved in [12, p. 4] a unique solution $u\in C^n(K)\cap\{x\in C^n(K)\colon B_i(x)=0,\ i=1,\dots,n\}$ of that problem. Then we define a mapping $T\colon C^{n-1}(K)\to C^n(K)$ by putting $T(w)=u$, where u is the mentioned solution.

For $T(w)$ we have the inequality

$$\|T(w)\|_{n-1}+\|(T(w))^{(n)}\|\leqq M_4\|f(t,w(t),\dots,w^{(n-1)}(t))\|,$$

which has been proved in [12, p. 5]. Its proof is based on (10). From this inequality we obtain

$$(12)\qquad \|T(w)\|_{n-1}\leqq M_4\|f(t,w(t),\dots,w^{(n-1)}(t)\|_0(b-a)^{1/2}.$$

By c) there exists an $r_0>0$ such that $Q_{r_0}=\dfrac{1}{M_4(b-a)^{1/2}}r_0$ and, thus if $\|w\|_{n-1}\leqq r_0$, then, by (12), $\|T(w)\|_{n-1}\leqq r_0$, too. Hence T maps the ball $B=\{w\in C^{n-1}(K)\colon \|w\|_{n-1}\leqq r_0\}$ into itself.

Continuity of T with respect to the $C^{n-1}(K)$ norm can be proved in a similar way as it has been done in [12, p. 6]. Using the same notations as in [12] from the inequality

$$\|u_i-u_0\|_{n-1}+\|u_i^{(n)}-u_0^{(n)}\|\leqq M_4\|(\alpha_i-\alpha_0)u_0+\beta_i-\beta_0\|$$

we get

$$\|u_i-u_0\|_{n-1}\leqq M_4r_0(b-a)^{1/2}\|\alpha_i-\alpha_0\|_0+M_4(b-a)^{1/2}\|\beta_i-\beta_0\|_0.$$

The uniform continuity of f and h on $K\times\{x\in R^n\colon |x|\leqq r_0\}$ implies that $\|\alpha_i-\alpha_0\|_0\to 0$ and $\|\beta_i-\beta_0\|_0\to 0$ as $i\to\infty$. But this gives that $\|u_i-u_0\|_{n-1}\to 0$, which means that T is continuous in the C^{n-1} topology.

Consider now the compactness of T. On the basis of (12) we have that if $\|w\|_{n-1}\leqq M$, then $\|T(w)\|_{n-1}\leqq M_5$, and by this, (11) implies that $\|(T(w))^{(n)}\|_0$ is bounded, too, which means that under the mapping T the image of a bounded set is relatively compact (in the C^{n-1} topology). The Schauder fixed point theorem completes the proof of Theorem 2.

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КЛАСС ДИФФЕРЕНЦИАЛЬНЫХ УРАВНЕНИЙ ПОДОБНЫХ ЛИНЕЙНЫМ УРАВНЕНИЯМ

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Резюме

В работе показано, что некоторые свойства, а особенно те, которые связаны с дифференциальными неравенствами (монотонность, неосцилляция), линейных дифференциальных уравнений можно перенести на класс нелинейных уравнений вида

$$x^{(n)}+\sum_{k=1}^{n}p_k(t, x, x', \ldots, x^{(n-1)})x^{(n-k)}=q(t, x, x', \ldots, x^{(n-1)})$$

Таким образом были обобщены один результат Хартмана–Винтнера и теорема Кэннана–-Локера, касающаяся существования решения одной нелинейной краеКэй задачи. В доказательствах применяются методы функционального анализа.