Book reviews

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BOOK REVIEWS

A. A. Martynyuk: QUALITATIVE METHODS IN NONLINEAR DYNAMICS. Novel Approaches to Liapunov's Matrix Functions. Marcel Dekker, New York, 2002, x + 300 pages, USD 150.–.

This book gives an account of new developments in the Lyapunov approach to nonlinear dynamics of systems. It concerns especially matrix-valued Lyapunov functions and the construction of Lyapunov functions in general.

Discrete systems and impulsive systems are also dealt with in the book. The final part of the book is devoted to the problem of estimating the domains of asymptotic stability in terms of matrix-valued Lyapunov functions.

Štefan Schwabik, Praha

B. G. Pachpatte: INEQUALITIES FOR FINITE DIFFERENCE EQUATIONS. Pure and Applied Mathematics Series, vol. 247. Marcel Dekker, New York, 2002, 551 pages.

In Introduction, the author writes: "The monograph contains a detailed account of basic finite difference inequalities which find important applications in the theory of finite difference equations and related fields. Many of the results contained here have recently been investigated and never appeared in book form before. The book contains a large number of applications in a variety of disciplines, which will provide a rich source of inspiration for future research." No doubts about the importance of inequalities in general and of (finite) difference inequalities in particular. But the book under review seems to be a little bit too much "monothematic", and my impression is that the author has used the occasion to collect here mainly his own results, which is supported also by the fact that from the 220 references, 86 are due to the author (three of them with a co-author). All five chapters are entitled "... finite difference inequalities" and differ only in the afore-standing adjective: Linear (Chap. 1), Nonlinear (Chap. 2 and Chap. 3), Linear multidimensional (Chap. 4), Nonlinear multidimensional (Chap. 5); every chapter contains at least two sections devoted to "Pachpatte's inequalities", a section about applications and a short section with historical and bibliografical notes.

Alois Kufner, Praha

G. Da Prato, L. Tubaro (eds.): STOCHASTIC PARTIAL DIFFERENTIAL EQUATIONS AND APPLICATIONS. Marcel Dekker, 2002, softcover, ISBN 0-8247-0792-3, 474 pages, USD 185.–.

These proceedings from the Fifth International Conference on Stochastic Partial Differential Equations and Applications held in Trento, Italy, are an overview of current research in the fields of the general theory of stochastic partial differential equations, specific stochastic partial differential equations, finite and infinite dimensional diffusion processes, stochastic calculus, filtering theory, stochastic quantization, mathematical finance, fluid dynamics, theory of interacting particles, quantum probability and stochastic control.

The book contains 25 contributions (of about twenty pages each) including new results as well as surveys and reviews of problems in question so that the reader can get a feeling of what is the up-to-date state of knowledge in the respective areas.

This is why the book can serve as a source for new ways of research as well as a digest for professionals working in SPDE's.

Martin Ondreját, Praha

V. Lakshmikantham, D. Trigiante: THEORY OF DIFFERNCE EQUATIONS NUMERICAL METHODS AND APPLICATIONS. Second edition. Pure and Applied Mathematics 251, Marcel Dekker, New York, 2002, ISBN 0-8247-0803-2, 300 pages, USD 150.–.

This is the second edition of the book published originally by Academic Press in 1998. Compared with the first edition, two new chapters have been added and all remaining chapters have been modified. The second edition consists of nine chapters and four appendices.

The first chapter presents some necessary preliminaries like difference calculus and related topics. The second chapter is devoted essentially to the theory of linear difference equations, the attention being paid also to the stability theory of these equations. Chapter three deals with linear systems of difference equations and extends the methods introduced in the previous chapter. The main concern of the fourth chapter is the stability theory where, in addition to the previous treatment, the comparison principle, the Lyapunov function method and other more special topics are discussed. Chapter five is new in comparison to the first addition and deals with the relationship between difference equations and banded matrices.

Chapters six and seven are directed to numerical aspects of the theory of difference equations. Attention is paid e.g. to Miller's, Olver's and Clenshaw's algorithm and to numerical aspects of solution of differential equations. Chapter eight deals with some applications of the previous results to such models of the real world phenomena as population dynamics, traffic in channels and other models. The last (new) chapter contains the treatment of classical difference equations of relevant historical interest. The appendices present useful facts on algebra and calculus of matrices, Schur criterion and Chebyshev polynomials. Each chapter contains at the end problems for solution; the solutions are given in the last appendix at the end of the book.

Compared with some other recent books on difference equations (e.g., R.P. Agarwal, *Difference Equations and Inequalities*, S. Elaydi, *An Introduction to Difference Equations*, W. G. Kelley, A. C. Peterson, *Difference Equations, an Introduction with Applications*) this book is directed more to numerical aspects of the theory of difference equations, but for the reader who is not familiar with the theory of difference equations represents a good introduction into the problem.

Ondřej Došlý, Brno

Y. Nievergelt: FOUNDATIONS OF LOGIC AND MATHEMATICS. Birkhäuser, Basel, 2002, 432 pages, EUR 90.–.

The first part is an elementary introduction to logic and classical set-theoretical foundations of mathematics. The second shorter part is an elementary introduction to discrete mathematics, mainly number theory and finite combinatorics. The book is intended for introductory undergraduate courses. Each chapter contains a lot of examples, tables and exercises (the total number of exercises is over one thousand). In the first part many proofs are written in a logical formalism. Throughout the book only simple theorems are proved and the proofs are detailed. (In order to teach students how to prove theorems, the author even gives a five-line proof the "theorem" that 2 is a prime.) The book contains a number of applications, ranging from automated theorem proving to the ENIGMA machine, which makes the book more attractive for students.

Students starting with mathematics are usually trained by computing a lot of problems in the calculus. Therefore, there are also many introductory textbooks about the calculus. For those who want to specialize in discrete mathematics, logic or computer science, this book may be a better choice.

Pavel Pudlák, Praha

E. Menzler-Trott: GENTZENS PROBLEM. Mathematische Logik im Nationalsozialistischen Deutschland. Birkhäuser, Basel, 2001, xviii+ 411 p., EUR 43.–.

The book presents a detailed biography of the mathematical logician, Gerhard Gentzen (1909–1945) along with a detailed description of mathematical logic in the Third Reich.

Gentzen's place in logic is given by his work in proof theory, especially by his work concerning the consistency of number theory from 1936, in the time when he served as an assistant to David Hilbert in Göttingen.

The author describes Gentzen as a young person without strong political views, doing his best to do his research in spite of the ideological struggle going on around him.

The story of Gentzen is in fact a case study helping to understand how individuals and especially scientists lived under National Socialism (and in various other totalitarian regimes as well).

There was an effort to establish a "German logic" in the frame of "German mathematics". Gentzen's work in logic was used by some Nazi mathematicians in power for supporting their racist views of mathematics.

During the war Gentzen served as an air force radio operator for a relatively short time and he attained the Dozent position at the German University of Prague in 1944 (all Czech universities had been closed by the Nazis in the period 1939–1945). After the defeat of Germany in May 1945 Gentzen was arrested and imprisoned in the hectic time of reprisals in Czechoslovakia. Shortly thereafter he died of starvation in captivity.

A fourteen-page essay by Jan von Plato on Gentzen and proof theory; three talks about his work by Gentzen; chronology, bibliographies, photographs are included in the appendix of the book.

The book of Menzler-Trott is a contribution to the historical memory concerning the barbarous times of the twentieth century which in many parts of the world affected people and, among others, also science very badly.

Štefan Schwabik, Praha

Hermann Sohr: THE NAVIER-STOKES EQUATIONS. An elementary functional analytic approach. Birkhäuser, Basel, 2001, 380 pages, EUR 104.–.

The aim of this monograph is to develop an elementary and self-contained approach to the mathematical theory of a viscous incompressible fluid in a domain $\Omega \in R^N$, described by the Navier-Stokes equations. The first two chapters include the basic functional analytic tools in Hilbert and Banach spaces. Their important objective is to formulate the theory for a completely general domain Ω, in particular for arbitrary unbounded, non-smooth domains. Therefore, the restriction to two and three dimensional case was necessary. In the third chapter the author deals with the stationary Navier-Stokes equations. First, he is interested in the linear case—Stokes problem (weak solution, the properties of the Stokes operator). Then he describes the stationary Navier-Stokes equations—weak solutions, and introduces two difficult terms—the convective term and the pressure. He formulates the existence result for weak solutions in bounded and unbounded domains, establishes the regularity properties, and uniqueness results. In the fourth chapter he describes the linearized nonsteady problem while the last chapter is devoted to fully nonlinear Navier-Stokes equations.

We can find there weak solutions, properties of the convective term, energy equality and strong continuity, Serrin's uniqueness condition, integrability properties of weak solutions in space and time—the scale of Serrin's quantity, the problem with pressure and regularity properties of weak solutions. He deals with an approximation of the Navier-Stokes equations and finally with the existence of weak solutions and strong solutions describing all known results. The book is mainly directed to the students familiar with basic tools in Hilbert and Banach spaces.

Šárka Nečasová, Praha

E. DiBenedetto: REAL ANALYSIS. Birkhäuser, Boston, 2002, ISBN-0-8176-4231-5, 485 pages, EUR 98.–.

The book is a self-contained introduction to real analysis. The preliminaries consist of a review of the notions of countable sets, a brief collection of the various notions of ordering, the Hausdorff maximal principle, Zorn's lemma, the well-ordering principle, and their fundamental connections. Chapter I deals with topologies and metric spaces. It includes Urysohn's lemma, the Tietze extension theorem, the Baire theorem and the Banach-Steinhauss theorem. Chapter II is a modern account of measure theory. Measures are constructed from outer measures by the Carathéodory process. The process is implemented in specific examples such as the Lebesgue-Stieltjes measures in $\mathbb{R}$ and the Hausdorff measure. The Radon measures and their regularity properties are studied. The Besicovitch covering theorem is proved for general Radon measures in $\mathbb{R}^{\mathbb{N}}$. Chapter III introduces the Lebesgue integral. The theory is preceded by the notions of measurable functions, convergence in measure, Egorov's theorem on selecting almost-everywhere convergent subsequences from sequences convergent in measure, and Lusin's theorem characterizing measurability in terms of quasicontinuity. The integral is represented in terms of the distribution function of its integrand. The theory of measures is completed by introducing the notion of signed measure and by proving Hahn's decomposition theorem. This leads to other decomposition theorems such as those of Jordan and Lebesgue. The Radon-Nikodým theorem is proved, too. Chapter IV studies measurable functions of real variables. The discussion starts from functions of bounded variation. Then the theory of differentiating a measure with respect to another one is presented. The Besicovitch-Lebesgue theorem is proved. The final part of the chapter contains a detailed discussion of the Stone-Weierstrass theorem. Chapter V introduces the theory of L^p spaces for $1 \leqslant p < \infty$. Chapter VI is an introduction to functional analysis. The first part of the chapter is devoted to Banach spaces. The Riesz lemma, the Hahn-Banach theorem and the open mapping theorem are proved. The last part of the chapter offers an introduction to Hilbert spaces. The Riesz representation theorem of a functional through the inner product is presented. Chapter VII is about spaces of real-valued, continuous functions, differentiable functions, infinitely differentiable functions with compact support in an open set, and weakly differentiable functions. The Riesz theorem on representation of linear functionals on the space of continuous fuctions with compact support is proved. Then the theory of distributions is developed. The notion of weak derivative in an open set is introduced as an aspect of the theory of distributions. The Mayers-Serrin theorem is proved. Chapter VIII deals with integrable functions of real variables. Vitali covering theorem is applied to the notion of maximal function, its properties and its related strong L^p estimates for $1 < p < \infty$. Weak estimates are also proved and used in the Marcinkiewicz interpolation theorem. Then the BMO space is studied. Chapter IX provides an array of embedding theorems for functions in Sobolev spaces.

Dagmar Medková, Praha

Israel Gohberg, Heinz Langer (eds.): LINEAR OPERATORS AND MATRICES. THE PETER LANCASTER ANNIVERSARY VOLUME. Birkhäuser, Basel, 2002, 292 pages, EUR 130,–.

The book is a contributed volume in honor of Peter Lancaster, an outstanding mathematician, on the occasion of his seventieth birthday.

It begins with a most interesting autobiographical paper of the honored person My life and Mathematics. This, together with commemorative articles of Richard K. Guy and Israel Gohberg, shows Peter Lancaster's scientific scope and influence in a worldwide scale. A complete list of his publications is also included.

The volume then presents 16 original research papers covering topics from operator theory, matrix theory and applications, topics in which Peter Lancaster was most active. Among the authors one finds such names as D. Alpay, Harm Bart, R. Bhatia, L. Elsner, I. Gohberg, M. A. Kaashoek, H. Langer, L. Lerer, I. Marek, L. Rodman, B. Silberman, and others.

Miroslav Fiedler, Praha

Philippe Blanchard, Erwin Brüning: MATHEMATICAL METHODS IN PHYSICS. Distributions, Hilbert Space Operators, and Variational Methods. Birkhäuser, Boston, 2003, xxiii+471 pages.

This text is a translated, considerably revised and extended version of the book *Distributionen und Hilbertraumoperatoren. Mathematische Methoden der Physik (Springer, Wien, 1993)* which arose on the basis of lectures for students in the fourth semester at the Faculty of Physics of the University of Bielefeld. As the authors mention in the introduction, the goal of this course is to provide basic mathematical knowledge and skills as they are needed for modern courses in quantum mechanics, relativistic quantum field theory and related areas.

The first part of the book is devoted to distributions. It starts with an introduction to basic linear functional analysis necessary for the Schwartz theory of distributions which is the focus of this part. The calculus for distributions is developed, tensor product and convolution for distributions with applications are discussed. A relation of distributions to analytic functions is described. Also spaces of generalized functions other than Schwartz distributions are introduced.

The second part is concerned with Hilbert spaces, mainly with the aspects of linear operators in Hilbert spaces. After an introduction concerning the history and a relation to physics, basic concepts of Hilbert spaces are explained. Direct sums, tensor products, separable spaces are presented and topological aspects (as compactness and weak topology) are discussed. The focus of this part are linear operators. The fundamental properties of single bounded linear operators and of the C^*-algebra $\mathcal{B}(H)$ of all bounded linear operators on a Hilbert space H are studied, a calculus in this C^*-algebra is explained. Special classes of bounded operators (projections, unitary operators, compact operators etc.) are described. Self-adjoint Hamilton operators are discussed. Elements of the spectral theory are presented, in particular, the spectral theory of compact operators is developed. The spectral theorem is explained, some applications of the spectral representation are described (functional calculus, decomposition of the spectrum, etc.)

The last part is devoted to variational methods. The core of this part is the chapter on direct methods which presents general existence results for extrema of functionals and explains minimization of special classes of functionals. Differential calculus in Banach spaces is introduced and the results needed for the classical approach of variational methods are shown. Further, the Lagrange multiplieres method is described in great generality and the

existence of such a multiplier is proved. This gives the existence of an eigenvalue of the partial differential operator. The spectrum of some linear second order partial differential operators is explicitly described. The spectral theorem for compact self-adjoint operators is proven. At last, the mathematical basis of the Hohenberg-Kohn density functional theory is given. It is based on the theory of Schrödinger operators for N-particle systems and is a starting point of some concrete methods used mainly in chemistry.

In an appendix, detailed proofs of some principles and results of functional analysis used are given.

The reader is assumed to have a solid background in analysis and linear algebra. The book is written in a very nice and understandable form and addresses mainly students with interest in the interaction between physics and mathematics.

Milan Kučera, Praha

S. Albeverio, N. Elander, W. Norrie Everitt, P. Kurasov (eds.): OPERATOR METHODS IN ORDINARY AND PARTIAL DIFFERENTIAL EQUATIONS. Birkhäuser, Basel, 2002, 434 pages, EUR 147.66,–.

This is the proceedings of the S. Kovalevsky symposium held in Stockholm in 2000. The first part of the book is devoted to the life of S. Kovalevsky—the famous female mathematician whose work influenced the development of mathematics in Europe in the 19-th century.

Part two of the book contains 23 contributions related to various topics in pure and applied mathematics: Many body interactions problems, hydroelasticity, spectral theory, the Schrödinger equation. Lyapunov exponents, families of spectral measures, singular perturbations, integral equations, relativistic scattering, the Laplace operator, resolvent estimates etc.

The book contains also two articles by J.-E. Björk describing the life and professorship of S. Kovalevsky in Stockholm.

Eduard Feireisl, Praha

A. Guzman: CONTINUOUS FUNCTIONS OF VECTOR VARIABLES. Birkhäuser, Boston, 2002, x + 207 pages, EUR 49.53.–.

The book contains the following chapters: 1. Euclidean space, 2. Sequences in normed spaces, 3. Limits and continuity in normed spaces, 4. Characteristics of continuous functions and 5. Topology in normed spaces.

Elementary topology is the basis for studying properties of real functions defined for n-dimensional vectors. The text of the book is constructed in a more or less axiomatic way, boundedness, extremes, uniform continuity and relations between open sets, connectedness, compactness and continuity of functions is presented.

The book is written very carefully with the evident aim of using it for teaching multivariable functions. A big amount of problems with solutions is presented at the end of the book.

The easy-to-read classical structure makes the book a very good source both for teaching and study.

Štefan Schwabik, Praha