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EKF–BASED DUAL SYNCHRONIZATION OF CHAOTIC COLPITTS CIRCUIT AND CHUA'S CIRCUIT

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In this paper, dual synchronization of a hybrid system containing a chaotic Colpitts circuit and a Chua's circuit, connected by an additive white Gaussian noise (AWGN) channel, is studied via numeric simulations. The extended Kalman filter (EKF) is employed as the response system to achieve the dual synchronization. Two methods are proposed and investigated. The first method treats the combination of a Colpitts circuit and a Chua's circuit as a higher-dimensional system, while the second method considers the Colpitts circuit and Chua's circuit separately and utilizes the cross-coupling scheme. The simulation results indicate that the proposed methods can effectively achieve and maintain dual synchronization of the hybrid system through an AWGN channel.

Keywords: chaos, dual synchronization, Colpitts circuit, Chua's circuit, EKF

AMS Subject Classification: 65P20, 34C15, 62M20

1. INTRODUCTION

Since the work of Pecora and Carroll on chaos synchronization [5], there have been considerable interests in this research topic for its potential usages in signal encryption, communications, radars, etc. [9, 11]. As a branch of chaos synchronization, the concept of dual synchronization was raised by Liu and Davids in 2000 [4], motivated by the study of Tsimring and Sushchik in 1996 [14] on multiplexing chaos using synchronization. According to Liu and Davids [4], dual synchronization refers to using a scalar signal to simultaneously synchronize two different pairs of chaotic oscillators. Dual synchronization offers a potential way to increase the channel capacity by sending multiple signals in one channel and then separating them in the receiver. From the viewpoint of secure communication, dual synchronization may enhance the security level since the transmitted signal is more complex than the signals generated by a single chaotic circuit.

Recently, dual synchronization of chaotic circuits has become a hot topic of research [8, 10, 15, 16]. In 2002, Yang et al. investigated the dual synchronization of Lorenz system pair and Rossle system pair and its application in communication, where the Lyapunov exponents of the error system were studied to achieve the

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synchronization [16]. In 2003, Uchida et al. demonstrated experimentally the dual synchronization of two pairs of chaotic Colpitts circuits [15]. More recently, dual synchronization of Colpitts circuit pair and Chua's circuit pair was investigated by using the linear cross-coupling method [10]. In most of the previous works, the channel between the drive and the response was assumed to be noiseless. However, in real-world communications, channel noises are unavoidable and have to be considered.

In this paper, we use EKF to investigate the dual synchronization of a hybrid system containing a chaotic Colpitts circuit and a Chua's circuit through an AWGN channel. Though EKF has been successfully applied to synchronization problems of some continuous and discrete chaotic systems [1, 3, 12], to the best of our knowledge, using EKF to study dual synchronization of chaotic circuit is new. In this paper we propose two EKF-based methods suitable for dual synchronization through an AWGN channel: One regards the combination of two chaotic systems as a higher-dimensional system, while the other considers the two systems separately and utilizes the cross-coupling method.

This paper is organized as follows. Section 2 is the problem formulation, and Section 3 describes the EKF algorithm and discusses the proposed methods. Section 4 presents the simulation results of the synchronization process and the synchronization performance subject to an AWGN channel. Section 5 concludes the paper.

2. PROBLEM FORMULATION

The configuration of the dual synchronization of chaotic Colpitts circuit and Chua's circuit with an AWGN channel is shown in Figure 1.

For the Colpitts oscillator, its state equations are as follows [7]:

$$\begin{cases} \dfrac{\mathrm{d}V_{C1,co}(t)}{\mathrm{d}t} = \dfrac{1}{C_{1,co}}\Big(- f_1(-V_{C2,co}(t)) + I_{co}(t)\Big) \\ \dfrac{\mathrm{d}V_{C2,co}(t)}{\mathrm{d}t} = \dfrac{1}{C_{2,co}}\Big(I_{co}(t) - \dfrac{V_{C2,co}(t) - V_{ee}}{R_{e,co}}\Big) \\ \dfrac{\mathrm{d}I_{co}(t)}{\mathrm{d}t} = \dfrac{1}{L_{co}}\Big(- V_{C1,co}(t) - V_{C2,co}(t) - I_{co}(t)R_{1,co} + V_{CC}\Big) \end{cases} \tag{1}$$

where $f_1(.)$ is the driving-point characteristic of the nonlinear resistor of the BJT, given in [6], described by

$$f_1(x) = I_s\left[\exp\left(\frac{x}{V_T}\right) - 1\right] \approx I_s\left[\exp\left(\frac{x}{V_T}\right)\right], \text{ if } x \gg V_T \tag{2}$$

In the Colpitts oscillator, this characteristic can be expressed as $I_E = f(V_{BE}) = f(-V_{C2})$. From Eq. (1) it follows that

$$f_1(-V_{C2}(t)) = I_s \exp\left(-\frac{V_{C2}(t)}{V_T}\right) \tag{3}$$

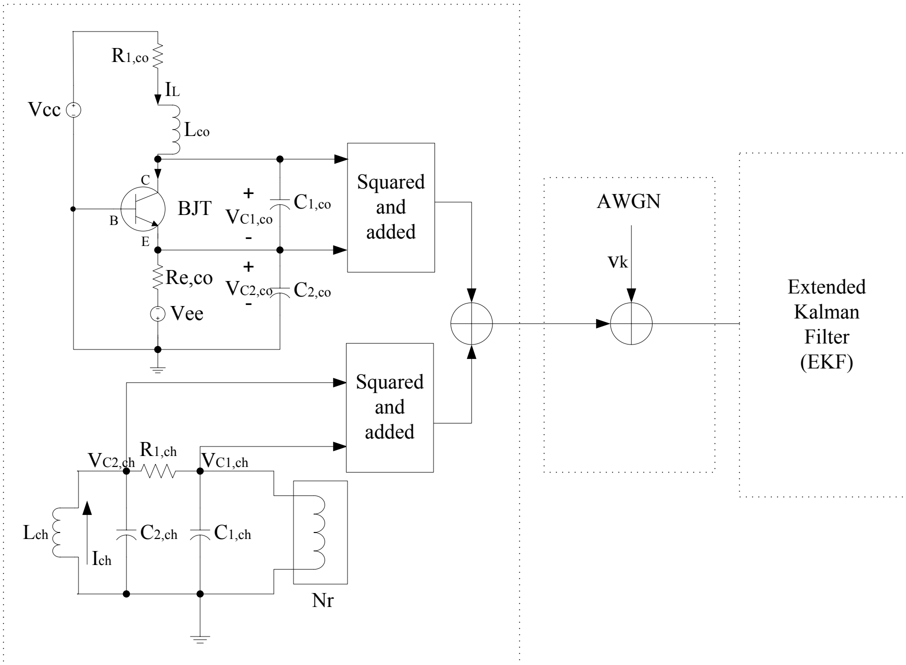

Fig. 1. Dual synchronization of chaotic Colpitts circuit and Chua's circuit.

The state equations for Chua's circuit are given in [2], as follows:

$$\begin{cases} \dfrac{\mathrm{d}V_{C1,ch}(t)}{\mathrm{d}t} = \dfrac{1}{C_{1,ch}}\Big(- f_2(V_{C1,ch}(t)) + \dfrac{1}{R_{1,ch}}(V_{C2,ch}(t) - V_{C1,ch}(t)\Big) \\ \dfrac{\mathrm{d}V_{C2,ch}(t)}{\mathrm{d}t} = \dfrac{1}{C_{2,ch}}\Big(I_{ch}(t) - \dfrac{V_{C2,ch}(t) - V_{C1,ch}(t)}{R_{1,ch}}\Big) \\ \dfrac{\mathrm{d}I_{ch}(t)}{\mathrm{d}t} = -\dfrac{1}{L_{co}}V_{C2,ch}(t) \end{cases} \tag{4}$$

where $f_2(.)$ is the nonlinear $I - V$ characteristic of the negative resistance Nr, and is described by

$$f_2(x) = \begin{cases} G_b x + (G_b - G_a)E & x < -E \\ G_a x & -E \leq x \leq E \\ G_b x + (G_a - G_b)E & x > E \end{cases} \tag{5}$$

where E is the saturation point of the nonlinear resistance.

The sum of chaotic signals $V^2_{C1,co}(t) + V^2_{C2,co}(t)$ from Colpitts oscillator and $V^2_{C1,ch}(t) + V^2_{C2,ch}(t)$ from Chua's oscillator is selected to be the transmitted signal. After the transmitted signal goes through the AWGN channel and reaches the receiver, it becomes

$$V_{\text{arrive}} = V^2_{C1,co}(t) + V^2_{C2,co}(t) + V^2_{C1,ch}(t) + V^2_{C2,ch}(t) + v(t) \tag{6}$$

where $v(t)$ is a Gaussian process with zero mean induced by the AWGN channel.

The analog signal is firstly sampled by an analog-digital-converter (ADC) with sampling time interval T, and then used to be the observed value for the EKF as the response system. According to Eq. (1) and Eq. (4), the discrete state equations in the EKF in Figure 1 for the Colpitts circuit and Chua's circuit are

$$\boldsymbol{x}_{k,co} = \boldsymbol{\Phi}_{co}\boldsymbol{x}_{k-1,co} + \boldsymbol{G}_{co} \tag{7}$$

$$\boldsymbol{x}_{k,ch} = \boldsymbol{\Phi}_{ch}\boldsymbol{x}_{k-1,ch} + \boldsymbol{G}_{ch} \tag{8}$$

where

$$\begin{aligned}
\boldsymbol{x}_{k,co} &= (V_{C1,co}(kT), V_{C2,co}(kT), I_{L,co}(kT))^T,\\
\boldsymbol{x}_{k,ch} &= (V_{C1,ch}(kT), V_{C2,ch}(kT), I_{L,ch}(kT))^T,\\
\boldsymbol{\Phi}_{co} &= \begin{pmatrix} 1 & 0 & \frac{T}{C_{1,co}} \\ 0 & 1-\frac{T}{(C_{2,co}R_{e,co})} & \frac{T}{C_{2,co}} \\ -\frac{T}{L_{co}} & -\frac{T}{L_{co}} & 1-\frac{R_{1,co}T}{L_{co}} \end{pmatrix},\\
\boldsymbol{G}_{co} &= \begin{pmatrix} -\frac{f_1(-V_{C2,co}((k-1)T))T}{C_{1,co}} \\ \frac{V_{ee}T}{(C_{2,co}R_{e,co})} \\ \frac{V_{cc}T}{L_{co}} \end{pmatrix},\\
\boldsymbol{\Phi}_{ch} &= \begin{pmatrix} 1-\frac{T}{(C_{1,ch}R_{1,ch})} & \frac{T}{(C_{1,ch}R_{1,ch})} & 0 \\ \frac{T}{C_{2,ch}R_{1,ch}} & 1-\frac{T}{(C_{2,ch}R_{1,ch})} & \frac{T}{C_{2,ch}} \\ 0 & -\frac{T}{L_{ch}} & 1 \end{pmatrix},\\
\boldsymbol{G}_{ch} &= \begin{pmatrix} -\frac{f_2(V_{C1,ch}((k-1)T))T}{C_{1,ch}} \\ 0 \\ 0 \end{pmatrix}.
\end{aligned}$$

The measurement model is

$$\boldsymbol{z}_k = (\boldsymbol{x}_{k,co}[1])^2 + (\boldsymbol{x}_{k,co}[2])^2 + (\boldsymbol{x}_{k,ch}[1])^2 + (\boldsymbol{x}_{k,ch}[2])^2 + v_k \tag{9}$$

where $\boldsymbol{x}_k[i]$ denotes the ith element of the vector $\boldsymbol{x}_k$.

In numerical simulations, the parameters of the Colpitts circuit are: $V_{CC} = 5\text{V}$, $V_{EE} = -5\text{V}$, $R_{1,co} = 74.5\Omega$, $L_{co} = 2.1\,\text{mH}$, $C_{1,co} = C_{2,co} = 237\,\text{nF}$, $R_{e,co} = 2000\Omega$. The parameters of Chua's circuit are: $L_{ch} = 18\,\text{mH}$, $C_{1,ch} = 10\,\text{nF}$, $C_{2,ch} = 100\,\text{nF}$, $R_{ch} = 1.8\,\text{k}\Omega$ and values of the parameters for the negative resistance Nr in Eq. (5) are $G_a = -8.33 \times 10^{(-4)}$, $G_b = -4.09 \times 10^{(-4)}$, $E = 1$. The time interval T is 0.1 μs. The transmitter exhibits chaotic oscillation with these parameters. Ref. [10] has shown that using the linear cross-coupling synchronization scheme, perfect synchronization can be achieved through an ideal channel. However, even when very small AWGN is exerted, synchronization becomes very hard to achieve.

3. SYNCHRONIZATION METHODS

The celebrated Kalman filter, rooted in the state-space formulation of linear dynamical systems, is a minimum mean-square (variance) estimator. For nonlinear models, one may use EKF, which is a first-order analytic approximation of Kalman filtering.

Consider a discrete-time dynamical system whose system model and observation model are given by

$$x_t = f(x_{t-1}) \tag{10}$$

$$y_t = h(x_t) + v_t \tag{11}$$

where $x_t \in R_n$ is an $n_x \times 1$ vector, f: $R_{n_x} \to R_{n_x}$ and h: $R_{n_x} \times R_{n_v} \to R_{n_y}$ are nonlinear functions, $v(t)$ is the zero-mean white Gaussian noise with $E(v_i, v_j) = R \cdot \delta_{ij}$, where $E(.)$ is the expectation operator and δ_{ij} is the Kronecker delta function.

The EKF-based synchronization method, which uses an EKF as the response system and produces a state estimate based on the observed signal, has been investigated by several researchers [12, 3, 1]. The basic idea of the EKF-based method is illustrated in Figure 2. It predicts the system state and then updates the state estimate upon receiving a new noisy measurement each time [13]. Therefore, the EKF essentially consists of two stages: prediction and update. The prediction stage uses the system model to obtain a priori estimate for the next time step. The update operation incorporates the measurements into the priori estimate so as to get an improved posteriori estimate.

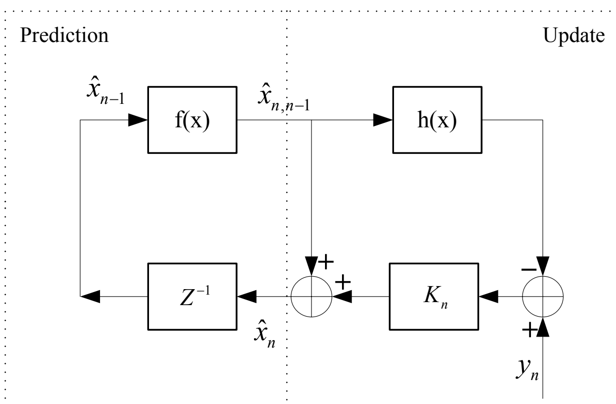

Fig. 2. Structure of the EKF-based response system.

For the synchronization problem formulated in Section 2, two methods are proposed and investigated below to obtain the dual synchronization.

3.1. Combining two circuits as a higher-dimensional system

In Figure 1, since the measurement is a combination of some chaotic signals generated by Colpitts circuit and Chua's circuit, we can regard the composite $\boldsymbol{x}_{k,co}$ and $\boldsymbol{x}_{k,ch}$ as a higher-dimension state vector $\boldsymbol{X}_k$. Based on the principle of EKF, the states of

$\boldsymbol{X}_k$ can be iteratively estimated from the observed noise-contaminated signal, and in this way dual synchronization can be achieved.

According to Eq. (1) and Eq. (4), the state equation for the six-dimensional combined chaotic system is

$$\boldsymbol{X}_k = \boldsymbol{\Phi}\boldsymbol{X}_{k-1} + \boldsymbol{G} \tag{12}$$

where $\boldsymbol{X}_k = [V_{C1,co}(kT), V_{C2,co}(kT), I_{L,co}(kT), V_{C1,ch}(kT), V_{C2,ch}(kT), I_{L,ch}(kT)]^T$,

$$\boldsymbol{\Phi} = \begin{pmatrix} 1 & 0 & \frac{T}{C_{1,co}} & 0 & 0 & 0 \\ 0 & 1-\frac{T}{(C_{2,co}R_{e,co})} & \frac{T}{C_{2,co}} & 0 & 0 & 0 \\ -\frac{T}{L_{co}} & -\frac{T}{L_{co}} & 1-\frac{R_{1,co}T}{L_{co}} & 0 & 0 & 0 \\ 0 & 0 & 0 & 1-\frac{T}{(C_{1,ch}R_{1,ch})} & \frac{T}{(C_{1,ch}R_{1,ch})} & 0 \\ 0 & 0 & 0 & \frac{T}{C_{2,ch}R_{1,ch}} & 1-\frac{T}{(C_{2,ch}R_{1,ch})} & \frac{T}{C_{2,ch}} \\ 0 & 0 & 0 & 0 & -\frac{T}{L_{ch}} & 1 \end{pmatrix}$$

and

$$\boldsymbol{G}_{co} = \begin{pmatrix} -\frac{f_1(-V_{C2,co}((k-1)T))T}{C_{1,co}} \\ \frac{V_{ee}T}{(C_{2,co}R_{e,co})} \\ \frac{V_{cc}T}{L_{co}} \\ -\frac{f_2(V_{C1,ch}((k-1)T))T}{C_{1,ch}} \\ 0 \\ 0 \end{pmatrix}.$$

The measurement equation for the chaotic system is

$$\boldsymbol{z}_k = (\boldsymbol{X}_k[1])^2 + (\boldsymbol{X}_k[2])^2 + (\boldsymbol{X}_k[4])^2 + (\boldsymbol{X}_k[5])^2 + v_k \tag{13}$$

where $\boldsymbol{X}_k[i]$ denotes the ith element of the vector $\boldsymbol{X}_k$.

3.2. Cross-coupling method

In the response system shown in Figure 1, the arrived signal is the sum of some chaotic signals generated by different chaotic oscillators. If we consider the Colpitts circuit and Chua's circuit separately and utilize the cross-coupling method as in [10], two EKFs can be constructed separately. As shown in Figure 3, for the Colpitts circuit, the observation signal can be regarded as $z_k - ((\tilde{V}_{C1,ch})^2 + (\tilde{V}_{C2,ch})^2)$, and for Chua's circuit, the measurement equation is $z_k - ((\tilde{V}_{C1,co})^2 + (\tilde{V}_{C2,co})^2)$, where $\tilde{V}_{C1,ch}, \tilde{V}_{C2,ch}, \tilde{V}_{C1,co}$ and $\tilde{V}_{C2,co}$ can be obtained separately from the priori estimates of the corresponding EKF. In this way, dual synchronization can be achieved by using two three-dimensional EKFs. Compared with the first method, it is obvious that this method can save a large amount of computing cost, because the inverses and multiplications of the square matrix in the EKFs have complexity $O(N^3)$, where N is the dimension of the matrix. For the above two methods, the corresponding value of N is 6 and 3 respectively.

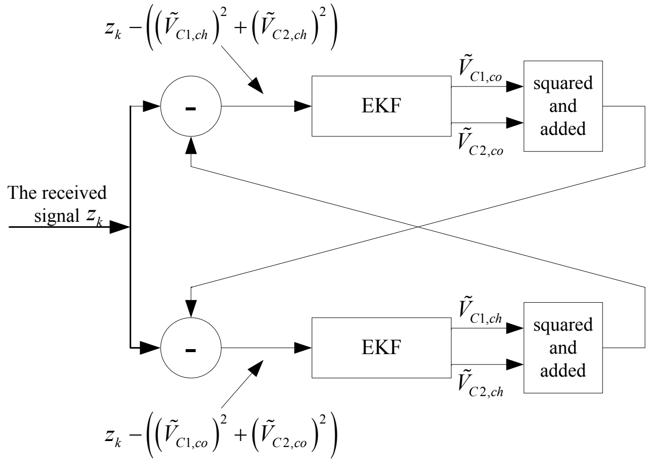

Fig. 3. The cross-coupling method.

4. SIMULATION AND DISCUSSIONS

We now show that dual synchronization can be achieved when the two proposed EKF-based methods are employed as the response system. The effects of different levels of AWGN on the synchronization performance are also investigated.

To evaluate the synchronization performance, we define the average attractor distance (AAD) between the drive system and the response system as

$$D = \lim_{t_s \to \infty} \frac{\int_{t_0}^{t_s} \sqrt{e_1^2 + e_2^2 + e_3^2} \mathrm{d}t}{t_s - t_0} \tag{14}$$

where $e_1 = V_{C1} - \tilde{V}_{C1}, e_2 = V_{C2} - \tilde{V}_{C2}, e_3 = I_L - \tilde{I}_L$, and t_0 denotes the settling time when the transient parts of the signals have passed. The value of D will be zero when the transmitter and receiver are in perfect synchronization. But if the synchronization performance is not very good, the variable D will be non-zero, and a bigger value of D means a worse synchronization performance.

4.1. Time evolution of synchronization

With the parameters listed in Section 2, the transmitter can exhibit chaotic oscillation. When the transmitted signal passes through an AWGN channel with noise variance $\sigma^2 = 1$ and reaches the receiver constructed by the EKF described in Section 3, the simulation results using the first and the second method are plotted in Figure 4 and Figure 5, respectively. The upper three sub-plots (a) – (c) show the error signals of the Colpitts circuit, while the below sub-plots (d) – (f) are of Chua's circuit. Figure 4 and Figure 5 show that the transmitter's state can be estimated and tracked, indicating that dual synchronization can be obtained and maintained by using both of the proposed two methods.

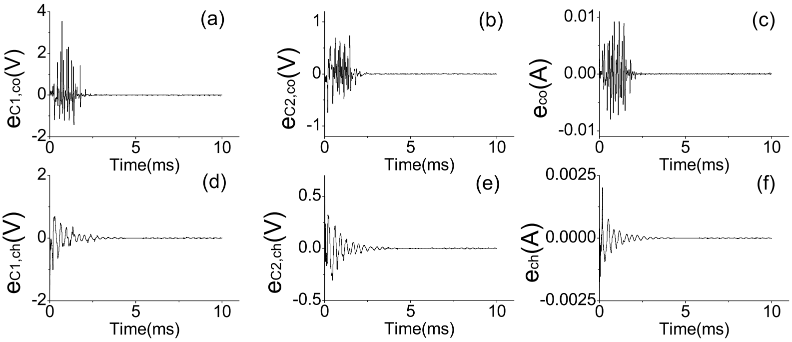

Fig. 4. With the first method, error signals of Colpitts circuit:
$(a) e_{C1,co} = V_{C1,co} - \tilde{V}_{C1,co}, (b) e_{C2,co} = V_{C2,co} - \tilde{V}_{C2,co}, (c) e_{co} = I_{co} - \tilde{I}_{co}$;
and error signals of Chua's circuit:
$(d) e_{C1,ch} = V_{C1,ch} - \tilde{V}_{C1,ch}, (e) e_{C2,ch} = V_{C2,ch} - \tilde{V}_{C2,ch}, (f) e_{ch} = I_{ch} - \tilde{I}_{ch}$.

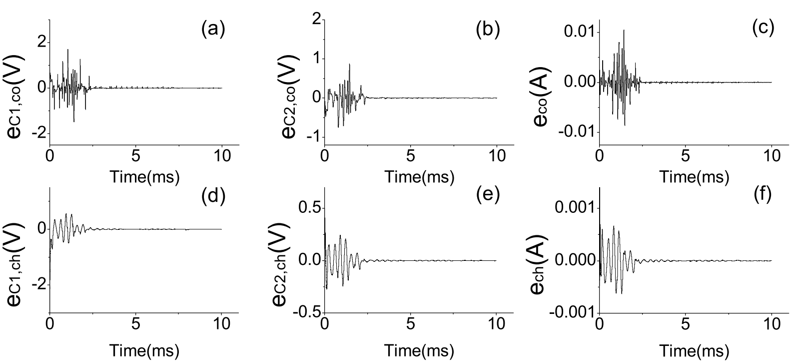

Fig. 5. With the second method, error signals of Colpitts circuit:
$(a) e_{C1,co} = V_{C1,co} - \tilde{V}_{C1,co}, (b) e_{C2,co} = V_{C2,co} - \tilde{V}_{C2,co}, (c) e_{co} = I_{co} - \tilde{I}_{co}$;
and error signals of Chua's circuit:
$(d) e_{C1,ch} = V_{C1,ch} - \tilde{V}_{C1,ch}, (e) e_{C2,ch} = V_{C2,ch} - \tilde{V}_{C2,ch}, (f) e_{ch} = I_{ch} - \tilde{I}_{ch}$.

4.2. Synchronization performance vs. noise level

In this subsection, the synchronization performance is studied, considering different levels of AWGN, by observing the value of AAD. The AAD vs. the noise level are plotted in Figure 6. Note that even when the AAD has a value of 0.05, from the time-domain waveform of the drive and response, we can see that synchronization performance is quite good.

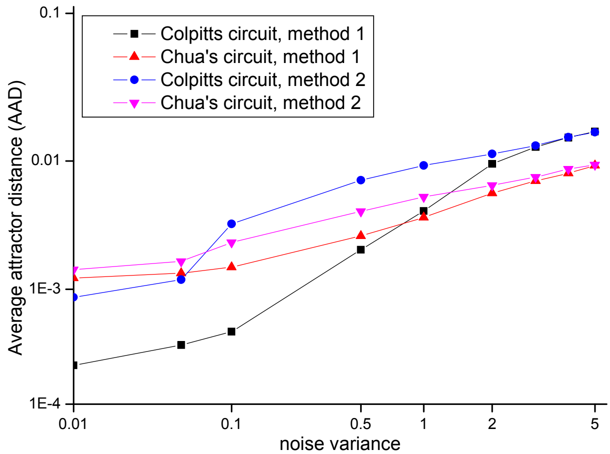

Fig. 6. AAD vs. noise variance.

From Figure 6, it is observed that with the increment of the noise variance σ^2, the synchronization performance degrades at a rather slow rate for both methods. It indicates that the two methods are effective for this dual synchronization problem with relatively big AWGN.

When the synchronization performances are compared, it can be seen form Figure 6 that the first method outperforms the second method, especially when the noise level is low. When the channel noise level is relatively high, the performances of the two methods approach each other. The reason can be explained as follows. When the cross-coupling method is used, the observation signal for each EKF contains not only the channel noise but also the synchronization error signal of the other circuit pair, which can be regarded as additional noise. Thus, the equivalent noise will be even higher than the actual channel noise. When the channel noise is small, the additional noise will become dominant, but when the channel noise is rather big, the additional noise will be negligible.

5. CONCLUSION

In this paper, two EKF-based methods for dual synchronization of chaotic Colpitts circuit and Chua's circuit through AWGN channel have been proposed. Numerical simulation results demonstrate that the proposed structures can effectively achieve robust dual synchronization of the chaotic circuits. In this way, the proposed methods can be used to chaotic signal separation and chaos-based communication.

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