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Dedicated to Professor Beloslav Riečan on the occasion of his 70th birthday

SEQUENTIAL CONVERGENCES ON PSEUDO MV-ALGEBRAS

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(*Communicated by Anatolij Dvurečenskij*)

ABSTRACT. According to a result of Dvurečenskij, each pseudo MV-algebra $\mathcal{A}$ can be represented as an interval of a unital lattice ordered group G. We denote by $\operatorname{Conv}\mathcal{A}$ and $\operatorname{Conv}G$ the system of all sequential convergences on $\mathcal{A}$ and on G, respectively. Both $\operatorname{Conv}\mathcal{A}$ and $\operatorname{Conv}G$ are partially ordered in a natural way. We prove that $\operatorname{Conv}\mathcal{A}$ is isomorphic to a subsystem $\operatorname{Conv}_b G$ of $\operatorname{Conv}G$. The system $\operatorname{Conv}\mathcal{A}$ is isomorphic to $\operatorname{Conv}G$ if each orthogonal subset of $\mathcal{A}$ is finite.

1. Introduction

The notion of pseudo MV-algebra (denoted also as generalized or noncommutative MV-algebra) has been introduced independently by Georgescu and Iorgulescu [7], [8] and by Rachůnek [13].

Dvurečenskij [4] proved that each pseudo MV-algebra $\mathcal{A}$ can be constructed by means of a unital lattice ordered group (G,u); analogously as in the theory of MV-algebras (cf. Cignoli, D'Ottaviano and Mundici [2]) we write $\mathcal{A}=\Gamma(G,u)$.

Sequential convergences on MV-algebras were investigated by the author [11]. The definition is analogous to that for lattice ordered groups (cf. Harminc [9] and the author [10]). A similar definition can be applied for pseudo MV-algebras.

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Let $\mathcal{A}$ and (G, u) be as above. We denote by $\operatorname{Conv} \mathcal{A}$ and $\operatorname{Conv} G$ the system of all sequential convergences on $\mathcal{A}$ or on G, respectively. (For the definitions, cf. Section 2 below.) Both the systems $\operatorname{Conv} \mathcal{A}$ and $\operatorname{Conv} G$ are partially ordered by the set-theoretical inclusion; they are meet-semilattices.

We define a subsystem $\operatorname{Conv}_b G$ of $\operatorname{Conv} G$; the elements of $\operatorname{Conv}_b G$ are called bounded sequential convergences on G.

We show that there exists an isomorphism of $\operatorname{Conv} \mathcal{A}$ onto the partially ordered system $\operatorname{Conv}_b G$. This generalizes a result from [11] concerning MV-algebras.

Let $\mathcal{F}$ be the class of all lattice ordered groups H such that each orthogonal subset of H is finite. Further, let $\mathcal{F}_1$ be the class of all pseudo MV-algebras $\mathcal{A}_1$ satisfying the analogous condition. The structure of lattice ordered groups belonging to $\mathcal{F}$ was described by Conrad [3]. If $\mathcal{A}$ and (G, u) are as above, then G belongs to $\mathcal{F}$ if and only if $\mathcal{A}$ belongs to $\mathcal{F}_1$.

We prove that if $\mathcal{A} \in \mathcal{F}_1$, then

(i) $\operatorname{Conv} \mathcal{A}$ is isomorphic to $\operatorname{Conv} G$;

(ii) $\operatorname{Conv} \mathcal{A}$ is a finite Boolean algebra.

We recall that sequential convergences in D-posets were systematically applied by Frič [6]. The notion of D-poset is due to Chovanec and Kôpka [1]; it is equivalent to the notion of effect algebra (Foulis and Bennet [5]). Each MV-algebra is a D-poset.

2. Preliminaries

For the sake of completeness, we recall the definition of a pseudo MV-algebra.

DEFINITION 2.1. Let A be a nonemtpy set. Let $\mathcal{A} = (A; \oplus, ^-, ^\sim, 0, 1)$ be an algebraic structure of type $(2,1,1,0,0)$. For $x, y \in A$ we put

$$y \odot x = (x^- \oplus y^-)^\sim .$$

$\mathcal{A}$ is a *pseudo MV-algebra* if the following axioms (A1)–(A8) are satisfied for each $x, y, z \in A$:

(A1) $x \oplus (y \oplus z) = (x \oplus y) \oplus z$;

(A2) $x \oplus 0 = 0 \oplus x = x$;

(A3) $x \oplus 1 = 1 \oplus x = 1$;

(A4) $1^\sim = 0$; $1^- = 0$;

(A5) $(x^- \oplus y^-)^\sim = (x^\sim \oplus y^\sim)^-$;

(A6) $x \oplus (x^\sim \odot y) = y \oplus (y^\sim \odot x) = (x \odot y^-) \oplus y = (y \odot x^-) \oplus x$;

(A7) $x \odot (x^- \oplus y) = (x \oplus y^\sim) \odot y$;

(A8) $(x^-)^\sim = x$.

For a pseudo MV-algebra $\mathcal{A}$ and $x, y \in A$ we set $x \leqq y$ if $x^- \oplus y = 1$. Then $(A; \leqq)$ is a lattice with the least element 0 and the greatest element 1; we denote $(A; \leqq) = \ell(\mathcal{A})$.

If the operation $\oplus$ in $\mathcal{A}$ is commutative, then $\mathcal{A}$ is an *MV-algebra*; in such a case $x^- = x^\sim$ for each $x \in A$.

Let G be a lattice ordered group. The group operation in G is denoted additively though it is not assumed to be commutative. Let $u \in G^+$ such that for each $g \in G$ there exists a positive integer n with $g \leqq nu$. The element u is a *strong unit* of G; we say that (G, u) is a unital lattice ordered group. For $x, y \in G$ we put

$$x \oplus y = (x + y) \wedge u\,,$$
$$x^- = u - x\,, \quad x^\sim = -x + u\,, \quad 1 = u\,.$$

Let A be the interval $[0, u]$ of G. Then $(A; \oplus, ^-, ^\sim, 0, 1)$ is a pseudo MV-algebra; it is denoted by $\Gamma(G, u)$.

THEOREM 2.2. (Cf. [4].) *For each pseudo MV-algebra $\mathcal{A}$ there exists a unital lattice ordered group (G, u) such that $\mathcal{A} = \Gamma(G, u)$.*

Let $\mathbb{N}$ be the set of all positive integers. An element of $A^{\mathbb{N}}$ will be denoted by $(x_n)_{n\in\mathbb{N}}$ or by (x_n); it is a *sequence* in A. If $x \in A$ and $x_n = x$ for each $n \in \mathbb{N}$, then we write $(x_n) = \operatorname{const} x$. Let $K \subseteq A^{\mathbb{N}} \times A$. A relation of the form $\big((x_n), x\big) \in K$ will be denoted by writing $x_n \to_K x$.

DEFINITION 2.3. A subset K of $A^{\mathbb{N}} \times A$ is a *sequential convergence in $\mathcal{A}$* if the following conditions are satisfied:

(i) If $x_n \to_K x$ and if (y_n) is a subsequence of (x_n), then $y_n \to_K x$.

(ii) If $(x_n) \in A^{\mathbb{N}}$, $x \in A$ and if for each subsequence (y_n) of (x_n) there is a subsequence (z_n) of (y_n) such that $z_n \to_K x$, then $x_n \to_K x$.

(iii) If $(x_n) \in A^{\mathbb{N}}$, $x \in A$, $(x_n) = \operatorname{const} x$, then $x_n \to_K x$.

(iv) If $x_n \to_K x$ and $x_n \to_K y$, then $x = y$.

(v) If $x_n \to_K x$ and $y_n \to_K y$, then $x_n \oplus y_n \to_K x \oplus y$, $x_n^- \to_K x^-$ and $x_n^\sim \to_K x^\sim$.

(vi) If $x_n \leqq y_n \leqq z_n$ is valid for each $n \in \mathbb{N}$ and if $x_n \to_K x$, $z_n \to_K x$, then $y_n \to_K x$.

We denote by $\operatorname{Conv} \mathcal{A}$ the system of all sequential convergences in $\mathcal{A}$. The system $\operatorname{Conv} \mathcal{A}$ is partially ordered by the set-theoretical inclusion.

If, in particular, $\mathcal{A}$ is an MV-algebra, then in view of [11; 1.1, 1.3], the definition of sequential convergence in $\mathcal{A}$ as defined in [11] coincides with that given in 2.3.

Let $K(0)$ be the set of all $\big((x_n), x\big) \in A^{\mathbb{N}} \times A$ such that there is $m \in \mathbb{N}$ with $x_n = x$ for each $n \geqq m$. It is easy to verify that $K(0)$ is the least element of $\operatorname{Conv} \mathcal{A}$.

Let I be a nonempty set and for each $i \in I$ let $K_i \in \operatorname{Conv} \mathcal{A}$. Then in view of 2.3, $\bigcap\limits_{i \in I} K_i \in \operatorname{Conv} \mathcal{A}$. This yields:

LEMMA 2.4. *$\operatorname{Conv} \mathcal{A}$ is a meet-semilattice. If $K \in \operatorname{Conv} \mathcal{A}$, then the interval $[K(0), K]$ of $\operatorname{Conv} \mathcal{A}$ is a complete lattice.*

Now let G be a lattice ordered group and $K \subseteq G^{\mathbb{N}} \times G$. Similarly as in the case of pseudo MV-algebras, we write $x_n \to_K x$ if $\big((x_n), x\big) \in K$.

DEFINITION 2.5. (Cf. [11].) A subset K of $G^{\mathbb{N}} \times G$ is a *sequential convergence in G* if the conditions (i)–(iv), (vi) from 2.3 are satisfied and if, moreover, the following conditions are valid:

(v(1)) If $x_n \to_K x$ and $y_n \to_K y$, then $x_n \wedge y_n \to_K x \wedge y$ and $x_n \vee y_n \to_K x \vee y$;
(v(2)) if $x_n \to_K x$ and $y_n \to_K y$, then $x_n + y_n \to_K x + y$ and $-x_n \to -x$.

The system of all sequential convergences in G will be denoted by $\operatorname{Conv} G$; it is partially ordered by the set-theoretical inclusion. Let $K(0)$ be defined analogously as in the case of $\operatorname{Conv} \mathcal{A}$. Similarly as in 2.4, we have:

LEMMA 2.6. *$\operatorname{Conv} G$ is a meet-semilattice. If $K \in \operatorname{Conv} G$, then the interval $[K(0), K]$ of $\operatorname{Conv} G$ is a complete lattice.*

3. Auxiliary results

Assume that $\mathcal{A}$ is a pseudo MV-algebra and that, under the notation as above, the relation $\mathcal{A} = \Gamma(G, u)$ is valid.

LEMMA 3.1. *Let $x, y \in A$, $x \leqq y$. Then $y - x = (x \oplus y^{\sim})^{-}$.*

Proof. We have

$$x \oplus y^{\sim} = \big(x + (-y + u)\big) \wedge u = \big((x - y) + u\big) \wedge u .$$

Since $x \leqq y$, we get $x - y \leqq 0$ and hence $(x-y)+u \leqq u$; thus $x \oplus y^{\sim} = (x-y)+u$. Then

$$(x \oplus y^{\sim})^{-} = u - (x \oplus y^{\sim}) = u - (x - y + u) = y - x .$$

□

Analogously we verify:

LEMMA 3.2. *Let $x, y \in A$, $x \leqq y$. Then $-x + y = (y^{-} \oplus x)^{\sim}$.*

Lemma 3.3. *Let $x, y \in A$. Then $x \vee y = y \oplus (x^- \oplus y)^\sim$.*

Proof. We have $(x^- \oplus y)^\sim = -(x^- \oplus y) + u$ and

$$x^- \oplus y = \big((u-x)+y\big) \wedge u\,,$$
$$-(x^- \oplus y) = (-y+x-u) \vee (-u)\,.$$

Thus we get

$$-(x^- \oplus y) + u = (-y+x) \vee 0\,,$$
$$y \oplus (x^- \oplus y)^\sim = \big(y + (-y+x) \vee 0\big) \wedge u = (x \vee y) \wedge u = x \vee y\,.$$

□

From 2.3 and 3.3 we conclude:

Lemma 3.4. *Let $K \in \operatorname{Conv} \mathcal{A}$, $x_n \to_K x$, $y_n \to_K y$. Then $x_n \vee y_n \to_K x \vee y$.*

Lemma 3.5. *Let $x, y \in A$. Then $x \wedge y = (x^- \vee y^-)^\sim$.*

Proof. We have

$$\begin{aligned}(x^- \vee y^-)^\sim &= -(x^- \vee y^-) + u = -\big((u-x) \vee (u-y)\big) + u\\ &= \big((x-u) \wedge (y-u)\big) + u = x \wedge y\,.\end{aligned}$$

□

Now, 2.3, 3.4 and 3.5 yield:

Lemma 3.6. *Let $K \in \operatorname{Conv} \mathcal{A}$, $x_n \to_K x$, $y_n \to_K y$. Then $x_n \wedge y_n \to_K x \wedge y$.*

Lemma 3.7. *Let $K \in \operatorname{Conv} \mathcal{A}$, $x_n \to_K x$, $y_n \to_K y$, $x_n \leqq y_n$ for each $n \in \mathbb{N}$. Then $x \leqq y$.*

Proof. For each $n \in \mathbb{N}$ we have $x_n = x_n \wedge y_n$. Hence in view of 3.6, $x_n \wedge y_n \to_K x \wedge y$. Thus according to 2.3(iv), $x = x \wedge y$. □

From 3.7, 3.1 and 3.2 we obtain:

Corollary 3.7.1. *Let $K, x, y, (x_n)$ and (y_n) be as in 3.7. Then*

$$y_n - x_n \to_K y - x\,, \qquad -x_n + y_n \to_K -x + y\,.$$

A sequence (x_n) in G is *bounded* if there is $m \in \mathbb{N}$ such that $-mu \leqq x_n \leqq mu$ for each $n \in \mathbb{N}$.

Let $K \in \operatorname{Conv} G$. We denote by K_b the system of all bounded sequences belonging to K. In view of the Definition 2.5 we obtain:

Lemma 3.8. *For each $K \in \operatorname{Conv} G$, K_b is an element of $\operatorname{Conv} G$.*

We put

$$\operatorname{Conv}_b G = \{K_b :\ K \in \operatorname{Conv} G\}\,.$$

4. The systems $\operatorname{Conv}_0 G$ and $\operatorname{Conv}_0 \mathcal{A}$

For a lattice ordered group G and $K \in \operatorname{Conv} G$ we put

$$K^0 = \{(x_n) \in G^{\mathbb{N}} : \ x_n \to_K 0 \ \text{and} \ x_n \geqq 0 \ \text{for each} \ n \in \mathbb{N}\}.$$

Further, we set

$$\operatorname{Conv}_0 G = \{K^0 : \ K \in \operatorname{Conv} G\}.$$

The system $\operatorname{Conv}_0 G$ is partially ordered by the set-theoretical inclusion.

We denote

$$\operatorname{Conv}_0^b G = \{K^0 : \ K \in \operatorname{Conv}_b G\}.$$

For the assertion (i) of the following lemma, cf. [9]; the assertion (ii) is easy to verify. (Cf. also [11], where the commutativity of G was assumed.)

Lemma 4.1.

(i) *Put* $\varphi_0(K) = K^0$ *for each* $K \in \operatorname{Conv} G$. *Then* φ_0 *is an isomorphism of* $\operatorname{Conv} G$ *onto* $\operatorname{Conv}_0 G$.

(ii) *Let* $K \in \operatorname{Conv} G$. *Then* $K \in \operatorname{Conv}_b G$ *if and only if* $K^0 \in \operatorname{Conv}_0^b G$.

Let $\mathcal{A}$ be a pseudo MV-algebra and $K \in \operatorname{Conv} \mathcal{A}$. Analogously as in the case of lattice ordered groups we put

$$K^0 = \{(x_n) \in A^{\mathbb{N}} : \ x_n \to_K 0\},$$
$$\operatorname{Conv}_0 \mathcal{A} = \{K^0 : \ K \in \operatorname{Conv} \mathcal{A}\}.$$

$\operatorname{Conv}_0 \mathcal{A}$ is partially ordered under the set-theoretical inclusion. Let $((x_n), x) \in A^{\mathbb{N}} \times A$. For each $n \in \mathbb{N}$ we denote $p_n = x_n \vee x$, $q_n = x_n \wedge x$, $t_n = p_n - q_n$, $t'_n = -q_n + p_n$.

Lemma 4.2. *Let* $K \in \operatorname{Conv} \mathcal{A}$. *Then, under the notation as above, the following conditions are equivalent:*

(i) $x_n \to_K x$;
(ii) $(t_n) \in K^0$ *and* $(t'_n) \in K^0$.

Proof.

a) Let (i) be valid. Since $\operatorname{const} x \in K$ we obtain

$$p_n \to_K x, \qquad q_n \to_K x.$$

Since $p_n \geqq q_n$ for each $n \in \mathbb{N}$, in view of 3.7.1 we get $t_n \to_K 0$ and $t'_n \to_K 0$ Thus (ii) holds.

b) Assume that (ii) is satisfied. Let $n \in \mathbb{N}$. We have

$$x_n = (x_n - q_n) + (q_n - x) + x.$$

From the definition of p_n and q_n we obtain

$$x_n - q_n = p_n - x\,.$$

Hence

$$x_n = (p_n - x) + \bigl(-(x - q_n) + x\bigr)\,.$$

Since $0 \leqq x - q_n \leqq x$, we infer

$$x - q_n \in A\,, \qquad -(x - q_n) + x \in A\,.$$

Therefore

$$x_n = (p_n - x) \oplus \bigl(-(x - q_n) + x\bigr)\,. \tag{1}$$

Further, $0 \leqq p_n - x \leqq p_n - q_n = t_n$, thus $(p_n - x) \to_K 0$. Also, $0 \leqq x - q_n \leqq p_n - q_n = t_n$, whence $(x - q_n) \to_K 0$. From this and from 3.7.1 we conclude

$$\bigl(-(x - q_n) + x\bigr) \to_K x\,.$$

Hence (1) yields $x_n \to_K x$. □

Let K and K' belong to $\operatorname{Conv}\mathcal{A}$. Then clearly

$$K \subseteq K' \implies K^0 \subseteq (K')^0\,. \tag{2}$$

Further, from 4.2 we obtain that the implication in (2) can be reversed. Hence we have:

COROLLARY 4.3. *For each $K \in \operatorname{Conv}\mathcal{A}$ put $\varphi_1(K) = K^0$. Then φ_1 is an isomorphism of $\operatorname{Conv}\mathcal{A}$ onto $\operatorname{Conv}_0\mathcal{A}$.*

We remark that the arguments in the proofs of [11; 3.1–3.14] dealing with MV-algebras remain valid for pseudo MV-algebras. Hence we have:

LEMMA 4.4. *The partially ordered systems $\operatorname{Conv}_0\mathcal{A}$ and $\operatorname{Conv}_0^b G$ are isomorphic.*

THEOREM 4.5. *Let $\mathcal{A}$ be a pseudo MV-algebra with $\mathcal{A} = \Gamma(G,u)$, where (G,u) is a unital lattice ordered group. Then the partially ordered systems $\operatorname{Conv}\mathcal{A}$ and $\operatorname{Conv}_b G$ are isomorphic.*

Proof. This is a consequence of 4.1, 4.3 and 4.4. □

Theorem 4.5 generalizes [11; Theorem 3.14] concerning MV-algebras.

5. On pseudo MV-algebras belonging to the class $\mathcal{F}_1$

In the present section we apply some results of [12]. We remark that the notation in [12] is different from that used above. Namely, let G be a lattice ordered group, $K \in \operatorname{Conv} G$ and let K^0 be as in Section 4. The symbol $\operatorname{Conv} G$ in [12] means, in fact, the system $\operatorname{Conv}_0 G$.

Again, we assume that $\mathcal{A}$ is a pseudo MV-algebra with $\mathcal{A} = \Gamma(G, u)$, where (G, u) is a unital lattice ordered group. Let $\mathcal{F}$ and $\mathcal{F}_1$ be as in Section 1.

LEMMA 5.1. *$\mathcal{A}$ belongs to $\mathcal{F}_1$ if and only if G belongs to $\mathcal{F}$.*

Proof. If G belongs to $\mathcal{F}$, then we obviously have $\mathcal{A} \in \mathcal{F}_1$. Conversely, suppose that $\mathcal{A}$ belongs to $\mathcal{F}_1$ and let $\{g_i\}_{i \in I}$ be an orthogonal subset of G such that $g_i > 0$ for each $i \in I$. Put $a_i = u \wedge g_i$ for $i \in I$. Then $\{a_i\}_{i \in I}$ is an orthogonal subset of A and $a_i > 0$ for each $i \in I$. Hence I is finite and thus $G \in \mathcal{F}$. □

Let X be a convex linearly ordered subgroup of G and let $K(X)$ be a sequential convergence on X. If $x_n \to_{K(X)} 0$, then from [12; Lemma 2.3] it follows that there exists $x_0 \in X^+$ having the property that $-x_0 \leqq x_n \leqq x_0$ for each $n \in \mathbb{N}$. This yields that the sequence (x_n) is bounded in G.

Now assume that the pseudo MV-algebra $\mathcal{A}$ belongs to $\mathcal{F}_1$. Hence in view of 5.1, G belongs to $\mathcal{F}$. Let $K \in \operatorname{Conv} G$.

Take any $(g_n) \in K^0$. In view of [12] there are convex linearly ordered subgroups $X_1, \dots, X_m$ of G, sequential convergences K_i on X_i, sequences (x_n^i) with $x_n^i \to_{K_i} 0$ $(i = 1, 2, \dots, m)$, and $k \in \mathbb{N}$, such that for each $n \in \mathbb{N}$, $n \geqq k$, the relation

$$g_n = x_n^1 + \dots + x_n^m \tag{1}$$

is valid.

Since for each $i \in \{1, 2, \dots, m\}$ the sequence (x_n^i) is bounded in G, (1) yields that the sequence (g_n) is bounded in G. Thus each sequence belonging to K^0 is bounded in G. From this we conclude that each sequence belonging to K is bounded in G. We obtain

$$\operatorname{Conv} G = \operatorname{Conv}_b G. \tag{2}$$

Hence from 4.5 we get:

THEOREM 5.2. *Let $\mathcal{A}$ be a pseudo MV-algebra belonging to $\mathcal{F}_1$. Then the partially ordered systems* $\operatorname{Conv} \mathcal{A}$ *and* $\operatorname{Conv} G$ *are isomorphic.*

THEOREM 5.3. *Let $\mathcal{A}$ be a pseudo MV-algebra belonging to $\mathcal{F}_1$. Then* $\operatorname{Conv} \mathcal{A}$ *is a finite Boolean algebra.*

Proof. In view of 5.1, $G \in \mathcal{F}$ (where G is as above). According to [12; Theorem 2.18], $\operatorname{Conv}_0 G$ is a finite Boolean algebra. In view of 4.1, $\operatorname{Conv}_0 G \simeq \operatorname{conv} G$.

Then (2) yields $\mathrm{Conv}_0 G \simeq \mathrm{conv}_b G$. Hence according to 4.5, $\mathrm{Conv}\,\mathcal{A}$ is a finite Boolean algebra. Then it follows from 4.1 and 4.5 that $\mathrm{Conv}\,\mathcal{A}$ is a finite Boolean algebra as well. □

For the case of MV-algebras we have the following stronger result.

THEOREM 5.4. *Let $\mathcal{A}$ be an MV-algebra. Then the following conditions are equivalent:*

(i) $\mathrm{Conv}\,\mathcal{A}$ *is a generalized Boolean algebra.*
(ii) $\mathrm{Conv}\,\mathcal{A}$ *is a Boolean algebra.*
(iii) $\mathrm{Conv}\,\mathcal{A}$ *is a finite Boolean algebra.*
(iv) $\mathcal{A} \in \mathcal{F}_1$.

Proof. According to 5.1, $\mathcal{A} \in \mathcal{F}_1 \iff G \in \mathcal{F}$. Now it suffices to apply [12; Theorem (A)], Lemma 4.1 and Theorem 4.5. □

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