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DISTRIBUTED H_∞ ESTIMATION FOR MOVING TARGET UNDER SWITCHING MULTI-AGENT NETWORK

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In this paper, the distributed H_∞ estimation problem is investigated for a moving target with local communication and switching topology. Based on the solution of the algebraic Riccati equation, a recursive algorithm is proposed using constant gain. The stability of the proposed algorithm is analysed by using the Lyapounov method, and a lower bound for estimation errors is obtained for the proposed common H_∞ filter. Moreover, a bound for the H_∞ parameter is obtained by means of the solution of the algebraic Riccati equation. Finally, a simulation example is employed to illustrate the effectiveness of the proposed estimation algorithm.

Keywords: multi-agent systems, distributed estimation, H_∞ filter, switching topology

Classification: 93E12, 62A10

1. INTRODUCTION

Recently, multi-agent systems have attracted much attention for its broad applications in many areas such as sensor networks, power grids, and public transportation. State estimation by multi-agent systems is one of the important problems related to the moving target tracking, signals or parameters estimation in sensor networks [17, 22, 24]. Because of the ability of tracking the fast moving target and non-stationary process, the distributed Kalman filtering have wildly used in distributed estimation for moving target in sensor networks, and many results have been obtained. For example, a decentralized Kalman filter was proposed in [14, 15], which made the sensor network to track the average of n sensors estimation by using two consensus distributed Kalman filter. Also, a gossip-based distributed Kalman filter was studied in [10], where each sensor occasionally and randomly exchanged information with only one neighbor. Furthermore, a diffusion distributed estimation problem was studied in [8]. However, for the limits of the sensor's capacity, not all the the sensors could connected to the target in practice. From this point of view, under switching topology, a distributed consensus Kalman-based estimation algorithm was studied in [24], where, with some wild assumptions (i. e., observability and connectivity), the upper and lower bound were obtained for estimation errors.

In the presence of the noises and uncertainties, a natural way is to use H_∞ estimation method. Many researches have investigated the distributed H_∞ estimation problem. In

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[20], by using the vector dissipativity theory, the distributed robust consensus filter was proposed via optimization of H_∞ disagreement between the nodes, subject to linear matrix inequalities (LMIs) constraints. Distributed H_∞ estimation under missing measurements and Markovian switching conditions were studied in [22] and [19], respectively. In [12], A PI consensus algorithm was proposed which enable decentralized implementation of H_∞ filters at each node of mobile network. Moreover, in [3], a distributed H_∞ filtering problem was discussed for a class of discrete-time Markovian jump nonlinear time-delay systems with deficient statistics of mode transitions. Notice that the sufficient conditions for the existence of distributed H_∞ estimation are all based on solutions of LMIs in some existing distributed H_∞ estimations [19–21]. However, there are no sufficient conditions dependent only on the system dynamics to guarantee the existence of solution to LMIs [18], and the time complexity of solving such an LMI is $O(n^2p^4)$, where n is the number of agents and p is the number of each node state [13]. Thus, in practice, it is not always feasible to design distributed H_∞ estimation by using LMI techniques.

The objective of this paper is to design a distributed H_∞ estimation algorithm for moving target using constant gain under switching topology. We assume that the target may not be measured by some sensors for the limits of sensor's capacity. Different from most existing works, our approach is designed by the solution of the algebraic Riccati equation (ARE) instead of general LMIs, and then a constant gain is obtained for distributed H_∞ estimation under switching connectivity. Based on the well-known detectability and connectivity, the distributed H_∞ estimation performance is guaranteed in this paper, and a lower bound for the total mean square estimation errors (TMSEE) is achieved. Moreover, in order to make sure the positive definiteness of the solution to ARE, a bound for H_∞ parameter is obtained, which is helpful to determine the H_∞ parameter.

The rest of this paper is organized as follows. In Section II, the formulation of distributed H_∞ estimation for moving target is presented, along with some preliminaries. In section III, the stability analysis for the proposed distributed H_∞ estimation algorithm and the bound for H_∞ parameter are given. In Section IV, a simulation example is showed. Finally, concluding remarks are found in Section V.

2. PROBLEM STATEMENT

In this section, we provide necessary preliminaries and then formulate the distributed H_∞ estimation problem.

A. Sensor Network Topology

Consider a multi-agent system that consists of n sensors that can only communication with its neighbors. The topology of the system can be described by graph theory [4]. An undirected graph denoted as $\mathscr{G} = (\mathscr{V}, \mathscr{E})$, where $\mathscr{V} = \{1, 2, \dots, n\}$ is the node set, and $\mathscr{E} = \{(i, j) : i, j \in \mathscr{V}\}$ is the edge set. If node i and node j connected by an edge, then these two vertices are called adjacent, and we define $\mathscr{N}_i(k) = \{j : (i, j) \in \mathscr{E}\}$ is the neighbor set of node i. In this paper, we can regard sensor i as node i, the communication link can be treated as edge. A path is a sequence of edges with the form $(i_1, i_2), (i_2, i_3), \dots$, where $i_j \in \mathscr{V}$. We call $\mathscr{G}$ is connected if there exists a path between any two vertices of graph $\mathscr{G}$. We call graph $\mathscr{G}' = (\mathscr{V}', \mathscr{E}')$ is the subgraph of $\mathscr{G} = (\mathscr{V}, \mathscr{E})$, if $\mathscr{V}' \subseteq \mathscr{V}, \mathscr{E}' \subseteq \mathscr{E}$. For $\mathscr{G}' \subseteq \mathscr{G}$, if $\mathscr{G}'$ is connected and there is no other vertices in $\mathscr{V} - \mathscr{V}'$ connected to the $\mathscr{G}'$, then we call $\mathscr{G}'$ is one maximal connected branches of $\mathscr{G}$. In this paper, we consider the system with n sensors and a target. The interaction among sensors could be described by an undirected graph $\mathscr{G}$. The interaction among n sensors and a target can be described as $\bar{\mathscr{G}}$. Then we have $\mathscr{G} \subseteq \bar{\mathscr{G}}$. We call $\bar{\mathscr{G}}$ is connected, if for every maximal connected branch of $\mathscr{G}$, there is at least one sensor that is connected to the target.

The weighted adjacency matrix of graph $\mathscr{G}$ defined as $G = (g_{ij})_{nn} \in R^{n\times n}$, where $g_{ii} = 0$ and $g_{ij} = g_{ji} > 0$. Then the degree matrix defined as $D^{(G)} = diag\{d_1^{(G)}, d_2^{(G)}, \dots, d_n^{(G)}\} \in R^{n\times n}$, where diagonal elements $d_i^{(G)} = \sum_{j=1}^n g_{ij}$ $(i = 1, 2, \dots, n)$. The Laplacian of the weighted graph defined as $L = D^{(G)} - G$.

Here, we consider the distributed H_∞ estimation problem with switching topology. The index set of all the possible interconnected graphs (involving n sensors and a target) denoted as $\mathcal{P} = \{1, 2, \dots, N\}$. The switching signal defined as $\tau : [0, \infty) \to \mathcal{P}$. Assume there exists an infinite sequence of bounded, non-overlapping, contiguous time-intervals $[k_i, k_{i+1})$ $(i = 0, 1, \dots)$ with $k_0 = 0$. Therefore, the neighbor set $\mathscr{N}_i$, weighted adjacency matrix G, degree matrix D are piece-wise constant at the time-interval $[k_i, k_{i+1})$ $(i = 0, 1, \dots)$ and Laplacian matrix L also is piece-wise constants. Then we can use $\mathscr{N}_i(k)$, $G(k) = (g_{ij}(k))_{nn}$, $D^{(G)}(k) = diag(d_1^{(G)}(k), d_1^{(G)}(k), \dots, d_n^{(G)}(k))$ and $L_p(k) = D^{(G)}(k) - G(k)$ to describe the time-varying cases.

Furthermore, in order to describe the connection between sensors, we assume that there are fixed constants α_{ij} $(i, j = 1, 2, \dots, n)$. If sensor i and j connected at k, $g_{ij}(k) = \alpha_{ij} = \alpha_{ji}$, otherwise $g_{ij}(k) = 0$. Denote $b_i(k)$ as the connection weight between sensor i and the target, and there exist constants β_i $(i = 1, 2, \dots, n)$ such that

$$b_i(k) = \begin{cases} \beta_i, & \text{if sensors } i \text{ is connected to the target at } k, \\ 0, & \text{otherwise.} \end{cases}$$

Let $B_p(k) = diag(b_1(k), b_2(k), \dots, b_n(k))$ and $H_p(k) = L_p(k) + B_p(k)$. Therefore, $B_p(k), H_p(k)$ are piece-wise constant matrices at the time-interval $[k_i, k_{i+1})$ $(i = 0, 1, \dots)$ and only take finite values at time-interval $[0, \infty)$.

The following assumption on the graphs for sensor network is used in the distributed estimation literatures [23, 24].

Assumption 1: (Connectivity) The graph $\bar{\mathscr{G}}$ is connected, i. e., for every maximal connected branch of $\mathscr{G}$, there exist at least one sensor that is connected to the target.

Under above assumption, we review a useful lemma of $H_p(k)$ in [5].

Lemma 2.1. Denote

$$\lambda_0 = \min\{\lambda_p(k) : H_p(k)\phi_p(k) = \lambda_p(k)\phi_p(k),\ \phi_p(k) \neq 0,\ p = 1, 2, \dots, N,\ \forall k\}.$$

If Assumption 1 holds, then $\lambda_0 > 0$.

B. Distributed H_∞ Estimation

The dynamics of the target is as follows,

$$x(k+1) = Ax(k) + w_0(k), \tag{1}$$

where $x(k) = \left[x(k,1) \cdots x(k,m)\right]^T \in R^{m\times 1}$ and $w_0(k)$ is the m-dimensional process noise.

Furthermore, the measurement of the moving target by sensor i ($i = 1, 2, \ldots, n$) is as follows,

$$y_i(k) = b_i(k)Cx(k) + w_{i1}(k), \tag{2}$$

where $C \in R^{q\times m}$ is the observable matrix. If sensor i is connected to the moving target, then the measurement obtained by sensor i is $y_i(k) = \beta_i Cx(k) + w_{i1}(k)$. Otherwise, the measurement of sensor i is $y_i(k) = w_{i1}(k)$.

For sensor i ($i = 1, 2, \ldots, n$), we construct a distributed estimation algorithm:

$$\hat{x}_i(k+1) = A\hat{x}_i(k) + AK(y_i(k) - b_i(k)C\hat{x}_i(k) + z_i(k)), \tag{3}$$

where $z_i(k)$ is the relative measurement errors of its neighbors, i. e.,

$$z_i(k) = \sum_{j\in\mathscr{N}_i(k)} g_{ij}(k)C(\hat{x}_j(k) - \hat{x}_i(k)) + w_{i2}(k),$$

and $w_{i2}(k)$ is $q-$dimensional communication noise.

The following assumption is adopted throughout the paper.

Assumption 2: (Detectability) The pair (A, C) is assumed to be detectable.

Let $\eta_i(k) = \hat{x}_i - x$ be the local estimation error at node i. From (1) and (2), we can obtain the local filter error satisfy the following equation:

$$\begin{aligned}\eta_i(k+1) &= \hat{x}_i(k+1) - x(k+1)\\ &= A\hat{x}_i(k) + AK(y_i(k) - b_i(k)C\hat{x}_i(k) + z_i(k)) - Ax(k) - w_0(k)\\ &= A\eta_i(k) + AK(b_i(k)Cx(k) - b_i(k)C\hat{x}_i(k) + \sum_{j\in\mathscr{N}_i(k)} g_{ij}(k)C(\hat{x}_j(k) - \hat{x}_i(k)))\\ &\quad + AK\bar{w}_{i1}(k) - w_0(k).\end{aligned} \tag{4}$$

where $\bar{w}_{i1}(k)$ defined in above equation. The following definition is arising in H_∞ filtering theory. This definition is also used in the literature [12] [19].

Definition 1. The filtering errors $\eta_i(k), (i = 1, 2, \ldots, n)$ are said to satisfy H_∞-consensus performance constraints if the following inequalities hold:

$$\frac{1}{n}\sum_{i=1}^{n}\|\eta_i\|_2^2 \le \frac{1}{\gamma}(\|x(0)\|_R^2 + \frac{1}{n}\sum_{i=1}^{n}(\|\bar{w}_{i1}\|_2^2 + \|w_0\|_2^2)), \tag{5}$$

where $\|\eta_i\|_2 = (\sum_{k=0}^{N-1}\|\eta_i(k)\|_2^2)^{\frac{1}{2}}$, for some given disturbance attenuation level $\gamma > 0$.

The distributed H_∞ estimation problem in our paper is to design a static gain K, such that filtering error η_i satisfies the H_∞-consensus performance constraints (5), and make the TMSEE

$$E(k) = \sum_{i=1}^{n} \mathbb{E}(\hat{x}_i(k) - x(k))^T(\hat{x}_i(k) - x(k))$$

lower bounded. Furthermore, we want to investigate the bound for γ, which helps to guarantee the existence of the solution to H_∞ problem. Obviously, the constant gain K in the algorithm has advantages of low computational complexity and simple design in the switching topology case.

3. MAIN RESULTS

In this section, firstly, we analysis the distributed H_∞ estimation algorithm for a moving target. Then we give a lower bound for TMSEE. Finally, the bound for γ is investigated.

First, we denote

$$\hat{X}(k) = [\hat{x}_1^T(k), \ldots, \hat{x}_n^T(k)]^T, \quad X(k) = [x^T(k), \ldots, x^T(k)]^T, \quad \eta(k) = [\eta_1^T(k), \ldots, \eta_n^T(k)]^T$$
$$D_1 = diag(I_q \quad I_q, \ldots, I_q \quad I_q), \quad D_2 = [I_m, \ldots, I_m]^T.$$

Then (4) can be written as

$$\begin{aligned}\eta(k+1) \ = \ & (I_n \otimes A)\eta(k) + (I_n \otimes (AK))((B_p(k)\mathbf{1}) \otimes (Cx(k)) \\ & - (H_p(k) \otimes C)\hat{X}(k)) + (I_n \otimes (AK))D_1 w_1(k) - D_2 w_0(k),\end{aligned} \tag{6}$$

where $w_1(k) = [w_{11}^T(k), w_{12}^T(k), \ldots, w_{n1}^T(k), w_{n2}^T(k)]^T$.
Denote $D = \begin{bmatrix}(I_n \otimes (AK))D_1 & -D_2\end{bmatrix}$, $w(k) = [w_1^T(k), w_0^T(k)]^T$, and then (6) can be rewritten as

$$\eta(k+1) = (I_n \otimes A)\eta(k) + (I_n \otimes (AK))((B_p(k)\mathbf{1}) \otimes (Cx(k)) - (H_p(k) \otimes C)\hat{X}(k)) + Dw(k).$$

Furthermore, by the properties of Kronecker product,

$$\begin{aligned}\eta(k+1) &= (I_n \otimes A)\eta(k) + (I_n \otimes (AK))((B_p(k)\mathbf{1}) \otimes (Cx(k)) - (H_p(k) \otimes C)\hat{X}(k)) + Dw(k) \\ &= (I_n \otimes A)\eta(k) + (I_n \otimes (AK))((B_p(k) \otimes C)X(k) - (H_p(k) \otimes C)\hat{X}(k)) + Dw(k) \\ &= (I_n \otimes A)\eta(k) + (I_n \otimes (AK))((H_p(k) \otimes C)X(k) - (H_p(k) \otimes C)\hat{X}(k)) + Dw(k) \\ &= (I_n \otimes A)\eta(k) - (I_n \otimes (AK))(H_p(k) \otimes C)\eta(k) + Dw(k) \\ &= ((I_n \otimes A) - H_p(k) \otimes (AKC))\eta(k) + Dw(k).\end{aligned} \tag{7}$$

For presentation convenience, the performance of distributed H_∞ estimation (5) can be written as

$$\|\eta\|^2 \le \frac{1}{\gamma}\{\|w\|_2^2 + \eta^T(0)R\eta(0)\}. \tag{8}$$

3.1. Analysis of distributed H_∞ filtering

Before giving the main results, the following two lemmas are helpful in subsequent analysis.

Lemma 3.1. (Kailath et al. [9]) Under Assumption 2, given a $\gamma > 0$, there exists a positive definite matrix P satisfied following algebraic Riccati equation:

$$P = A(P^{-1} + C^T R^{-1} C - \gamma I_m)^{-1} A^T + Q, \tag{9}$$

subject to $P^{-1} + C^T R^{-1} C - \gamma I_m > 0$.

Lemma 3.2. (Marshall et al. [11]) Given any matrix X and any positive definite matrix P, the following matrix inequalities are equivalent:

$$X^T P X - P < 0; \quad X P^{-1} X^T - P^{-1} < 0. \tag{10}$$

The following theorem provides a constant gain K for the distributed H_∞ estimation algorithm (3).

Theorem 3.3. Under Assumptions 1 and 2, given a positive $\gamma > 0$, and a positive definite matrix $R^T = R > 0$, there exists a static gain

$$K = \max\{\frac{1}{\lambda_0}, 1\} M C^T (C M C^T + R)^{-1}$$

with initial condition $\eta_i^T(0) P^{-1} \eta_i(0) \leq \frac{1}{\gamma} \eta_i^T(0) R \eta_i(0)$, subject to $I_m - \gamma P^{-1} > 0$, where $M = (P^{-1} - \gamma I_m)^{-1}$, $\Phi = \frac{1}{\gamma} I_{mn} - D^T(I_n \otimes P^{-1}) D > 0$ and P is the solution of ARE (9), such that the filter error $\eta(k)$ satisfied H_∞ performance (8).

Proof. Since $H_p(k) = L_p(k) + B_p(k)$ is a symmetric matrix, there exists an unitary matrix $U_p(k)$ satisfying

$$U_p(k) H_p(k) U_p^T(k) = \Lambda_p(k) = diag(\lambda_{p1}(k), \ldots, \lambda_{pn}(k)).$$

By Lemma 2.1, $\lambda_{pr}(k) \geq \lambda_0 > 0, r = 1, 2, \ldots, n$.

Denote $\bar{\eta}(k) = (U_p(k) \otimes I_n) \eta(k)$, then

$$\bar{\eta}^T(k) \bar{\eta}(k) = \eta^T(k) \eta(k).$$

Let $V(k) = \bar{\eta}^T(k)(I_n \otimes P^{-1}) \bar{\eta}(k)$. Therefore, it follows that

$$\begin{aligned}
&V(k+1) - V(k) + \|\eta(k)\|^2 - \frac{1}{\gamma}\|w(k)\|^2 \\
&= \bar{\eta}^T(k)\{(I_n \otimes A - \Lambda_p(k) \otimes (AKC))^T (I_n \otimes P^{-1})(I_n \otimes A - \Lambda_p(k) \otimes (AKC)) + I_{mn}\} \bar{\eta}(k) \\
&\quad + 2\bar{\eta}^T(k)(I_n \otimes A - \Lambda_p(k) \otimes (AKC))^T (I_n \otimes P^{-1})(U_p(k) \otimes I_n) D w(k) \\
&\quad - w^T(k)(\frac{1}{\gamma} I_{mn} - D^T (U_p(k) \otimes I_n)(I_n \otimes P^{-1}) D) w(k).
\end{aligned}$$

Completing the square to $w(k)$, the above equation can be written as

$$\begin{aligned}
&V(k+1) - V(k) + \|\eta(k)\|^2 - \frac{1}{\gamma}\|w(k)\|^2 \\
&= \bar{\eta}^T(k)\{(I_n \otimes A - \Lambda_p(k) \otimes (AKC))^T \Gamma (I_n \otimes A - \Lambda_p(k) \otimes (AKC)) \\
&\quad + (I_n \otimes A - \Lambda_p(k) \otimes (AKC))^T (I_n \otimes P^{-1})(I_n \otimes A - \Lambda_p(k) \otimes (AKC)) + I_n\}\bar{\eta}(k) \\
&\quad - (w(k) - w^*(k))^T \tilde{\Phi}(w(k) - w^*(k)),
\end{aligned} \tag{11}$$

where $w^*(k) = \Phi^{-1} D(I_n \otimes P^{-1})(I_n \otimes A - \Lambda_p(k) \otimes (AKC))\bar{\eta}$,
$\Gamma = (I_n \otimes P^{-1}) D \Phi^{-1} D^T (I_n \otimes P^{-1})$.

Take $K = \mu K_0$, where $\mu = \max\{\frac{1}{\lambda_0}, 1\}$, $K_0 = MC^T(CMC^T + R)^{-1}$. For any p and k, we always have $\lambda_p(k)\mu > 1$. From ARE (9), it is not hard to obtain

$$\begin{aligned}
(A - AK_0C)P(A - AK_0C)^T - P \\
&= -Q - A(I_m - K_0C)(P^{-1} - \gamma I_m)^{-1} C^T K_0^T A^T \\
&\quad - \gamma(A - AK_0C)P(I_m - \gamma P^{-1})^{-1} P(A - AK_0C)^T.
\end{aligned}$$

Due to $(P^{-1} - \gamma I_m)^{-1} = P + \gamma P(I_m - \gamma P^{-1})^{-1} P$ and the condition $I_m - \gamma P^{-1} > 0$, we have

$$(A - \lambda_p(k)\mu AK_0C)P(A - \lambda_p(k)\mu AK_0C)^T - P \leq -Q.$$

By Lemma 3.1, there exists a constant $\alpha > 0$ such that

$$(A - \lambda_p(k)\mu AK_0C)^T P^{-1}(A - \lambda_p(k)\mu AK_0C) - P^{-1} < -\alpha I_m,$$

and there exists a constant $\rho > 0$ such that $\Gamma \leq \rho(I_n \otimes P^{-1})$, then we have

$$\begin{aligned}
&V(k+1) - V(k) + \mathbb{E}(\|\eta(k)\|^2 - \frac{1}{\gamma}\|w(k)\|^2) \\
&\leq -\alpha\lambda_{\min}(P)V(k) - \alpha\rho\lambda_{\min}(P)V(k) + \mathbb{E}(-(w(k) - w^*(k))^T \Phi(w(k) - w^*(k))) \\
&\leq -\alpha(1+\rho)\lambda_{\min}(P)V(k) + \mathbb{E}(-(w(k) - w^*(k))^T \Phi(w(k) - w^*(k))) \\
&\leq 0.
\end{aligned}$$

As a result,

$$\mathbb{E}\{\sum_{k=1}^{N-1} \|\eta(k)\|^2\} \leq \frac{1}{\gamma}\sum_{k=1}^{N-1} \|w(k)\|^2 - \mathbb{E}\{\eta^T(N)(I_n \otimes P^{-1})\eta(N)\} + \eta^T(0)(I_n \otimes P^{-1})\eta(0),$$

with the initial condition $\eta_i^T(0)P^{-1}\eta_i(0) \leq \frac{1}{\gamma}\eta_i^T(0)R\eta_i(0)$, which implies that the H_∞ performance (8) is satisfied, as long as $P > 0$. $\square$

Remark 3.4. In the above analysis, a static gain K for distributed H_∞ estimation is obtained. Notice that the distributed H_∞ estimation we presented has a connection with the distributed estimation which use the Kalman filter. As $\gamma \to 0, M = (P^{-1} - \gamma I_m)^{-1} \to P$, this is consistent with the case in [23], and moreover, if there is no noise, the result is consistent with [6].

Motivated by Theorem 2 in [23], we can also obtain a lower bound for TMSEE in the following theorem.

Theorem 3.5. Under Assumptions 1 and 2, a lower bound for $E(k)$ is $tr(Z(k))$, where $tr(Z(k))$ is solution of the following difference equation:

$$Z(k+1) = (I_n \otimes A)(Z^{-1}(k) + (H_p(k) \otimes C)^T (D_1 D_1^T)^{-1} (H_p(k) \otimes C) - \gamma I_{mn})^{-1} (I_n \otimes A)^T + D_2 D_2^T. \tag{12}$$

Proof. First, we can construct an estimation algorithm based on common H_∞ filtering. For sensor i ($i = 1, 2, \ldots, n$), we consider the following estimation algorithm,

$$\bar{x}_i(k+1) = A\bar{x}_i(k) + AK(k)(y_i(k) - C\bar{x}_i(k) + \bar{z}_i(k)), \tag{13}$$

where $\bar{z}_i(k)$ has the same definition with $z_i(k)$ in (3).

Denote $\bar{\eta}_i(k) = \bar{x}_i(k) - x(k)$, $\bar{X}(k) = [\bar{x}_1^T(k), \ldots, \bar{x}_n^T(k)]^T$, $\bar{\eta}(k) = [\bar{\eta}_1^T(k), \ldots, \bar{\eta}_n^T(k)]^T$, then we can obtain the following compact form

$$\bar{\eta}(k+1) = ((I_n \otimes A) - (I_n \otimes A)K(k)(H_p(k) \otimes C))\bar{\eta}(k) + D(k)w_i(k) - D_2 w_0(k), \tag{14}$$

where $D(k) = (I_n \otimes A)K(k)D_1$, $K(k) = [K_1^T(k), K_2^T(k), \ldots, K_n^T(k)]^T$. Based on the H_∞ estimation theory [9], we can taking

$$K(k) = (Z^{-1}(k) - \gamma I_{mn})(I_n \otimes C)^T ((I_n \otimes C)(Z^{-1}(k) - \gamma I_{mn})(I_n \otimes C)^T + D_1 D_1^T)^{-1},$$

where $Z(k)$ is the solution of the following difference Riccati equation,

$$Z(k+1) = (I_n \otimes A)(Z^{-1}(k) + (H_p(k) \otimes C)^T (D_1 D_1^T)^{-1} (H_p(k) \otimes C) - \gamma I_{mn})^{-1} (I_n \otimes A)^T + D_2 D_2^T.$$

Denote $\bar{E}(k) = \mathbb{E}\{\bar{\eta}^T(k)\bar{\eta}(k)\}$, and $tr(\bar{E}(k)) \geq tr(Z(k))$ for any dynamic gain $K(k)$. If we take $K(k) = diag(K, K, \ldots, K)$, and then the algorithm (13) has the same form as (3). Therefore, $E(k) \geq tr(Z(k))$. Thus the lower bound of TMSEE achieved by $tr(Z(k))$. □

Remark 3.6. When $\gamma \to 0$, the lower bound for TMSEE in our distributed H_∞ estimation will be the case in [23].

3.2. Bound for H_∞ certain level

In the preceding subsection, a static constant gain K and a lower bound for TMSEE were obtained. Notice that, if γ is too large, the distributed H_∞ estimator may not have a solution. Thus, a bound for γ is helpful to design distributed H_∞ estimator. In what follows, we present a bound for γ.

At first, we give two lemmas as follows.

Lemma 3.7. (Horn and Johnson [7]) Suppose $S, T \in S^n$, and $P \in R^{n\times m}$. Then we have:

a. if $S \geq T$, then $\lambda(S) \geq \lambda(T)$,

b. $\lambda_n(S)I \leq S \leq \lambda_1(S)I$,

c. if $S \geq T$, then $P^T SP \geq P^T TP$,

d. if $S \geq T > 0$, then $T^{-1} \geq S^{-1}$,

where λ_1 and λ_n denote the maximum and minimum of eigenvalue, respectively.

Lemma 3.8. (Horn and Johnson [7]) If $A, B \in C^{n\times n}$ are real symmetric matrix, then

a. $\lambda_t(A+B) \geq \max_{i+j=t+n}(\lambda_i(A)+\lambda_j(B))$,

b. $\lambda_t(A+B) \leq \max_{i+j=t+1}(\lambda_i(A)+\lambda_j(B))$,

c. $\sigma_t(AB) \geq \max_{i+j=t+n}((\sigma_i(A)\sigma_j(B))$,

d. $\sigma_t(AB) \leq \max_{i+j=t+1}(\sigma_i(A)\sigma_j(B))$,

where σ_1 and σ_n denote the maximum and minimum of singular value, respectively.

Then we give the main result of this section.

Theorem 3.9. If there is a positive definite matrix $\bar{P}$ satisfied following equation,

$$\bar{P} = A\bar{P}A^T - A\bar{P}C^T(C\bar{P}C^T+R)^{-1}C\bar{P}A^T + Q, \tag{15}$$

then the bound for the γ can be described as $0 \leq \gamma \leq \lambda_1(W) - \Theta$ where $W = C^T R^{-1} C, \Theta = \frac{\sigma_n^2(A)\lambda_n(\bar{P})-\lambda_n(\bar{P})+\lambda_n(Q)}{\lambda_n^2(\bar{P})-\lambda_n(Q)\lambda_n(\bar{P})}$.

P r o o f. As $\gamma \to 0$, (9) reduces to (15). Under Assumption 2, there exists a positive definite matrix $\bar{P}$ satisfying (15).

Since W is positive, the algebraic Riccati equation of H_∞ estimation (9) can be written as

$$\bar{P} = A(\bar{P}^{-1} - \gamma I + W)^{-1}A^T + Q. \tag{16}$$

Next, we will show that γ is bounded by the the solution of (15). By Lemmas 3.7 and 3.8, we have

$$\begin{aligned}\lambda_n(\bar{P}) &= \lambda_n(A(\bar{P}^{-1}-\gamma I+W)^{-1}A^T+Q)\\ &\geq \lambda_n(A(\bar{P}^{-1}-\gamma I+W)^{-1}A^T)+\lambda_n(Q)\\ &\geq \sigma_n^2(A)\lambda_n(\bar{P}^{-1}-\gamma I+W)^{-1}+\lambda_n(Q)\\ &= \sigma_n^2(A)\lambda_n^{-1}(\bar{P}^{-1}-\gamma I+W)+\lambda_n(Q)\\ &\geq \frac{\sigma_n^2(A)}{\lambda_1(\bar{P}^{-1})+\lambda_1(W-\gamma I)}+\lambda_n(Q)\\ &\geq \frac{\sigma_n^2(A)}{\frac{1}{\lambda_n(\bar{P})}+\lambda_1(W-\gamma I)}+\lambda_n(Q)\\ &= \frac{\sigma_n^2(A)\lambda_n(\bar{P})}{1+\lambda_n(\bar{P})\lambda_1(W-\gamma I)}+\lambda_n(Q).\end{aligned}$$

Since $\lambda_n(\bar{P})\geq\lambda_n(Q)$, it is not hard to obtain

$$\lambda_1(W-\gamma I)\geq\frac{\sigma_n^2(A)\lambda_n(\bar{P})-\lambda_n(\bar{P})+\lambda_n(Q)}{\lambda_n^2(\bar{P})-\lambda_n(Q)\lambda_n(\bar{P})}=\Theta.$$

Combining $\lambda_1(W)+\lambda_1(-\gamma I)\geq\lambda_1(W-\gamma I)$, we can obtain $0\leq\gamma\leq\lambda_1(W)-\Theta$. □

Remark 3.10. We can treat γ as a disturbance to (15), and thus, the positive definiteness of solution can be guaranteed under the disturbance.

4. SIMULATIONS

In this section, we present a numerical example to illustrate the effectiveness of the proposed algorithm, and verify the bound for γ.

We consider a 2-dimensional tracking problem which used in [24]. The target moves along a line with the constant velocity, which can be described by

$$x(k+1)=\begin{bmatrix}1 & T\\ 0 & 1\end{bmatrix}x(k)+\begin{bmatrix}0.1\\ 0.1\end{bmatrix}w_0(k),$$

where $w_0(k)$ is 1-dimensional Gaussian noises and T is the filter period, $x(k)=[x_1(k)\ x_2(k)]^T$, and then $x_1(k)$ and $x_2(k)$ can be treated as the position and velocity of the target, respectively.

Three sensors and the target consist of the graph $\bar{\mathscr{G}}$, which is switching periodically among $\bar{\mathscr{G}}_1$,$\bar{\mathscr{G}}_2$,$\bar{\mathscr{G}}_3$ and $\bar{\mathscr{G}}_4$ (Figure 1). The switching time from one graph to another one is 1 second.

Considering that we can only obtain position information, then measurement obtained by sensor i ($i=1,2$) is as follows,

$$y_i(k)=b_i(k)[1\ 0]x(k)+0.1w_{i1}(k), \tag{17}$$

where $w_{11}(k)$, $w_{21}(k)$ are white noises. In this case, we take

$$\begin{cases}b_1(k)=b_2(k)=b_3(k)=1,\ k=4nT,\ n=0,1,\dots\\ b_1(k)=1,\ b_2(k)=b_3(k)=0,\ k=4nT+T,\ n=0,1,\dots\\ b_1(k)=b_2(k)=1,\ b_3(k)=0,\ k=4nT+2T,\ n=0,1,\dots\\ b_1(k)=b_2(k)=0,\ b_3(k)=1,\ k=4nT+3T,\ n=0,1,\dots\end{cases}.$$

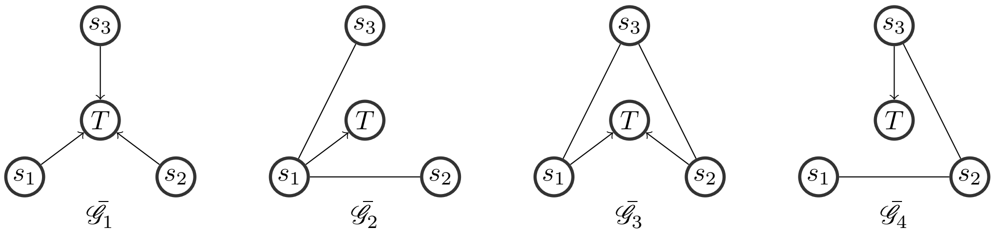

Fig. 1. Switching Graphs.

The relative measurement errors are as follows,

$$\begin{aligned} z_{12}(k) &= [1\ 0](g_{12}(k)(\hat{x}_2(k) - \hat{x}_1(k)) + g_{13}(k)(\hat{x}_3(k) - \hat{x}_1(k))) + 0.1w_{12}(k), \\ z_{22}(k) &= [1\ 0](g_{21}(k)(\hat{x}_1(k) - \hat{x}_2(k)) + g_{23}(k)(\hat{x}_3(k) - \hat{x}_2(k))) + 0.1w_{22}(k), \\ z_{32}(k) &= [1\ 0](g_{31}(k)(\hat{x}_1(k) - \hat{x}_3(k)) + g_{32}(k)(\hat{x}_2(k) - \hat{x}_3(k))) + 0.1w_{32}(k), \end{aligned} \tag{18}$$

where $\hat{x}_1(k), \hat{x}_2(k)$ and $\hat{x}_3(k)$ are the estimations by sensors, and $w_{12}(k)$, $w_{22}(k)$ and $w_{32}(k)$are white noises. We take $\alpha_{ij} = \alpha_{ji}$, and then

$$\begin{aligned} &g_{12}(k) = g_{21}(k) = 0,\ g_{13}(k) = g_{31}(k) = 0,\ g_{23}(k) = g_{32}(k) = 0,\ k = 4nT,\ n = 0, 1, \ldots \\ &g_{12}(k) = g_{21}(k) = 1,\ g_{13}(k) = g_{31}(k) = 1,\ g_{23}(k) = g_{32}(k) = 0,\ k = (4n+1)T,\ n = 0, 1, \ldots \\ &g_{12}(k) = g_{21}(k) = 0,\ g_{13}(k) = g_{31}(k) = 1,\ g_{23}(k) = g_{32}(k) = 1,\ k = (4n+2)T,\ n = 0, 1, \ldots \\ &g_{12}(k) = g_{21}(k) = 1,\ g_{13}(k) = g_{31}(k) = 0,\ g_{23}(k) = g_{32}(k) = 1,\ k = (4n+3)T,\ n = 0, 1, \ldots \end{aligned}$$

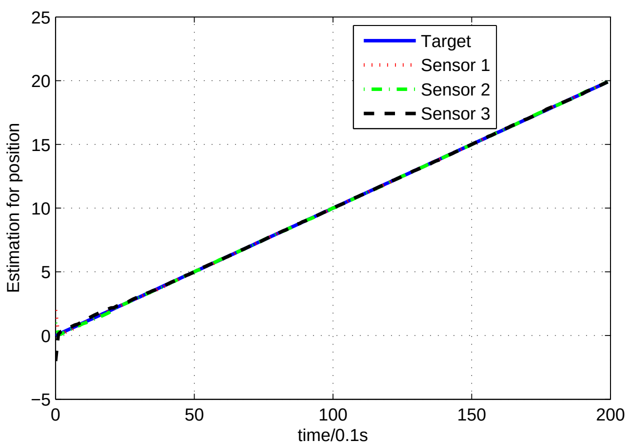

Fig. 2. Estimation for position.

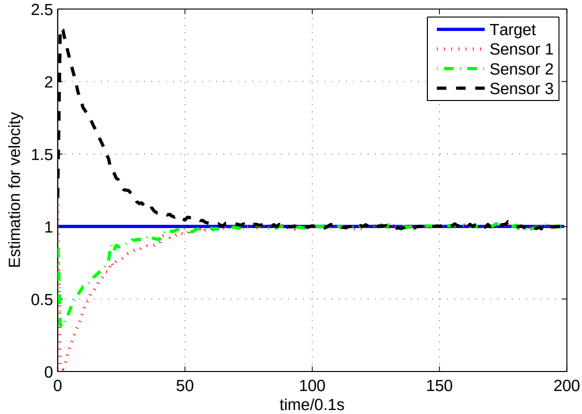

Fig. 3. Estimation for velocity.

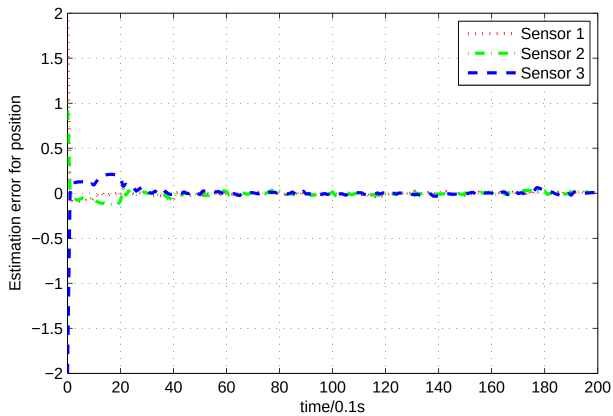

Fig. 4. Filtering errors for position.

We first choose the initial error of the target state as $\eta_1(0) = [2 \;\; 0.2]^T$, $\eta_2(0) = [1 \;\; -0.1]$, $\eta_3(0) = [-2 \;\; 0.2]$. In this case, we obtain $0 \le \gamma \le 0.4874$, where we choose $\gamma = 0.1$ and $Q = \begin{bmatrix} 0.01 & 0 \\ 0 & 0.01 \end{bmatrix}$. We can obtain the constant gain $K = \begin{bmatrix} 0.9376 & 0.5884 \end{bmatrix}^T$. The simulation sample time is $0.1s$. Figures 2 and 3 demonstrate that the estimations of position and velocity of the target. The filtering errors are given in Figures 4 and 5. Lower bound for TMSEE is illustated in Figure 6, which verifies Theorem 3.5. Denote P is the solution of (9). When we take $0 \le \gamma \le 0.4874$, the minimun eigenvalue of P satisfies $\lambda_{\min}(P) > 0$. However, when we take $\gamma = 0.49$, the minimun eigenvalue of P becomes $\lambda_{\min}(P) = -37.23$. The minimum eigenvalue of P is showed in Figure 7, which verifies the Theorem 3.9.

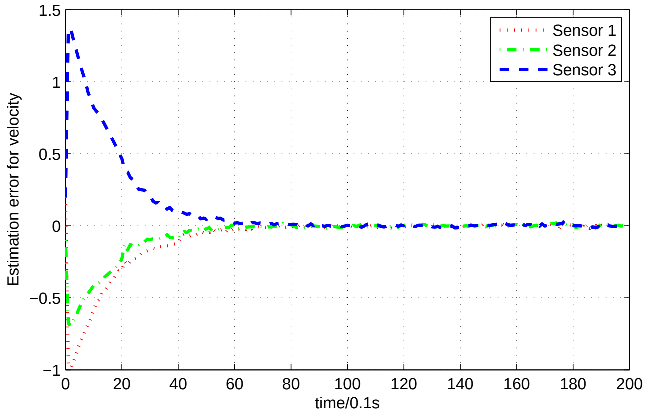

Fig. 5. Filtering errors for velocity.

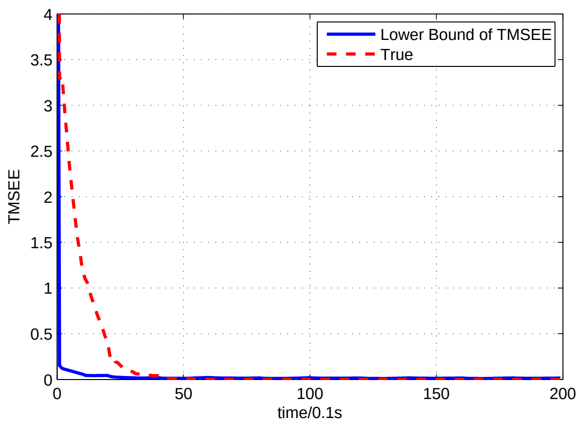

Fig. 6. Total mean square estimation error.

5. CONCLUSIONS

This paper studied the distributed H_∞ estimation problem for moving target with switching topology, and the capacity of sensors take into consideration. By solving ARE instead of solving general LMIs, we presented a distributed H_∞ estimation algorithm with a constant gain, which was of low computational complexity and simple design. Under the conditions of detectability and connectivity, we showed a constant gain K and a lower bound for TMSEE were showed. Moreover, we obtained a bound for the H_∞ parameter by solving ARE.

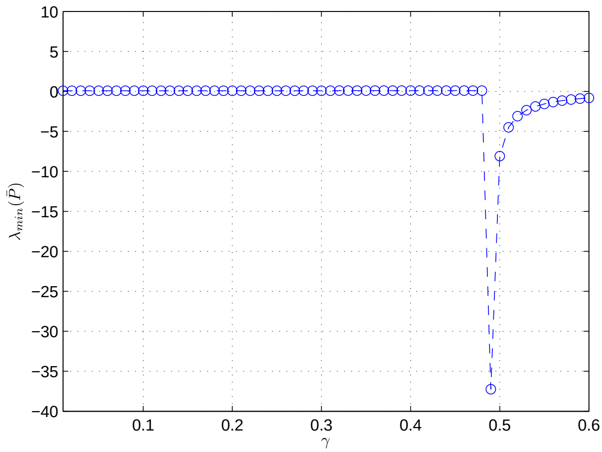

Fig. 7. Minimum eigenvalue.

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