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SYNCHRONIZATION OF FRACTIONAL CHAOTIC COMPLEX NETWORKS WITH DELAYS

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The synchronization of fractional-order complex networks with delay is investigated in this paper. By constructing a novel Lyapunov-Krasovskii function V and taking integer derivative instead of fractional derivative of the function, a sufficient criterion is obtained in the form of linear matrix inequalities to realize synchronizing complex dynamical networks. Finally, a numerical example is shown to illustrate the feasibility and effectiveness of the proposed method.

Keywords: fractional complex networks, delays, Lyapunov-Krasovskii theorem, synchronization

Classification: 34D06, 93D05

1. INTRODUCTION

In the past few decades, many efforts have been devoted to complex networks due to the potential applications in various fields, such as World Wide Web, social networks, neural networks, gene networks, metabolic networks, power grid networks,information science and so on [2, 6, 15, 20, 31]. As an interesting and significant dynamic behavior, synchronization has always been a hot research topic [19, 29, 30, 32, 33]. In the past few years, much research effort has been dedicated to unveiling the influence of interaction topologies on the onset of network synchronization and its stability. For example, Li studied pinning synchronization of complex dynamical networks with mixed coupling [5, 11]. Tang and Chen studied synchronizing two complex networks with nonidentical topological structures by adaptive control [18].

However, it should be noted that most of the studies are mainly concentrated on the integer-order complex networks. In recent years, fractional calculus, as a generalization of ordinary differentiation and integration, has received much attention due to its application in physics and engineering [8, 26]. The behavior of many systems can be elegantly described by fractional differential systems, such as viscoelasticity, dielectric polarization, quantum evolution of complex systems and fractional kinetics [1, 17, 27]. Especially, since fractional derivatives are nonlocal and have weakly singular kernels, fractional derivatives provide an excellent tool for describing the memory and hereditary properties of various materials and processes. Moreover, it would be more accurate

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if practical systems are described by fractional-order dynamical models rather than integer-order ones. Therefore, it is essential to study the synchronization of fractional order complex dynamical networks [13, 23].

Recently, the dynamics and the synchronization of fractional-order complex networks have become a hot topic [21, 22]. The majority of the existing research results relative to fractional-order networks have been concerned without delay. Due to the limited information channels and large-scale interconnected complex networks, time-delayed coupling extensively exists in many physical systems. This implies that a system with time delay becomes more complicated and interesting [10, 12, 28]. The same is to fractional complex networks. Even though the synchronization of integer-order complex network with delay has been intensively studied by various control schemes in the past few years, the synchronization of fractional order complex dynamical network with delay has received less attention in spite of its practical significance. To our knowledge, Dai and Si studied the adaptive lag synchronization of delayed fractional complex networks [7, 14]. Wang and Yang studied cluster synchronization of fractional-order coupled-delay complex network via adaptive pinning control [25]. Hu and Lu realized synchronizing fractional complex networks with distributed delays by constructing an extending item based on the stability theorem of fractional system without delay [9].

Compared with the synchronization of integer complex networks, there is fewer achievements in the synchronization of fractional complex networks. Although the fractional Lyapunov-Krasovskii stable theorem present a novel approach to study the stability of fractional system with delay by designing a positive function V_1 and a semi positive extending function V_2 and calculating the fractional derivative of the positive function $V_1 + V_2$ [4, 16], it is usually very difficult to construct a semi positive extending function V_2 as fractional derivative is nonlocal and have weakly singular kernels. Aimed at this problem, we proposed a novel approach to analyze the stability and synchronization of fractional complex networks with delay by designing a positive function V_1 and a semi positive extending function V_2 and taking integer derivative instead of fractional derivative in this paper. A numerical example is finally presented to demonstrate the effectiveness of the theoretical result.

This paper is organized as follows: Sections 2 introduces some definitions, lemmas, and properties of fractional calculus and complex networks; In Sections 3, the main approach is proposed to synchronize fractional complex networks with delay; Numerical examples are presented in Section 4; Finally, a conclusion is drawn in Section 5.

2. PRELIMINARIES AND DEFINITIONS

2.1. Fractional calculus

We first recall some definitions and properties related to fractional derivatives that will be used in this paper.

There are some definitions for fractional derivative. The commonly used definitions are Grunwald–Letnikov(GL), Riemann–Liouville(RL) and Caputo(C) definition. In this paper we mainly use the Caputo fractional operators. The Caputo definition of fractional derivative, which sometimes is called smooth fractional derivative, expressed as [25]:

$$ {}_a^C D_t^\alpha f(t) = \frac{1}{\Gamma(n-\alpha)} \times \int_a^t (t-\tau)^{-\alpha+n-1} f^{(n)}(\tau)\,\mathrm{d}\tau \tag{1} $$

where n is the first integer which is not less than α, i.e. $n-1 < \alpha \le n$ and $\Gamma(\cdot)$ is the Gamma function.

When $0 < \alpha \le 1$, we can get:

$$ \begin{aligned} {}_a^C D_t^\alpha f(t) &= \frac{1}{\Gamma(1-\alpha)} \times ((t)^{-\alpha} u(t)) * (f'(t)u(t-a)) \\ &= \frac{1}{\Gamma(1-\alpha)} \times ((t)^{-\alpha} u(t-a)) * ((f(t)u(t-a))' - f(a)\delta(t-a)) \\ &= \frac{1}{\Gamma(1-\alpha)} \times ((t)^{-\alpha} u(t)) * ((f(t)-f(a))u(t-a))' \\ &= \frac{1}{\Gamma(1-\alpha)} \times \frac{\mathrm{d}}{\mathrm{d}t}((t)^{-\alpha} u(t)) * ((f(t)-f(a))u(t-a)) \\ &= \frac{1}{\Gamma(1-\alpha)} \frac{\mathrm{d}}{\mathrm{d}t}((t)^{-\alpha} u(t)) * (f(t)u(t-a)) - ((t)^{-\alpha} u(t)) * (f(a)u(t-a))) \\ &= \frac{1}{\Gamma(1-\alpha)} (\frac{\mathrm{d}}{\mathrm{d}t}((t)^{-\alpha} u(t)) * (f(t)u(t-a)) - f(a)((t-a)^{-\alpha}\delta(t-a)) \end{aligned} \tag{2} $$

and

$$ \int_a^t {}_a^C D_\tau^\alpha f(\tau)\,\mathrm{d}\tau = \frac{1}{\Gamma(1-\alpha)} \times ((t)^{-\alpha} u(t-a)) * ((f(t)u(t-a)) - f(a)u(t-a)) \tag{3} $$

where $u(t)$ is the unit step function and the symbol $*$ represents convolution operation.

Lemma 1. (Aquila-Camacho et al. [3]) Let $x(t) = [x_1(t), \ldots, x_n(t)]^T \in \mathbb{R}^n$ be a differentiable vector-value function. Then for any time instant $t \ge t_0$

$$ {}_a^C D_t^\alpha [x^T(t)Px(t)] \le x^T(t)P {}_a^C D_t^\alpha x(t) + {}_a^C D_t^\alpha (x^T(t))Px(t) \tag{4} $$

where $\alpha \in (0,1]$ and $P \in \mathbb{R}^{n\times n}$ is a symmetric positive definite matrix.

2.2. Fractional complex networks

A. Notations

Throughout this paper, $\mathbb{R}^n$ shall denote the $n-$dimensional Euclidean space and $\mathbb{R}^{n\times n}$ denotes the set of all $n\times n$ real matrices. For a real matrix A, let A^T be its transpose and $A^s = (A + A^T)/2$ be its symmetric part. Let I_n be the n-dimensional identity matrix. For symmetric matrix A, the notation $A > 0$ (respectively, < 0) shall mean that A is a positive-definite (respectively, negative-definite) matrix. The symbol $\otimes$ denotes the Kronecker product.

Lemma 2. (Tang et al. [19]) If matrix $Q \in \mathbb{R}^{n\times n}$ satisfies $q_{ij} = q_{ji}$ and $q_{ii} = -\sum_{j=1,i\neq j}^{n} q_{ij}, i, j = 1, 2, \dots, n$, then

$$u^T Q \otimes Pv = \sum_{j=1}^{n}\sum_{i=1}^{n} u_i q_{ij} P v_j = -\sum_{j>i} q_{ij}(u_i - u_j)P(v_i - v_j)$$

for all matrices $P \in \mathbb{R}^{n\times n}$ and vectors $u = [u_1^T, u_2^T, \dots, u_n^T]^T$ and $v = [v_1^T, v_2^T, \dots, v_n^T]^T$, where $u_i = [u_{i1}, u_{i2}, \dots, u_{im}]^T$ and $v_i = [v_{i1}, v_{i2}, \dots, v_{im}]^T$.

Lemma 3. [29] For matrices A, B, C and D with appropriate dimensions, one has

(1) $(A \otimes B)^T = A^T \otimes B^T$

(2) $(A + B) \otimes C = A \otimes C + B \otimes C$

(3) $(A \otimes B)(C \otimes D) = (AC) \otimes (BD)$.

Lemma 4. [12] For matrices X and Y with appropriate dimensions, the following inequality holds for any $\eta > 0$

$$X^T Y + Y^T X \leq \eta X^T X + \frac{1}{\eta} Y^T Y. \tag{5}$$

B. Fractional complex networks model

Consider a complex network consisting of N coupled nodes with delays, in which each node is an n-dimensional fractional chaotic dynamical subsystem expressed as:

$$\begin{aligned} {}_{t_0}^{C}D_t^{\alpha} x_i(t) =& f(x_i(t)) + \sum_{j=1,i\neq j}^{N} c_1 a_{ij}(g(x_j(t)) - g(x_i(t))) \\ &+ \sum_{j=1,i\neq j}^{N} c_2 b_{ij}(h(x_j(t-\tau)) - h(x_i(t-\tau))) \end{aligned} \tag{6}$$

where $x_i(t) = (x_{i1}(t), x_{i2}(t), \dots, x_{in}(t))^T \in \mathbb{R}^n$ is the state vector of the ith node of the network at time t, $f(x_i(t)) = [f_1(x_i(t)), f_2(x_i(t)), \dots, f_n(x_i(t))]^T$ is a continuous vector-valued function, $A = [a_{ij}] \in \mathbb{R}^{N\times N}$ and $B = [b_{ij}] \in \mathbb{R}^{N\times N}$ are the outer coupling matrices of the network at time t and $t - \tau$ respectively such that $a_{ij} \geq 0$ for $a_{ii} = -\sum_{j=1,j\neq i}^{N} a_{ij}$, $b_{ii} = -\sum_{j=1,j\neq i}^{N} b_{ij}$, c_1 and c_2 represent the coupling strength, τ is the delay time and the nonlinear coupling functions $g(\cdot) : \mathbb{R}^n \to \mathbb{R}^n$ and $h(\cdot) : \mathbb{R}^n \to \mathbb{R}^n$ are continuous and of the form $g(x_i(t)) = [g_1(x_i(t)), g_2(x_i(t)), \dots, g_n(x_i(t))]^T$ and $h(x_i(t-\tau)) = [h_1(x_{i1}(t-\tau)), h_2(x_{i2}(t-\tau)), \dots, h_n(x_{in}(t-\tau))]^T$.

The objective of this paper is to control fractional chaotic dynamical network (6) so that it stays in the trajectory $s(t) \in \mathbb{R}^n$ of the system

$${}_{t_0}^{C}D_t^{\alpha} s(t) = f(s(t)). \tag{7}$$

Define the synchronization errors as: $e_i(t) = x_i(t) - s(t)$. From $a_{ii} = -\sum_{j=1,j\neq i}^{N} a_{ij}$ and $b_{ii} = -\sum_{j=1,j\neq i}^{N} b_{ij}$, we can get the synchronizing error system as:

$$\begin{aligned}
{}_{t_0}^{C}D_t^{\alpha} e_i(t) = {} & f(x_i(t)) - f(s(t)) + \sum_{j=1,i\neq j}^{N} c_1 a_{ij}(g(x_j(t)) - g(x_i(t))) \\
& + \sum_{j=1,i\neq j}^{N} c_2 b_{ij}(h(x_j(t-\tau)) - h(x_i(t-\tau))) + u_i(t) \\
= {} & f(x_i(t)) - f(s(t)) + \sum_{j=1}^{N} c_1 a_{ij} g(x_j(t)) - \sum_{j=1}^{N} c_1 a_{ij} g(s(t)) \\
& + \sum_{j=1}^{N} c_2 b_{ij} h(x_j(t-\tau)) - \sum_{j=1}^{N} c_2 b_{ij} h(s(t-\tau))] + u_i(t) \\
= {} & f(x_i(t)) - f(s(t)) + \sum_{j=1}^{N} c_1 a_{ij}(g(x_j(t)) - g(s(t))) \\
& + \sum_{j=1}^{N} c_2 b_{ij}(h(x_j(t-\tau)) - h(s(t-\tau))) + u_i(t) \\
& i = 1, 2, \ldots, N,
\end{aligned}$$

where $u_i = \eta_i e_i(t)$ is the controller to be designed.

That is to say:

$$ {}_{t_0}^{C}D_t^{\alpha} e(t) = F(e(t)) + (c_1 A \otimes I_n)G(e(t)) + (c_2 B \otimes I_n)H(e(t-\tau)) + \Theta e(t) \tag{8}$$

where $F(e(t)) = [f(x_1(t)), f(x_2(t)), \ldots, f(x_N(t))]^T - [f(s(t)), f(s(t)), \ldots, f(s(t))]^T$, $G(e(t)) = [g(x_1(t)), g(x_2(t)), \ldots, g(x_N(t))]^T - [g(s(t)), g(s(t)), \ldots, g(s(t))]^T$, $H(e(t-\tau)) = [h(x_1(t-\tau)), h(x_2(t-\tau)), \ldots, h(x_N(t-\tau))]^T - [h(s(t-\tau)), h(s(t-\tau)), \ldots, h(s(t-\tau))]^T$ and $\Theta = diag\{\eta_1(t), \eta_2(t), \ldots, \eta_N(t)\} \otimes I_n$.

Definition 1. The complex network (6) is said to be realized synchronization if the solution of system (2.2) is such that

$$\sum_{i=1}^{N} \lim_{t\to\infty} \|e_i(t)\| = 0 \tag{9}$$

for any initial conditions.

Assumption 1. For each function $f(\cdot)$, $g(\cdot)$, $h(\cdot)$ in network (6), there exist positive definite matrixes $\Phi \in \mathbb{R}^{N\times N}$ and $P \in \mathbb{R}^{n\times n}$, and positive constants l_1, l_2 and l_3 such that

$$(f(x(t)) - f(y(t)))^T \Phi \otimes P(f(x(t)) - f(y(t))) \leq l_1(x(t) - y(t))^T \Phi \otimes P(x(t) - y(t)) \tag{10}$$

$$(g(x(t))-g(y(t)))^T\Phi\otimes P(g(x(t))-g(y(t)))\le l_2(x(t)-y(t))^T\Phi\otimes P(x(t)-y(t)) \tag{11}$$

$$(h(x(t))-h(y(t)))^T\Phi\otimes P(h(x(t))-h(y(t)))\le l_3(x(t)-y(t))^T\Phi\otimes P(x(t)-y(t)). \tag{12}$$

3. THE MAIN RESULT

In this section, we derive a sufficient condition for synchronizing fractional complex networks with delay.

Theorem 1. The controlled fractional complex networks (6) globally asymptotically synchronizes to the isolated node (7), if there exist positive definite matrixes $\Phi\in\mathbb{R}^{N\times N}$, $P\in\mathbb{R}^{n\times n}$, a semi positive definite matrix $Q\in\mathbb{R}^{n\times n}$ and $k_1,k_2,k_3\ge 0$, and then the synchronizing errors network given by (2.2) satisfy:

$$\xi^T(t)\begin{bmatrix}\Xi_{11}+\Phi\otimes Q & \Xi_{12} & \Xi_{13} & 0 & \Xi_{15}\\ * & \Xi_{22} & 0 & 0 & 0\\ * & * & \Xi_{33} & 0 & 0\\ * & * & * & \Xi_{44}-\Phi\otimes Q & \Xi_{45}\\ * & * & * & * & \Xi_{55}\end{bmatrix}\xi(t)\le 0 \tag{13}$$

where

$\Xi_{11}=\Phi\otimes P\Theta+l_1k_1\Phi\otimes P+l_2k_2\Phi\otimes P+l_3k_3\Phi\otimes P$,
$\Xi_{12}=\frac{1}{2}\Phi\otimes P$,
$\Xi_{13}=\frac{1}{2}c_1(\Phi A)\otimes P$,
$\Xi_{15}=\frac{1}{2}c_2(\Phi B)\otimes P$,
$\Xi_{22}=-k_1\Phi\otimes P$,
$\Xi_{33}=-k_2\Phi\otimes P$,
$\Xi_{44}=k_3l_3\Phi\otimes P$,
$\Xi_{45}=-\frac{1}{2}k_3(\Phi B)\otimes P$,
$\Xi_{55}=-k_3(\Phi B)\otimes P$,
$\xi(t)=[e^T(t),F^T(e(t)),G^T(e(t)),e^T(t-\tau)),H^T(e(t-\tau))]^T$.

Proof. Construct a positive definite Lyapunov function $V=V_1+V_2$, where

$$V_1=\frac{1}{2}e(t)^T\Phi\otimes Pe(t) \tag{14}$$

and

$$V_2=\int_{t-\tau}^{t}e(\epsilon)^T\Phi\otimes Qe(\epsilon)\,\mathrm{d}\epsilon. \tag{15}$$

Take fractional derivative of the function V_1 and get:

$$\begin{aligned}{}^C_{t_0}D^\alpha_t V_1 &\le e(t)^T\Phi\otimes P\,{}^C_{t_0}D^\alpha_t e(t)\\ &= e(t)^T\Phi\otimes PF(e(t))+e(t)^T(\Phi\otimes P)\Theta e(t)+c_1e(t)^T(\Phi A)\otimes PG(e(t))\\ &+c_2e(t)^T(\Phi B)\otimes PH(e(t-\tau))\end{aligned}$$

$$
\begin{aligned}
&\leq e(t)^T\Phi\otimes PF(e(t))+e(t)^T(\Phi\otimes P)\Theta e(t)\\
&+k_1(l_1e(t)^T\Phi\otimes Pe(t)-F^T(e(t))\Phi\otimes PF(e(t)))\\
&+c_1e(t)^T(\Phi A)\otimes PG(e(t))+k_2(l_2e(t)^T\Phi\otimes Pe(t)-G^T(e(t))\Phi\otimes PG(e(t)))\\
&+c_2e(t)^T(\Phi B)\otimes PH(e(t-\tau))\\
&+k_3(l_3e(t-\tau)^T\Phi\otimes Pe(t-\tau)-H^T(e(t-\tau)\Phi\otimes PH(e(t-\tau)) \qquad (16)\\
&=\xi^T(t)\begin{bmatrix}\Xi_{11} & \Xi_{12} & \Xi_{13} & 0 & \Xi_{15}\\ * & \Xi_{22} & 0 & 0 & 0\\ * & * & \Xi_{33} & 0 & 0\\ * & * & * & \Xi_{44} & \Xi_{45}\\ * & * & * & * & \Xi_{55}\end{bmatrix}\xi(t)
\end{aligned}
$$

Take integer derivative of the function V_2 and get:

$$\dot{V}_2=e^T(t)\Phi\otimes Qe(t)-e^T(t-\tau)\Phi\otimes Qe(t-\tau). \qquad (17)$$

From 16 and 17, we can get:

$$
{}_{t_0}^{C}D_t^{\alpha}V_1+\dot{V}_2\leq\xi^T(t)\begin{bmatrix}\Xi_{11}+\Phi\otimes Q & \Xi_{12} & \Xi_{13} & 0 & \Xi_{15}\\ * & \Xi_{22} & 0 & 0 & 0\\ * & * & \Xi_{33} & 0 & 0\\ * & * & * & \Xi_{44}-\Phi\otimes Q & \Xi_{45}\\ * & * & * & * & \Xi_{55}\end{bmatrix}\xi(t)\leq 0. \qquad (18)
$$

As ${}_{a}^{C}D_t^{\alpha}V_1(t)+\frac{\mathrm{d}}{\mathrm{d}t}V_2(t)\leq 0$, there must exist $\xi(t)\geq 0$ satisfying:

$${}_{a}^{C}D_t^{\alpha}V_1+\frac{\mathrm{d}}{\mathrm{d}t}V_2(t)+\xi(t)=0. \qquad (19)$$

As:

$$
\begin{aligned}
{}_{a}^{C}D_t^{\alpha}V_1&=\frac{1}{\Gamma(1-\alpha)}\int_{-\infty}^{\infty}((t-\tau)^{-\alpha}u(t-\tau)V_1'(\tau)u(\tau)\,\mathrm{d}\tau\\
&=\frac{1}{\Gamma(1-\alpha)}(tu(t))^{-\alpha}*(V_1(t)u(t))'-\frac{1}{\Gamma(1-\alpha)}V_1(0)(tu(t))^{-\alpha}.
\end{aligned} \qquad (20)
$$

We can get:

$$
\begin{aligned}
0&\leq\frac{1}{\Gamma(1-\alpha)}(tu(t))^{-\alpha}*(V_1(t)u(t))+V_2(t)\\
&=V_2(0)-\int_0^t\xi(\tau)+\frac{1}{\Gamma(1-\alpha)}V_1(0)(\tau u(\tau))^{-\alpha}\mathrm{d}\tau.
\end{aligned} \qquad (21)
$$

As $\xi(t)+\frac{1}{\Gamma(1-\alpha)}V_1(0)(tu(t))^{-\alpha}\geq 0$ for any time $t>0$, then

$$
\begin{aligned}
&\lim_{t\to\infty}\frac{\mathrm{d}}{\mathrm{d}t}(V_2(0)-\int_0^t\xi(\tau)+\frac{1}{\Gamma(1-\alpha)}V_1(0)(\tau u(\tau))^{-\alpha}\mathrm{d}\tau)\\
&=\lim_{t\to\infty}-\xi(t)+\frac{1}{\Gamma(1-\alpha)}V_1(0)(tu(t))^{-\alpha}\\
&=\lim_{t\to\infty}-\xi(t)+0\leq 0.
\end{aligned} \qquad (22)
$$

We can draw a conclusion:

$$\begin{aligned}
&\lim_{t\to\infty} 0 \leq \frac{1}{\Gamma(1-\alpha)}(tu(t))^{-\alpha} * (V_1(t)u(t)) + V_2(t) \\
&= \lim_{t\to\infty} V_2(0) - \int_0^t \xi(\tau) + \frac{1}{\Gamma(1-\alpha)} V_1(0)(\tau u(\tau))^{-\alpha}\mathrm{d}\tau = 0
\end{aligned} \tag{23}$$

So, $\lim_{t\to\infty} V_1(t) = 0$ and $\lim_{t\to\infty} V_2(t) = 0$. The proof of Theorem 1 is completed. □

4. NUMERICAL EXAMPLES

In the section, we present an example to illustrate the main results obtained in this paper. Consider the following fractional Lorenz system

$$\begin{aligned}
{}_a^C D_t^\alpha x_1(t) &= a(x_2(t) - x_1(t)) + x_4(t) \\
{}_a^C D_t^\alpha x_2(t) &= bx_1(t) - x_2(t) - x_1(t)x_3(t) \\
{}_a^C D_t^\alpha x_3(t) &= x_1(t)x_2(t) - cx_3(t) \\
{}_a^C D_t^\alpha x_4(t) &= x_2(t)x_3(t) - x_4(t)
\end{aligned} \tag{24}$$

where $a = 10, b = 28, c = 8/3$. With the initial value of the fractional chaotic system $x_1(0) = 1, x_2(0) = 2, x_3(0) = 3, x_4(0) = 4$ the fractional order $\alpha = 0.995$, the fractional order chaotic system (24) exhibits the chaotic behavior as shown in Figure 1 [24].

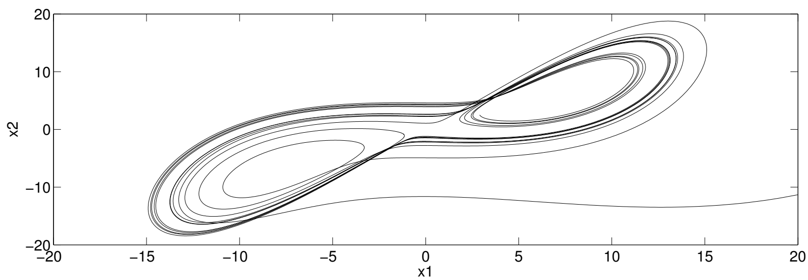

Fig. 1. The chaotic attractor x_1 - x_2 of fractional Lorenz hyperchaotic system in system (24).

Let complex network be with 6 nodes and the coupling configuration matrixes are given as follows, respectively.

$$A = (a_{ij})_{6\times 6} = \begin{bmatrix} -4 & 1 & 0 & 1 & 0 & 2 \\ 1 & -6 & 2 & 1 & 1 & 1 \\ 0 & 2 & -5 & 0 & 1 & 1 \\ 1 & 1 & 0 & -4 & 1 & 1 \\ 0 & 1 & 1 & 1 & -4 & 1 \\ 2 & 1 & 1 & 1 & 1 & -6 \end{bmatrix}$$

$$B=(b_{ij})_{6\times 6}=\begin{bmatrix} -2 & 1 & 0 & 1 & 0 & 0\\ 1 & -3 & 2 & 0 & 0 & 0\\ 0 & 2 & -5 & 0 & 1 & 2\\ 1 & 0 & 0 & -5 & 2 & 1\\ 0 & 0 & 1 & 2 & -4 & 1\\ 0 & 0 & 2 & 1 & 1 & -4 \end{bmatrix}.$$

Then, we can get the synchronizing error network:

$$\begin{aligned} {}_{t_0}^{C}D_t^{\alpha}e_i(t) &= f(x_i(t))-f(s(t))+\sum_{j=1}^{N}c_1a_{ij}(g(x_j(t))-g(s(t)))\\ &+\sum_{j=1}^{N}c_2b_{ij}(h(x_j(t-\tau))-h(s(t-\tau)))+u_i(t)\\ &i=1,2,\ldots,N. \end{aligned} \tag{25}$$

Define $f(e_i(t))=f(x_i(t))-f(s(t))$ and the controller $u_i(t)=k_ie_i(t)$ and we can get:

$$\begin{aligned} {}_{a}^{C}D_t^{\alpha}e_{i1}(t) &= 10e_{i2}(t)-10e_{i1}(t)+e_{i4}(t)+k_ie_{i1}\\ {}_{a}^{C}D_t^{\alpha}e_{i2}(t) &= 28e_{i1}(t)-e_{i2}(t)-e_{i1}(t)x_{i3}(t)-x_{i1}(t)e_{i3}(t)+k_ie_{i2}\\ {}_{a}^{C}D_t^{\alpha}e_{i3}(t) &= e_{i1}(t)x_{i2}(t)+x_{i1}(t)e_{i2}(t)-\frac{8}{3}e_{i3}(t)+k_ie_{i3}\\ {}_{a}^{C}D_t^{\alpha}e_{i4}(t) &= e_{i2}(t)x_{i3}(t)+x_{i2}(t)e_{i3}(t)-e_{i4}(t)+k_ie_{i4}. \end{aligned} \tag{26}$$

Then,

$$\begin{aligned} &e_i^T(t)f(e_i(t))=38e_{i2}(t)e_{i1}(t)+(k_i-10)e_{i1}^2(t)\\ &+e_{i4}(t)e_{i1}(t)+(k_i-1)e_{i2}^2(t)-e_{i1}(t)x_{i3}(t)e_{i2}(t)\\ &e_{i1}(t)x_{i2}(t)e_{i3}(t)+(k_i-\frac{8}{3})e_{i3}^2(t)+e_{i2}(t)x_{i3}(t)e_{i4}(t)+x_{i2}(t)e_{i3}(t)e_{i4}(t)+(k_i-1)e_{i4}^2(t)\\ &\le(9.5+|0.5x_{i3}(t)|+0.5|x_{i2}(t)|+k_i)e_{i1}^2(t)+(18+0.5|x_{i3}(t)|+0.5|x_{i3}(t)|+k_i)e_{i2}^2(t)\\ &+(0.5|x_{i2}(t)|-\frac{8}{3}+0.5|x_{i2}(t)|+k_i)e_{i3}^2(t)+(0.5|x_{i3}(t)|+0.5|x_{i2}(t)|-0.5+k_i)e_{i4}^2(t). \end{aligned} \tag{27}$$

In formula (24), taking the initial condition as $x_0=[0.1,0.2,0.4,0.5]^T$, by numerical simulation and removing the transient process, we can get:

$$\frac{e_i^T(t)f(e_i(t))}{e_i^T(t)e_i(t)}\le 63.724+k_i. \tag{28}$$

According to theorem 1, when the delay time is taken as $\tau=0.2$, the error system is gradually stable to zero according to theorem 1 when $k_i\le -87.724$.

In numerical simulations, the Caputo fractional definition is adopted, the feedback strength is selected as $k_i=-88(i=1,2,\ldots,6)$ and the initial conditions of the numerical simulations are taken as :$x_{i0}=[0.1i,0.2i,0.4i,0.5i]^T$. As the delay time $\tau=0.2$, we add the control input after $t=0.2$. The synchronizing errors $e_i(t)(1\le i\le 6)$ of the network with distributed delays is shown in Figure 2. It obviously that the synchronizing errors is gradually stable to zero after $t = 0.2$. The simulation results show the correctness of Theorem 1.

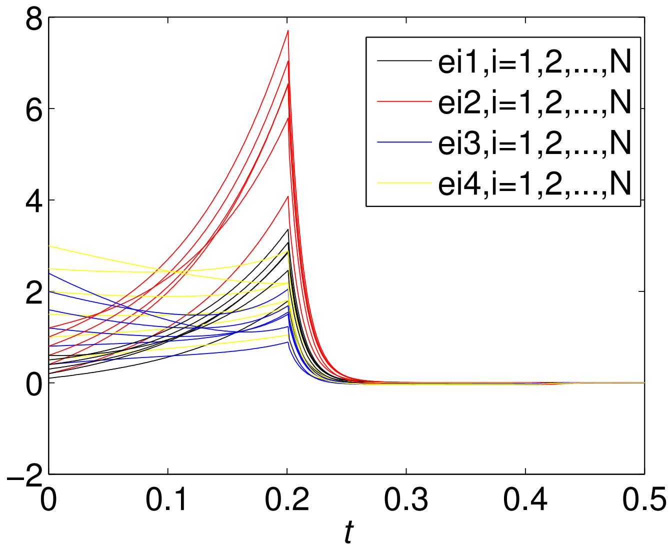

Fig. 2. The synchronizing errors e_{i1}, e_{i2}, e_{i3} and e_{i4} with time t.

5. CONCLUSION

In this paper, we extend Lyapunov-Krasovskii theorem to delayed fractional systems by constructing a novel positive function and the sufficient condition of synchronizing fractional complex networks with delays is derived by taking the integer derivative instead of fractional derivative of the positive function. The numerical example has been given to show the effectiveness of the approach.

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