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INVERSE OPTIMAL CONTROL FOR LINEARIZABLE NONLINEAR SYSTEMS WITH INPUT DELAYS

Xiushan Cai, Jie Wu, Xisheng Zhan and Xianhe Zhang

We consider inverse optimal control for linearizable nonlinear systems with input delays based on predictor control. Under a continuously reversible change of variable, a nonlinear system is transferred to a linear system. A predictor control law is designed such that the closed-loop system is asymptotically stable. We show that the basic predictor control is inverse optimal with respect to a differential game. A mechanical system is provided to illustrate the effectiveness of the proposed method.

Keywords: nonlinear systems, inverse optimality, predictor control, input delays

Classification: 93Cxx, 93Dxx

1. INTRODUCTION

Predictor-based techniques have been developed for stabilization linear/nonlinear systems with input delays [1, 2, 3, 7, 8, 10, 14, 18], tracking control [26, 27], optimal performance analysis of networked control systems [23, 24, 25], as well as observer design for a class of nonlinear system in cascade with counter-convecting transport dynamics [9].

The inverse optimality concept is of practical importance since it allows the design of optimal control laws, which may minimize/maximize a physical quantity of interest and which may possess certain robustness margins, without the need to solve a Hamilton–Jacobi–Isaacs partial differential equation (PDE) [19].

Inverse optimality, as an design objective for delay systems was pursued by Jankovic [12, 13]. Inverse-optimal re-design of the predictor-based feedback law was presented by using a low-pass filter in [17]. Input-to-state stability (ISS) and inverse optimality of linear time-varying-delay predictor feedbacks have been investigated in [5]. The method in [5] is extended to multi-input linear systems [6]. Inverse optimal control for strict-feedforward nonlinear systems with input delays also exists [11].

If we can find coordinate changes to transform a class of nonlinear systems into linear systems, it will be very meaningful. It is revealed that the family of feedforward systems contains a substantial class that is linearizable by a diffeomorphic coordinate change in [15, 16]. An algorithm along with explicit transformations that linearizes a class of

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feedforward nonlinear systems is in [21]. Sufficient and necessary conditions are given for a control-affine mechanical system that can be transformed into a linear system by a diffeomorphism coordinate change in [20].

In this paper, we extend the result of [17] to inverse optimal control design for a class of linearizable nonlinear systems with input delays. First, using a continuously reversible change of variable, a nonlinear system is transferred to a linear system. A predictor control law is designed such that the closed-loop system is asymptotically stable. It is shown that the control law is inverse optimal with respect to a meaningful differential game. Inverse optimal control for a class of linearizable nonlinear systems with input delays has never been published from the knowledge of authors.

Notation. We use the common definitions of class $\mathcal{K}$, $\mathcal{K}_\infty$, $\mathcal{KL}$ functions from [18]. $\lambda_{\max}$ and $\lambda_{\min}$ are the maximum and minimum eigenvalues, respectively of the corresponding matrices. For a vector $X \in R^n$, $|X|$ denotes its usual Euclidean norm. For a scalar function $u(\cdot,t) \in L_2\,(0,1)$, $\|u(t)\|$ denotes $\left(\int_0^1 u^2(x,t)\,\mathrm{d}x\right)^{1/2}$. For a scalar function $U \in L_2\,(0,D)$, $\|U(t)\|$ denotes $\left(\int_0^D U^2(\theta)\,\mathrm{d}\theta\right)^{1/2}$.

2. SYSTEM DESCRIPTION AND BACKSTEPPING TRANSFORMATION

Consider the system with input delay

$$\dot{Z}(t) \;=\; \varphi(Z(t), U(t-D)) \tag{1}$$

where $Z \in R^n$ is the state, $U \in R$ is the input signal delayed by D units of time, and $\varphi : R^n \times R \to R^n$ is continuously differentiable. Suppose that there exists a continuously reversible coordinate change

$$X(t) \;=\; f(Z(t)) \tag{2}$$

such that system (1) is converted into a linear system as

$$\dot{X}(t) \;=\; AX(t) + B_1 U(t-D) \tag{3}$$

with $A \in R^{n\times n}, B_1 \in R^n$.

Remark 1. Sufficient and necessary conditions are given for a control-affine mechanical system that can be transformed into a linear system by a coordinate change in Theorem 1 of [20]. More information can be found from [4] for linearization of mechanical control systems and [22] for linearization of Hamiltonian and gradient systems.

Remark 2. In [21], for a class feedforward systems

$$\dot{x} = f(x) + g(x)u \tag{4}$$

where $f = [f_1, f_2, \cdots, f_n]^T$, $g = [g_1, g_2, \cdots, g_n]^T$, and $f_j(x) = x_j \overline{f}_j(x_{j+1}, \cdots, x_n) + \widehat{f}_j(x_{j+1}, \cdots, x_n)$, $g_j(x) = x_j \overline{g}_j(x_{j+1}, \cdots, x_n) + \widehat{g}_j(x_{j+1}, \cdots, x_n)$, $\widehat{f}_j(0) = 0, \widehat{g}_j(0) = 0, f_n = 0, \overline{f}_j(0), g_n \in R/\{0\}$ can be transformed into a linear controllable system via a coordinate change.

Using the transport PDE to express the delay, system (3) can be rewritten as

$$\begin{aligned} \dot{X}(t) &= AX(t) + B_1 u(0,t) && (5)\\ u_t(x,t) &= u_x(x,t),\ x \in [0,D] && (6)\\ u(D,t) &= U(t) && (7) \end{aligned}$$

where $u(x,t) = U(x+t-D)$.

The infinite-dimensional backstepping transformation is defined as

$$w(x,t) = u(x,t) - ke^{Ax}X(t) - k\int_0^x e^{A(x-y)}B_1 u(y,t)\,\mathrm{d}y, \tag{8}$$

for all $x \in [0,D]$. The gain vector k is selected so that $A + B_1 k$ is hurwitz.

Under the backstepping transformation, system (5)–(7) is transferred as

$$\begin{aligned} \dot{X}(t) &= (A + B_1 k)X(t) + B_1 w(0,t) && (9)\\ w_t(x,t) &= w_x(x,t),\ x \in [0,D] && (10)\\ w(D,t) &= U(t) - ke^{AD}X(t) - k\int_0^D e^{A(D-y)}B_1 u(y,t)\,\mathrm{d}y. && (11) \end{aligned}$$

The inverse backstepping transformation of w is defined as follows:

$$u(x,t) = w(x,t) + ke^{(A+B_1k)x}X(t) + k\int_0^x e^{(A+B_1k)(x-y)}B_1 w(y,t)\,\mathrm{d}y, \tag{12}$$

for all $x \in [0,D]$. With (12), system (9)–(11) is transferred to system (5)–(7).

3. ASYMPTOTICAL STABILIZATION FOR LINEARIZABLE NONLINEAR SYSTEMS

The control law for system (1) is designed as follows:

$$U(t) = \frac{c}{c+1}U_1(t) = U^*(t) \tag{13}$$

where

$$U_1(t) = ke^{AD}f(Z(t)) + k\int_{t-D}^t e^{A(t-\theta)}B_1 U(\theta)\,\mathrm{d}\theta \tag{14}$$

and $c > 0$ is sufficiently large, and the vector k is selected so that $A + B_1 k$ is hurwitz.

Theorem 3.1. Consider the closed-loop system (1), (13) and (14), there exist $c^* > 0$, and a class of $\mathcal{KL}$ function $\beta(s,t)$ such that for all $c > c^*$,

$$\Gamma(t) \le \beta(\Gamma(0),\, t), \quad \text{for all } t \ge 0, \tag{15}$$

with

$$\Gamma(t) = |Z(t)| + \|U(t)\|. \tag{16}$$

The proof of Theorem 3.1 is based on a series of technical lemmas which are given next.

First, we will prove that system (5)–(7) under the control law (13) and

$$U_1(t) = ke^{AD}X(t) + k\int_0^D e^{A(D-y)}B_1u(y,t)\,\mathrm{d}y \tag{17}$$

where k is given by (14), is exponentially stable.

Lemma 3.2. Consider system (5)–(7), together with the control law (13) and (17), for any $0 < \mu < 1$, there exist

$$\overline{\lambda} = \frac{\mu\min\{\frac{\lambda_{\min}(Q)}{2}, \frac{a_1 b}{2}\}}{\max\{\lambda_{\max}(P), \frac{a_1 e^{bD}}{2}\}}, \tag{18}$$

$$R = \frac{\max\{\overline{\beta}_1, 1+\overline{\beta}_2\}}{\min\{\lambda_{\min}(P), \frac{a_1}{2}\}}\max\{\lambda_{\max}(P), \frac{a_1 e^b}{2}\}\max\{\overline{\alpha}_1, 1+\overline{\alpha}_2\}, \tag{19}$$

and

$$c^* = \frac{\sqrt{e^{bD}\overline{a}\max\left\{\frac{a_1}{\lambda_{\min}(Q)}, \frac{1}{b}\right\}}}{\sqrt{1-\mu}}, \tag{20}$$

such that for all $c > c^*$, it holds

$$\Omega(t) \leq R\Omega(0)e^{-\overline{\lambda}t}, \text{ for } t \geq 0, \tag{21}$$

with

$$\Omega(t) = |X(t)|^2 + \|u(t)\|^2. \tag{22}$$

Proof. The proof is similar to that in [5], it is omitted. □

Noting that $u(y,t) = U(y+t-D)$, it is not difficult to find that (17) can be rewritten as

$$U_1(t) = ke^{AD}X(t) + k\int_{t-D}^t e^{A(t-\theta)}B_1U(\theta)\,\mathrm{d}\theta. \tag{23}$$

Lemma 3.3. Consider the closed-loop system (3), (13), (23), for any $0 < \mu < 1$, there exist $\overline{\lambda} > 0$, $R > 0$ and $c^* > 0$, which are given by (18), (19), (20), respectively, such that for all $c > c^*$, it holds

$$\widetilde{\Omega}(t) \leq R\widetilde{\Omega}(0)e^{-\overline{\lambda}t}, \quad \text{for all } t \geq 0, \tag{24}$$

with

$$\widetilde{\Omega}(t) = |X(t)|^2 + \|U(t)\|^2. \tag{25}$$

Proof. The proof is similar to that in [5], it is omitted. □

Lemma 3.4. Under the condition (2), there exist class $\mathcal{K}_\infty$ functions α_1, α_2 such that

$$|X(t)| + \|U(t)\| \leq \alpha_1(|Z(t)| + \|U(t)\|), \tag{26}$$

$$|Z(t)| + \|U(t)\| \leq \alpha_2(|X(t)| + \|U(t)\|). \tag{27}$$

Proof. With the help of (2), $f(\cdot)$ is continuously reversible, there exist class $\mathcal{K}_\infty$ functions $\overline{\alpha}_1$, $\overline{\alpha}_2$ such that

$$|f(Z(t))| \leq \overline{\alpha}_1(|Z(t)|), \tag{28}$$

and

$$|f^{-1}(X(t))| \leq \overline{\alpha}_2(|X(t)|). \tag{29}$$

So it can be deduced that

$$\begin{aligned} &|X(t)| + \|U(t)\| \\ &= |f(Z(t))| + \|U(t)\| \leq \overline{\alpha}_1(|Z(t)|) + \|U(t)\| \leq \alpha_1(|Z(t)| + \|U(t)\|) \end{aligned} \tag{30}$$

and

$$\begin{aligned} &|Z(t)| + \|U(t)\| \\ &= |f^{-1}(X(t))| + \|U(t)\| \leq \overline{\alpha}_2(|X(t)|) + \|U(t)\| \leq \alpha_2(|X(t)| + \|U(t)\|) \end{aligned} \tag{31}$$

where $\alpha_1(s) = \overline{\alpha}_1(s) + s$, $\alpha_2(s) = \overline{\alpha}_2(s) + s$. The proof is completed. □

Proof of the Theorem 3.1. Under the condition (2), it is easy to see that (23) can be described as (14). With (24), (26), (27), we get

$$\begin{aligned} |Z(t)| + \|U(t)\| &\leq \alpha_2(|X(t)| + \|U(t)\|) \\ &\leq \alpha_2\left(\sqrt{2(|X(t)|^2 + \|U(t)\|^2)}\right) \\ &\leq \alpha_2\left(\sqrt{2R(|X(0)|^2 + \|U(0)\|^2)e^{-\overline{\lambda}t}}\right) \\ &\leq \alpha_2\left(\sqrt{2R(|X(0)| + \|U(0)\|)^2 e^{-\overline{\lambda}t}}\right) \\ &\leq \alpha_2\left(\sqrt{2R}(|X(0)| + \|U(0)\|)e^{\frac{-\overline{\lambda}t}{2}}\right) \\ &\leq \alpha_2\left(\sqrt{2R}\alpha_1(|Z(0)| + \|U(0)\|)e^{\frac{-\overline{\lambda}t}{2}}\right) \\ &= \beta(\Gamma(0), t), \quad \text{for all } t \geq 0, \end{aligned} \tag{32}$$

where $\beta(s,t) = \alpha_2(\sqrt{2R}\alpha_1(s)e^{\frac{-\overline{\lambda}t}{2}})$. The proof is completed. □

4. INVERSE OPTIMAL DESIGN FOR LINEARIZABLE NONLINEAR SYSTEM

Theorem 4.1. Consider the closed-loop system (1), (13), (14), there exists a $c^{**} \geq c^*$ such that for all $c > c^{**}$, the control law (13), (14) minimizes the cost functional

$$J = \lim_{t\to\infty} \left(2\beta V(t) + \int_0^t \left(L(\tau) + \frac{\beta a_1 e^{bD}}{c} U^2(\tau) \right) \mathrm{d}\tau \right), \tag{33}$$

where L is a functional of $(Z(t), U(\theta))$, for all $t - D \leq \theta \leq t$, such that

$$L(t) \geq \beta\alpha_3(\Gamma(t)), \tag{34}$$

for arbitrary $\beta > 0, b > 0$, $a_1 = \frac{1}{4}\frac{\lambda_{\max}(PB_1B_1^TP)}{\lambda_{\min}(Q)}$, α_3 is a class $\mathcal{K}_\infty$ function, Γ is given by (16) and V is

$$V(t) = f(Z(t))^T Pf(Z(t)) + \frac{a_1}{2}\int_0^D e^{bx}w^2(x,t)\,\mathrm{d}x. \tag{35}$$

Proof. Choose

$$\begin{aligned} L(t) = &-\tfrac{\beta a_1 e^{bD}}{c+1}U_1(t)^2 + 2\beta X^T(t)QX(t) - 4\beta X^T(t)PB_1w(0,t) \\ &+a_1\beta w^2(0,t) + a_1\beta\textstyle\int_0^D be^{bx}w^2(x,t)\,\mathrm{d}x \end{aligned} \tag{36}$$

where $a_1 = \frac{1}{4}\frac{\lambda_{\max}(PB_1B_1^TP)}{\lambda_{\min}(Q)}$, U_1, w are given by (23), (8), respectively, and b, β are arbitrary positive scalars. Using (8) and (12), after some calculations, we get

$$\frac{|X(t)|^2 + \|u(t)\|^2}{\max\{\overline{\beta}_1, 1+\overline{\beta}_2\}} \leq |X(t)|^2 + \|w(t)\|^2 \leq \max\{\overline{\alpha}_1, 1+\overline{\alpha}_2\}(|X(t)|^2 + \|u(t)\|^2) \tag{37}$$

where

$$\begin{aligned} \overline{\alpha}_1 &= 3\left(1 + |k|^2|B_1|^2\tfrac{e^{2|A|D}-1}{2|A|}\right), \\ \overline{\alpha}_2 &= 3|k|^2\tfrac{e^{2|A|D}-1}{2|A|}, \\ \overline{\beta}_1 &= 3\left(1 + |k|^2|B_1|^2\tfrac{e^{2|A+B_1k|D}-1}{2|A+B_1k|}\right), \\ \overline{\beta}_2 &= 3|k|^2\tfrac{e^{2|A+B_1k|D}-1}{2|A+B_1k|}. \end{aligned} \tag{38}$$

With (17), (37), we have

$$U_1(t)^2 \leq \overline{a}(|X(t)|^2 + \|w(t)\|^2) \tag{39}$$

where $\overline{a} = 2|k|^2e^{2|A|D}(1+|B_1|^2)\max\{\overline{\beta}_1, 1+\overline{\beta}_2\}$. With (36), (39), we get

$$\begin{aligned} L(t) \geq &-\beta\tfrac{\overline{a}a_1e^{bD}}{c+1}(|X(t)|^2 + \|w(t)\|^2) + 2\beta\lambda_{\min}(Q)|X(t)|^2 \\ &-\tfrac{4\beta}{a_1}|X^T(t)PB_1|^2 + a_1b\beta\textstyle\int_0^D e^{bx}w^2(x,t)\,\mathrm{d}x \\ \geq &-\beta\tfrac{\overline{a}a_1e^{bD}}{c+1}(|X(t)|^2 + \|w(t)\|^2) + \beta\lambda_{\min}(Q)|X(t)|^2 + a_1b\beta\textstyle\int_0^D e^{bx}w^2(x,t)\,\mathrm{d}x \\ \geq &\;\beta\left(-\tfrac{\overline{a}a_1e^{bD}}{c+1} + \min\{\lambda_{\min}(\mathrm{Q}), \mathrm{a_1b}\}\right)(|X(t)|^2 + \|w(t)\|^2). \end{aligned} \tag{40}$$

Choose $c > c^{**}$ where c^{**} is such that

$$c^{**} \geq \max\left\{ \frac{2\overline{a}a_1e^{bD}}{\min\{\lambda_{\min}(Q), a_1b\}}, \frac{\sqrt{e^{bD}\overline{a}\max\left\{\frac{a_1}{\lambda_{\min}(Q)}, \frac{1}{b}\right\}}}{\sqrt{1-\mu}} \right\}, \tag{41}$$

for some $0 < \mu < 1$. By (37) and (41), we get from (40) that

$$\begin{aligned} L(t) &\geq \tfrac{\beta}{2}\min\{\lambda_{\min}(Q), a_1b\}(|X(t)|^2 + \|w(t)\|^2) \\ &\geq \tfrac{\beta\min\{\lambda_{\min}(Q), a_1b\}}{2\max\{\overline{\beta}_1, 1+\overline{\beta}_2\}}(|X(t)|^2 + \|u(t)\|^2) \\ &= \tfrac{\beta\min\{\lambda_{\min}(Q), a_1b\}}{2\max\{\overline{\beta}_1, 1+\overline{\beta}_2\}}(|X(t)|^2 + \|U(t)\|^2). \end{aligned} \tag{42}$$

With (26), one has

$$|X(t)| + \|U(t)\| \geq \alpha_1^{-1}(|Z(t)| + \|U(t)\|). \tag{43}$$

By (42), (43), we get

$$\begin{aligned} L(t) &\geq \tfrac{\beta\min\{\lambda_{\min}(Q), a_1b\}}{2\max\{\overline{\beta}_1, 1+\overline{\beta}_2\}}(|X(t)|^2 + \|U(t)\|^2) \\ &\geq \tfrac{\beta\min\{\lambda_{\min}(Q), a_1b\}}{2\max\{\overline{\beta}_1, 1+\overline{\beta}_2\}}\tfrac{(|X(t)|+\|U(t)\|)^2}{2} \\ &\geq \tfrac{\beta\min\{\lambda_{\min}(Q), a_1b\}}{2\max\{\overline{\beta}_1, 1+\overline{\beta}_2\}}\tfrac{(\alpha_1^{-1}(|Z(t)|+\|U(t)\|)^2}{2}. \end{aligned} \tag{44}$$

So (34) is obtained with $\alpha_3(s) = \frac{\min\{\lambda_{\min}(Q), a_1b\}}{2\max\{\overline{\beta}_1, 1+\overline{\beta}_2\}}\frac{(\alpha_1^{-1}(s))^2}{2}$.

With the help of (35), (36), (13), and the fact that $U^*(t) = \frac{c}{c+1}U_1(t)$, we have

$$\begin{aligned} L(t) &= -\tfrac{\beta c a_1 e^{bD}}{(c+1)^2}U_1(t)^2 + \beta a_1 e^{bD}\left(w(D,t)^2 - \tfrac{U_1(t)^2}{(c+1)^2}\right) - 2\beta\dot{V}(t) \\ &= -\tfrac{\beta c a_1 e^{bD}}{(c+1)^2}U_1(t)^2 + \beta a_1 e^{bD}\left((U(t) - U_1(t))^2 - \tfrac{U_1(t)^2}{(c+1)^2}\right) - 2\beta\dot{V}(t) \\ &= \tfrac{\beta a_1 e^{bD}}{c}(U^*(t))^2 + \beta a_1 e^{bD}\left((U(t) - U^*(t))^2 - \tfrac{2U(t)U^*(t)}{c}\right) - 2\beta\dot{V}(t). \end{aligned} \tag{45}$$

It can be deduced that

$$\begin{aligned} &\textstyle\int_0^t (L(\tau) + \frac{\beta a_1 e^{bD}}{c}U^2(\tau))\,\mathrm{d}\tau \\ &= -2\beta V(t) + 2\beta V(0) + \textstyle\int_0^t \beta a_1 e^{bD}\left(1 + \frac{1}{c}\right)(U(\tau) - U^*(\tau))^2\,\mathrm{d}\tau. \end{aligned} \tag{46}$$

We get

$$J = 2\beta V(0) + \int_0^\infty \beta a_1 e^{bD}\left(1 + \tfrac{1}{c}\right)(U(\tau) - U^*(\tau))^2\,\mathrm{d}\tau. \tag{47}$$

So the minimum of (47) is reached with

$$U(t) = U^*(t) \tag{48}$$

such that

$$J = 2\beta V(0). \tag{49}$$

The proof is completed. □

5. EXAMPLE

Consider the following mechanical system given by [20] as follows:

$$\dot{Z}_1(t) = Z_2(t) \tag{50}$$

$$\dot{Z}_2(t) = U(t-D) \tag{51}$$

$$\dot{Z}_3(t) = Z_4(t) \tag{52}$$

$$\dot{Z}_4(t) = Z_1(t)(1+Z_1(t)) + \frac{Z_1(t)Z_4(t)}{1+Z_1(t)} \tag{53}$$

where $Z_1, Z_2, Z_3, Z_4 \in R$ with $Z_1 > -1$ are the states and $U \in R$ is the input signal delayed by D units of time. Denote $X(t) = [X_1(t), X_2(t), X_3(t), X_4(t)]^T$, $Z(t) = [Z_1(t), Z_2(t), Z_3(t), Z_4(t)]^T$, with a change of variable

$$X(t) = f(Z(t)) = \begin{bmatrix} Z_1(t) \\ Z_2(t) \\ Z_3(t) - \frac{1}{2}(\frac{Z_4(t)}{1+Z_1(t)})^2 \\ \frac{Z_4(t)}{1+Z_1(t)} \end{bmatrix} \tag{54}$$

the plant (50)–(53) can be converted into

$$\dot{X}_1(t) = X_2(t) \tag{55}$$

$$\dot{X}_2(t) = U(t-D) \tag{56}$$

$$\dot{X}_3(t) = X_4(t) \tag{57}$$

$$\dot{X}_4(t) = X_1(t). \tag{58}$$

Denote

$$A = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 0 \end{bmatrix}, \ B_1 = \begin{bmatrix} 0 \\ 1 \\ 0 \\ 0 \end{bmatrix}, \tag{59}$$

we have

$$e^{At} = \begin{bmatrix} 1 & t & 0 & 0 \\ 0 & 1 & 0 & 0 \\ \frac{t^2}{2} & \frac{t^3}{6} & 1 & t \\ t & \frac{t^2}{2} & 0 & 1 \end{bmatrix}. \tag{60}$$

By (54), we have

$$Z(t) = f^{-1}(X(t)) = \begin{bmatrix} X_1(t) \\ X_2(t) \\ X_3(t) + \frac{1}{2}X_3(t)^2 \\ X_4(t)(1+X_1(t)) \end{bmatrix}. \tag{61}$$

Choosing

$$k = [-9, \ -5, \ -2, \ -7] \tag{62}$$

then $A + B_1 k$ is a hurwitz matrix. Using Theorem 3.1, the control law

$$U(t) = \frac{c}{c+1} U_1(t) = U^*(t) \tag{63}$$

where $c > 0$ is sufficiently large, and

$$U_1(t) = ke^{AD} f(Z(t)) + k \int_{t-D}^{t} e^{A(t-\theta)} B_1 U(\theta)\,\mathrm{d}\theta \tag{64}$$

asymptotically stabilizes system (50)–(53). Solving the matrix equation

$$\begin{bmatrix} 0 & 1 & 0 & 0 \\ -9 & -5 & -2 & -7 \\ 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 0 \end{bmatrix}^T P + P \begin{bmatrix} 0 & 1 & 0 & 0 \\ -9 & -5 & -2 & -7 \\ 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 0 \end{bmatrix} = \begin{bmatrix} -1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & -1 \end{bmatrix} \tag{65}$$

we get

$$P = \begin{bmatrix} 3.5833 & 0.4722 & 1.6389 & 3.7500 \\ 0.4722 & 0.1944 & 0.2500 & 0.5278 \\ 1.6389 & 0.2500 & 2.8056 & 3.1944 \\ 3.7500 & 0.5278 & 3.1944 & 6.4167] \end{bmatrix}. \tag{66}$$

By calculating, one has $a_1 = 0.1505$, and

$$\begin{aligned} V(t) = {} & \begin{bmatrix} Z_1(t) \\ Z_2(t) \\ Z_3(t) - \frac{1}{2}(\frac{Z_4(t)}{1+Z_1(t)})^2 \\ \frac{Z_4(t)}{1+Z_1(t)} \end{bmatrix}^T \\ & \times \begin{bmatrix} 3.5833 & 0.4722 & 1.6389 & 3.7500 \\ 0.4722 & 0.1944 & 0.2500 & 0.5278 \\ 1.6389 & 0.2500 & 2.8056 & 3.1944 \\ 3.7500 & 0.5278 & 3.1944 & 6.4167] \end{bmatrix} \\ & \times \begin{bmatrix} Z_1(t) \\ Z_2(t) \\ Z_3(t) - \frac{1}{2}(\frac{Z_4(t)}{1+Z_1(t)})^2 \\ \frac{Z_4(t)}{1+Z_1(t)} \end{bmatrix} + \frac{0.1505}{2} \int_0^D e^x w^2(x,t)\,\mathrm{d}x, \end{aligned} \tag{67}$$

$$\begin{aligned} L(t) = {} & -\frac{0.1505\beta e^D}{c+1} U_1(t)^2 + 2\beta Z_1(t)^2 + 2\beta Z_2(t)^2 + 2\beta \left(Z_3(t) - \frac{1}{2} \left(\frac{Z_4(t)}{1+Z_1(t)} \right)^2 \right)^2 \\ & + 2\beta \left(\frac{Z_4(t)}{1+Z_1(t)} \right)^2 - 4\beta \left(0.4722 Z_1(t) + 0.1944 Z_2(t) + 0.25 \left(Z_3(t) - \frac{1}{2} (\frac{Z_4(t)}{1+Z_1(t)})^2 \right) \right. \\ & \left. + 0.5278 \frac{Z_4(t)}{1+Z_1(t)} \right) w(0,t) + 0.1505\beta w^2(0,t) + 0.1505\beta \int_0^D e^x w^2(x,t)\,\mathrm{d}x. \end{aligned} \tag{68}$$

By Theorem 4.1, the control law (63), (64) minimizes the cost functional

$$J = \lim_{t\to\infty} \left(2\beta V(t) + \int_0^t \left(L(\tau) + \frac{0.1505\beta e^D}{c} U^2(\tau) \right) \mathrm{d}\tau \right) \tag{69}$$

for arbitrary $\beta > 0$ and $c > 0$ is sufficiently large and $V(t)$, $L(t)$ are given by (67), (68), respectively.

Responses of the states and the control law of the closed-loop system (50)–(53), (63), (64) are shown for $D = 3, c = 100$ in Fig.1. One can observe that the closed-loop system is asymptotically stable and the control law (63), (64) is inverse optimal with respect to the cost functional (69).

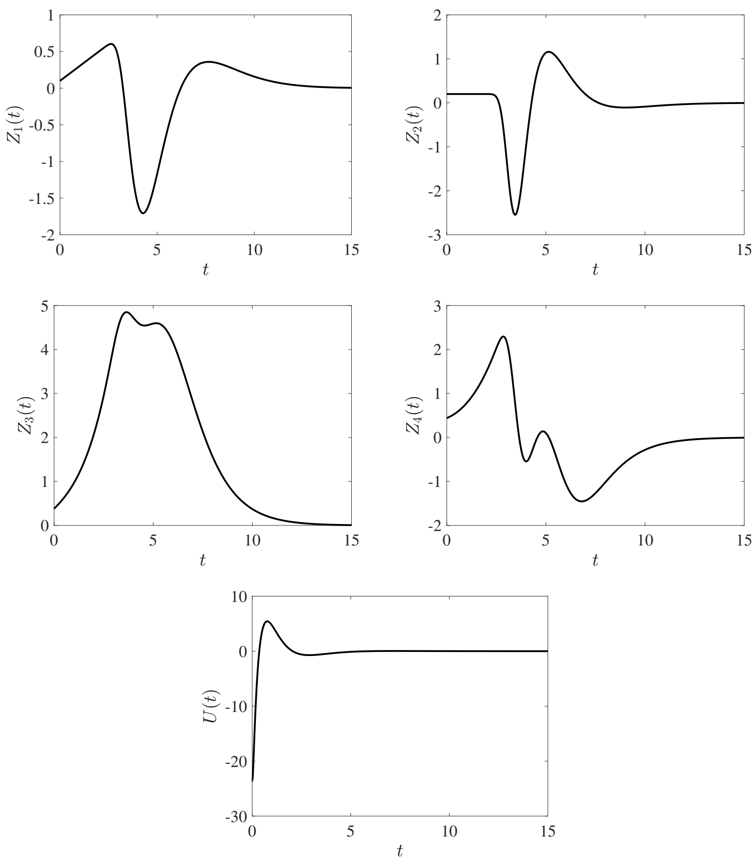

Fig. 1. Responses of the states $Z_1(t), Z_2(t), Z_3(t), Z_4(t)$ and the control law $U(t)$ of the closed-loop system (50)–(53),(63), (64) for initial conditions $Z_1(0) = 0.1, Z_2(0) = 0.2, Z_3(0) = 1.1, Z_4(0) = 0.44$ and $U(\theta) = 0$, for $\theta \in [0, 3]$.

6. CONCLUSIONS

We consider inverse optimal control design for linearizable nonlinear systems with input delays based on predictor control. A nonlinear system is transferred to a linear system with a continuously reversible coordinate change. We show that the basic predictor control is inverse optimal with respect to a meaningful differential game. A mechanical system is given to illustrate the validity of the proposed method.

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