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Abstract. We introduce new concept of almost demi Dunford–Pettis operators. Let E be a Banach lattice. An operator T from E into E is said to be almost demi Dunford–Pettis if, for every sequence $\{x_n\}$ in E_+ such that $x_n \to 0$ in $\sigma(E,E')$ and $\|x_n - Tx_n\| \to 0$ as $n \to \infty$, we have $\|x_n\| \to 0$ as $n \to \infty$. In addition, we study some properties of this class of operators and its relationships with others known operators.

Keywords: almost demi Dunford–Pettis operator; Banach lattice; positive Schur property

Classification: 46A40, 46B40, 46B42

1. Introduction

Throughout this paper X and Y will denote real Banach spaces, E and F will denote real Banach lattices. Let $\mathcal{B}_X$ be the closed unit ball of X and $\operatorname{sol}(A)$ denote the solid hull of a subset A of a Banach lattice. The positive cone of E will be denoted by $E_+ = \{x \in E\colon\ 0 \le x\}$.

W. Wnuk in [12] introduced the class of almost Dunford–Pettis operators. Let us recall an operator T from a Banach lattice E into a Banach space Y is said to be almost Dunford–Pettis, if the sequence $\{\|Tx_n\|\}$ converges to 0 for every weakly null sequence $\{x_n\}$ consisting of pairwise disjoint elements in E. After that, B. Aqzzouz and A. Elbour in [3] established an important characterization of this class in terms of positive weakly null sequences. More precisely, they proved that an operator $T\colon E \longrightarrow Y$ is almost Dunford–Pettis, if $\|Tx_n\| \to 0$ for every sequence $\{x_n\} \subset E_+$ such that $x_n \to 0$ in $\sigma(E,E')$ as $n \to \infty$, see [3, Theorem 2.2].

Recall from [11] that an operator $T\colon \mathcal{D}(T) \subseteq X \longrightarrow X$, where $\mathcal{D}(T)$ is a subspace of X, is said to be demicompact if for every bounded sequence (x_n) in the domain $\mathcal{D}(T)$ such that $(x_n - Tx_n)$ converges to $x \in X$, there is a convergent subsequence of (x_n). Note that each compact operator is demicompact, but the opposite is not always true. In fact, let $\mathrm{Id}_X\colon X \longrightarrow X$ be the identity operator

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of a Banach space X of infinite dimension. It is clear that $-\mathrm{Id}_X$ is demicompact but it is not compact. The concept of demicompactness has emerged in literature since 1966 in order to address fixed points. It was introduced by W. V. Petryshyn in [11].

Recall from [7] that an operator $T\colon X \longrightarrow X$ is said to be demi Dunford–Pettis (DDP in short), if for every sequence $\{x_n\}$ in X such that $x_n \to 0$ in $\sigma(X, X')$ and $\|x_n - Tx_n\| \to 0$ as $n \to \infty$, we have $\|x_n\| \to 0$ as $n \to \infty$. In [7, Theorem 2.4], the authors showed that each operator T from X into X is demi Dunford–Pettis if and only if X has the Schur property.

Recently, a series of papers introduced some operators on Banach lattices involving demi criteria. More precisely, order weakly demicompact operators, see [5], L-weakly and M-weakly demicompact operators, see [6], and B-weakly demicompact operators, see [8].

The purpose of this work is to pursue this analysis in order to define a new class of operators on Banach lattices related to the class of almost Dunford–Pettis that we call almost demi Dunford–Pettis.

This paper is organized in the following way. In Section 2, we shall introduce a new concept of almost demi Dunford–Pettis operators, see Definition 2.1. Note that the class of almost demi Dunford–Pettis operators involves that of almost Dunford–Pettis operators, see Proposition 2.2. Subsequently, we shall illustrate our analysis by some outstanding properties, see Theorems 2.4 and 2.9. In Section 3, we concentrate to solve the following problem "almost demi Dunford–Pettis operators and their relationships with other known operators" such as demi Dunford–Pettis operators, see Propositions 3.1 and 3.3, order weakly demicompact operators, Proposition 3.4 and Corollary 3.6, and L-weakly and M-weakly demicompact operators, see Propositions 3.8, 3.9 and Corollary 3.10.

To state our results, we need to fix some notations and recall some definitions. A vector lattice E is an ordered vector space in which $\sup(x, y)$ exists for every $x, y \in E$. Let E be a vector lattice for each $x, y \in E$ with $x \leq y$, the set $[x, y] = \{z \in E\colon\ x \leq z \leq y\}$ is called an order interval. A subset of E is said to be order bounded if it is included in some order interval. A vector $e \in E_+$ is said to be an order unit, if for each $x \in E$ there exists some $\lambda \geq 0$ such that $|x| \leq \lambda e$. A Banach lattice is a Banach space $(E, \|\cdot\|)$ such that E is a vector lattice and its norm satisfies the following property: for each $x, y \in E$ such that $|x| \leq |y|$, we have $\|x\| \leq \|y\|$. If E is a Banach lattice, its topological dual E', endowed with the dual norm and the dual order, is also a Banach lattice. A norm $\|\cdot\|$ of a Banach lattice E is order continuous if for each generalized sequence $\{x_\alpha\}$ such that $x_\alpha \downarrow 0$ in E, the sequence $\{x_\alpha\}$ converges to 0 for the norm $\|\cdot\|$, where the notation $x_\alpha \downarrow 0$ means that the sequence $\{x_\alpha\}$ is decreasing, its infimum exists and $\inf\{x_\alpha\} = 0$. A Banach lattice E is said to be an AM-space if for each $x, y \in E$ such that $\inf\{x, y\} = 0$, we have $\|x + y\| = \max\{\|x\|, \|y\|\}$. A Banach lattice E has the following properties:

- The Schur property if every weakly convergent sequence to 0 in E is norm convergent to zero.
- The positive Schur property if each weakly null sequence with positive terms in E converges to zero in norm.
- The order positive Schur property if every order bounded weakly convergent sequence to 0 in E_+ is norm convergent to zero, see [5, Definition 2.2].
- The weakly sequentially continuous lattice operations whenever $x_n \to 0$ in $\sigma(E, E')$ implies $|x_n| \to 0$ in $\sigma(E, E')$. We use the term operator $T\colon E \longrightarrow F$ between two Banach lattices to mean a bounded linear mapping. It is positive if $T(x) \geq 0$ in F whenever $x \in E_+$.

We refer the reader to the monographs [2], [10] for ambiguous terminology from Banach lattices and positive operators theory.

2. Definition and properties

We start this section by the following definition.

Definition 2.1. Let E be a Banach lattice. An operator $T\colon E \longrightarrow E$ is said to be almost demi Dunford–Pettis if for every sequence $\{x_n\}$ in E_+ such that $x_n \to 0$ in $\sigma(E, E')$ and $\|x_n - Tx_n\| \to 0$ as $n \to \infty$, we have $\|x_n\| \to 0$ as $n \to \infty$.

The next proposition gives examples of almost demi Dunford–Pettis operators.

Proposition 2.2. *Let E be a Banach lattice. The following operators are almost demi Dunford–Pettis:*

(1) $\alpha \mathrm{Id}_E$ *for all* $\alpha \neq 1$ *such that* $\mathrm{Id}_E\colon E \longrightarrow E$ *be the identity operator;*
(2) *almost Dunford–Pettis operators* $T\colon E \longrightarrow E$.

Proof: (1) Let $\{x_n\}$ be a sequence in E_+ such that $x_n \to 0$ in $\sigma(E, E')$ and $\|(1 - \alpha)x_n\| \to 0$ as $n \to \infty$. This implies that $\|x_n\| \to 0$ as $n \to \infty$ and the proof of the assertion is finished.

(2) Let $\{x_n\}$ be a sequence in E_+ such that $x_n \to 0$ in $\sigma(E, E')$ and $\|x_n - Tx_n\| \to 0$ as $n \to \infty$. Since T is almost Dunford–Pettis, we obtain $\|Tx_n\| \to 0$. From the following inequality

$$\|x_n\| \leq \|x_n - Tx_n\| + \|Tx_n\|$$

for each n, it follows that $\|x_n\| \to 0$ as $n \to \infty$. □

Remark 2.3. (i) In general, the assertion (1) in Proposition 2.2 is not true in case $\alpha = 1$. For example, let Id_{l^2} be the identity operator of l^2. Consider $\{e_n\}$ to be the sequence in l^2 where e_n is the sequence with the nth entry equals 1 and others are zero. So that the sequence $\{e_n\}$ is weakly null and $\|e_n - \mathrm{Id}_{l^2} e_n\| = 0$. Note that $\|e_n\| = 1$. This proves that the norm of $\{e_n\}$ does not converge to zero. Hence Id_{l^2} is not almost demi Dunford–Pettis.

(ii) The converse of the assertion (2) in Proposition 2.2 is false in general. Take $E = c$, the Banach lattice of all convergent sequences and Id_c to be the identity operator from c into itself. Clearly $-\mathrm{Id}_c$ is almost demi Dunford–Pettis, see Proposition 2.2 (1). On the other side, since c does not have the positive Schur property, $-\mathrm{Id}_c$ is not almost Dunford–Pettis, see [3, Theorem 2.2].

The following result gives a new characterization of positive Schur property in terms of almost demi Dunford–Pettis identity operator.

Theorem 2.4. *Let E be a Banach lattice, then the following assertions are equivalent:*

(1) *Each operator $T\colon E \longrightarrow E$ is almost demi Dunford–Pettis.*
(2) *The identity operator of E is almost demi Dunford–Pettis.*
(3) *The Banach lattice E has the positive Schur property.*

Proof: (1) $\Longrightarrow$ (2) Obvious.

(2) $\Longrightarrow$ (3) We have to show that $\|x_n\| \to 0$ as $n \to \infty$ for every $\{x_n\}$ sequence of E_+ such that $x_n \to 0$ in $\sigma(E, E')$. Let $\{x_n\}$ be such a sequence. It is clear that $\|x_n - \mathrm{Id}_E x_n\| = 0$. The fact that the identity operator $\mathrm{Id}_E\colon E \longrightarrow E$ is almost demi Dunford–Pettis, it follows that $\|x_n\| \to 0$ as $n \to \infty$.

(3) $\Longrightarrow$ (1) Since E has the positive Schur property, it follows from [3, Theorem 2.2] that T is almost Dunford–Pettis. Hence Proposition 2.2 implies that T is almost demi Dunford–Pettis. □

Remark 2.5. (i) It follows from Theorem 2.4 that the topological dual E' has the order positive Schur property if and only if its identity operator is almost demi Dunford–Pettis.

(ii) There exists an operator from a Banach lattice E into E that is almost demi Dunford–Pettis however E does not have the positive Schur property. In fact, if we take $E = c_0$, then $-\mathrm{Id}_E\colon E \longrightarrow E$ is almost demi Dunford–Pettis, see Proposition 2.2 (1), but c_0 does not have the positive Schur property.

Note that the class of almost demi Dunford–Pettis operators lacks the vector space structure with the sum and with the external product.

Example 2.6. Consider $E = c$ and $\mathrm{Id}_E\colon E \longrightarrow E$ be the identity operator. Clearly, $T = 2\mathrm{Id}_E$ and $S = -\mathrm{Id}_E$ are almost demi Dunford–Pettis operators, see

Proposition 2.2 (1). But, since c does not have the positive Schur property, then the operators $T+S=\mathrm{Id}_E$ and $-S=\mathrm{Id}_E$ are not, see Theorem 2.4.

The following proposition asserts that an almost Dunford–Pettis perturbation of an almost demi Dunford–Pettis is almost demi Dunford–Pettis operator.

Proposition 2.7. *Let $T\colon E\longrightarrow E$ be an almost demi Dunford–Pettis operator. If $S\colon E\longrightarrow E$ is almost Dunford–Pettis, then the operator $T+S$ is almost demi Dunford–Pettis.*

PROOF: Let $\{x_n\}$ be a sequence in E_+ such that $x_n\to 0$ in $\sigma(E,E')$ and $\|x_n-(T+S)x_n\|\to 0$ as $n\to\infty$. Since S is almost Dunford–Pettis from [3, Theorem 2.2], we infer that $\|Sx_n\|\to 0$ as $n\to\infty$. From the following inequality

$$\|x_n-Tx_n\|=\|x_n-Tx_n-Sx_n+Sx_n\|\le\|x_n-(T+S)x_n\|+\|Sx_n\|,$$

then $\|x_n-Tx_n\|\to 0$ as $n\to\infty$. Since T is almost demi Dunford–Pettis, we obtain $\|x_n\|\to 0$ as $n\to\infty$. This implies that $T+S$ is almost demi Dunford–Pettis. □

Remark 2.8. Note that the set of all almost demi Dunford–Pettis is not norm closed in $\mathbb{L}(E)$. In fact, consider $T_n=((n+1)/n)\mathrm{Id}_{l^2}$ for each $n\in\mathbb{N}^*$ where Id_{l^2} be the identity operator on l^2. Clearly $\{T_n\}$ is a sequence of almost demi Dunford–Pettis operators, see Proposition 2.2 (1). On the other side, from the following equalities:

$$\|T-\mathrm{Id}_{l^2}\|=\Big\|\frac{n+1}{n}\,\mathrm{Id}_{l^2}-\mathrm{Id}_{l^2}\Big\|=\Big\|\frac{2}{n}\,\mathrm{Id}_{l^2}\Big\|=\frac{2}{n},$$

we obtain $\lim_{n\to\infty}\|T-\mathrm{Id}_{l^2}\|=0$. Thus, $\{T_n\}$ is norm convergent to Id_{l^2}. As l^2 does not have the positive Schur property, it follows from Theorem 2.4 that Id_{l^2} is not almost demi Dunford–Pettis.

Contrarily to almost Dunford–Pettis operators, see [3, Corollary 2.3], the domination problem for positive almost demi Dunford–Pettis operators has in general a negative answer. In fact, consider $E=c_0$. Let S and T be two operators on E such that $S=\mathrm{Id}_0$ and $T=2\mathrm{Id}_0$. Clearly $0\le S\le T$. From Proposition 2.2 (1), T is almost demi Dunford–Pettis. But, S is not almost demi Dunford–Pettis because c_0 does not have the positive Schur property, see Theorem 2.4.

Let us recall that an operator $T\colon E\longrightarrow E$ is called central if it is dominated by a multiple of the identity operator, that is, T is a central operator if and only if there exists some scalar $\lambda>0$ such that $|Tx|\le\lambda|x|$ holds for all $x\in E$, see [1]. Similarly to [7, Theorem 3.11], we obtain that the dominance problem for central almost demi Dunford–Pettis operators is valid.

Theorem 2.9. *Let $S, T\colon E \longrightarrow E$ be two positive operators on a Banach lattice such that $0 \leq S \leq T \leq I$ holds. If T is an almost demi Dunford–Pettis operator, then S is almost demi Dunford–Pettis.*

As the set of almost Dunford–Pettis operators [4], the class of almost demi Dunford–Pettis operators does not satisfy the duality property, that is, there exist almost demi Dunford–Pettis operators whose adjoints are not almost demi Dunford–Pettis. In fact, the identity operator of the Banach lattice $L^1[0,1]$ is almost demi Dunford–Pettis, but its adjoint, which is the identity operator of the Banach lattice $L^\infty[0,1]$, is not almost demi Dunford–Pettis. And conversely, there exist operators that are not almost demi Dunford–Pettis but their adjoints are almost demi Dunford–Pettis. In fact, the identity operator of the Banach lattice c_0 is not almost demi Dunford–Pettis but its adjoint, which is the identity operator of the topological dual l^1, is almost demi Dunford–Pettis.

3. Relationships with some other operators

In this section, we study some relationships of almost demi Dunford–Pettis operators with other operators on Banach lattices.

First of all, we give the obvious relation, the class of almost demi Dunford–Pettis operators contains that of demi Dunford–Pettis operators. It is an easy to obtain this by using their definitions.

Proposition 3.1. *Let E be a Banach lattice. Every demi Dunford–Pettis operator $T\colon E \longrightarrow E$ is almost demi Dunford–Pettis.*

Remark 3.2. It is worth noting that the converse of Proposition 3.1 is false in general. In fact, the operator $\mathrm{Id}_{L^1([0,1])}$ is almost demi Dunford–Pettis because the Banach lattice $L^1([0,1])$ has the positive Schur property, see Theorem 2.4. But, since $L^1([0,1])$ does not have the Schur property and in view of [7, Theorem 2.4], it is not demi Dunford–Pettis.

Recall from [1] that an operator $T\colon E \longrightarrow E$ is called central if it is dominated by a multiple of the identity operator. That is, T is a central operator if and only if there exists some scalar, $\lambda > 0$ such that $|Tx| \leq \lambda |x|$ holds for all $x \in E$. The collection of all central operators is denoted by $\mathcal{Z}(E)$. The following result proves under which condition the classes of demi Dunford–Pettis operators coincide with the class of almost demi Dunford–Pettis operators.

Proposition 3.3. *Let E be a Banach lattice such that its lattice operations are weakly sequentially continuous. Consider $T\colon E \longrightarrow E$ be an operator such that $I - T \in \mathcal{Z}(E)$ and $I - T \geq 0$. If T is almost demi Dunford–Pettis, then T is a demi Dunford–Pettis.*

Proof: Let $\{x_n\}$ be a sequence in E such that $x_n \to 0$ in $\sigma(E,E')$ and $\|x_n - Tx_n\| \to 0$ as $n \to \infty$. We have to show that $\|x_n\| \to 0$ as $n \to \infty$. Since the lattice operations in E are weakly sequentially continuous, then $|x_n| \to 0$ in $\sigma(E,E')$. By hypothesis $I - T \in \mathcal{Z}(E)$ and in view of [1, Theorem 3.30], we obtain

$$|x_n - Tx_n| = |(I-T)(|x_n|)| = |I-T|(|x_n|)|$$

for all n. Since $I - T \geq 0$, $|I-T|(|x_n|)| = (I-T)(|x_n|)$ for all n. Therefore, $\|\,|x_n| - T|x_n|\,\| \to 0$ as $n \to \infty$. The fact that T is almost demi Dunford–Pettis, we obtain that $\|\,|x_n|\,\| = \|x_n\| \to 0$ as $n \to \infty$. Hence, T is a demi Dunford–Pettis operator. □

Recall from [5] that an operator T from E into E is said to be order weakly demicompact if, for every order bounded sequence $\{x_n\}$ in E_+ such that $x_n \to 0$ in $\sigma(E,E')$ and $\|x_n - Tx_n\| \to 0$ as $n \to \infty$, we have $\|x_n\| \to 0$ as $n \to \infty$. In [5, Theorem 2.1], H. Benkhaled et al. proved that each operator T from E into E is order weakly demicompact if and only if E has the order positive Schur property. Our following result proves that the class of order weakly demicompact operators includes that of almost demi Dunford–Pettis operators.

Proposition 3.4. *Let E be a Banach lattice. Every almost demi Dunford–Pettis operator $T\colon E \longrightarrow E$ is order weakly demicompact.*

Proof: Let T be an operator and $\{x_n\}$ be an order bounded sequence of E_+ such that $x_n \to 0$ in $\sigma(E,E')$ and $\|x_n - Tx_n\| \to 0$ as $n \to \infty$. By the fact that T is almost demi Dunford–Pettis, $\|x_n\| \to 0$ as $n \to \infty$ and the proof is finished. □

There exists an order weakly demicompact operator from a Banach lattice E into E that is not almost demi Dunford–Pettis. In fact:

Example 3.5. Let $E = c_0$ and $\mathrm{Id}_E\colon E \longrightarrow E$ be the identity operator. Since c_0 has the order positive Schur property, then Id_E is order weakly demicompact, see [5, Theorem 2.1]. On the other hand, c_0 does not have the positive Schur property. Hence, Id_E is not almost demi Dunford–Pettis, see Theorem 2.4.

Recall that a Banach lattice is said to be an AM-space with unit if an addition to being an AM-space, E also has an order unit. If Banach lattice E is an AM-space with unit, then the class of almost demi Dunford–Pettis operators on E coincides with that of order weakly demicompact operators on E.

Corollary 3.6. *Let E be an AM-space with unit, then each order weakly demicompact operator from E into E is almost demi Dunford–Pettis.*

Proof: Let $\{x_n\}$ be a sequence in E_+ such that $x_n \to 0$ in $\sigma(E,E')$ and $\|x_n - Tx_n\| \to 0$ as $n \to \infty$. Since E is AM-space with unit so from [2, Theorem 12.20] its closed unit ball is like an order interval. Then we have $\{x_n\}$ is order bounded in E_+. In view of T being order weakly demicompact, we can deduce that $\|x_n\| \to 0$ as $n \to \infty$. □

Recall [6] that an operator $T\colon E \longrightarrow E$ is called M-weakly demicompact if for every norm bounded disjoint sequence (x_n) in E such that $\|x_n - Tx_n\| \to 0$, we have $\|x_n\| \to 0$.

There exist operators which are not almost demi Dunford–Pettis nor M-weakly demicompact. In fact, the identity operator $\mathrm{Id}_{L^2([0,1])}\colon L^2([0,1]) \longrightarrow L^2([0,1])$ is not almost demi Dunford–Pettis nor M-weakly demicompact. Also, since the Banach space $L^1([0,1])$ admits the positive Schur property, its identity operator $\mathrm{Id}_{L^1([0,1])}\colon L^1([0,1]) \longrightarrow L^1([0,1])$ is almost demi Dunford–Pettis but not M-weakly demicompact.

To give a condition under which the classes of demi almost Dunford–Pettis operators coincide with the class of M-weakly demicompact operators, we need to present the following theorem:

Theorem 3.7 ([6, Theorem 5.9]). *Let E be a Banach lattice such that both E and E' have order continuous norms, and $T\colon E \longrightarrow E$ be an operator. Then, the following assertions are equivalent:*

(1) *T is M-weakly demicompact.*
(2) *For each norm bounded sequence $\{x_n\}$ in E_+ such that $x_n \to 0$ in $\sigma(E, E')$ and $\|x_n - Tx_n\| \to 0$, we have $\|x_n\| \to 0$.*

Proposition 3.8. *Let E be a Banach lattice such that both E and E' have order continuous norms, and $T\colon E \longrightarrow E$ be an operator. Then, the following assertions are equivalent:*

(1) *T is M-weakly demicompact.*
(2) *T is almost demi Dunford–Pettis.*

Proof: This follows from Definition 2.1 and Theorem 3.7. □

Let A be a nonempty bounded subset of a Banach lattice E. Then A is said to be L-weakly compact if $\lim \|x_n\| = 0$ for every disjoint sequence $\{x_n\}$ contained in the solid hull of A. Recall from [6] that an operator $T\colon E \longrightarrow E$ is called L-weakly demicompact if for every norm bounded sequence (x_n) in $\mathcal{B}_E$ such that $\{x_n - Tx_n\colon n \in \mathbb{N}\}$ is L-weakly compact subset of E, we have $\{x_n\colon n \in \mathbb{N}\}$ is L-weakly compact subset of E. To prove our next result, we need to establish the following proposition:

Proposition 3.9 ([6, Theorem 5.8]). *Let E be a Banach lattice. Every L-weakly demicompact operator $T\colon E \longrightarrow E$ is M-weakly demicompact.*

Corollary 3.10. *Let E be a Banach lattice such that both E and E' have order continuous norms. Every L-weakly demicompact operator $T\colon E \longrightarrow E$ is almost demi Dunford–Pettis.*

PROOF: In view of Propositions 3.8 and 3.9, we can deduce the result. □

We end this section by presenting the relationship between strictly singular operators and almost demi Dunford–Pettis operators. Recall that an operator $T\colon X \longrightarrow Y$ is said to be strictly singular if there is no infinite dimensional subspace M of X such that the restriction of T to M is an isomorphism. Strictly singular operators were introduced by Y. A. Abramovich, and C. D. Aliprantis in [1].

Proposition 3.11. *Every strictly singular operator $T\colon E \longrightarrow E$ is almost demi Dunford–Pettis.*

PROOF: Suppose that T is not almost demi Dunford–Pettis. Then there exists a weakly null sequence $\{x_n\}$ of E_+ such that $\|x_n - Tx_n\| \to 0$ as $n \to \infty$ and $\|x_n\| = 1$ for all $n \in \mathbb{N}$. Thus, by [9, Theorem I.1.10], $\{x_n\}$ has a basic subsequence $\{y_n\}$ with basic constant K. By passing to a subsequence we can assume that

$$\sum_{n=1}^{\infty} \|y_n - Ty_n\| < \infty.$$

Next, according to [9, Proposition I.1.15], there exists a subsequence $\{y_{n_i}\}$ of $\{y_n\}$ equivalent to $\{Ty_{n_i}\}$. From [9, Theorem I.1.13], we have

$$\left\| \sum_{i=1}^{\infty} \alpha_i T y_{n_i} \right\| \geq C \left\| \sum_{i=1}^{\infty} \alpha_i y_{n_i} \right\|$$

for some $C > 0$. Hence, T is not strictly singular. □

References

[1] Abramovich Y. A., Aliprantis C. D., *An Invitation to Operator Theory*, Graduate Studies in Mathematics, 50, American Mathematical Society, Providence, 2002.

[2] Aliprantis C. D., Burkinshaw O., *Positive Operators*, Pure and Applied Mathematics, 119, Academic Press, Orlando, 1985.

[3] Aqzzouz B., Elbour A., *Some characterizations of almost Dunford–Pettis operators and applications*, Positivity **15** (2011), no. 3, 369–380.

[4] Aqzzouz B., Elbour A., Wickstead A. W., *Positive almost Dunford–Pettis operators and their duality*, Positivity **15** (2011), no. 2, 185–197.

[5] Benkhaled H., Elleuch A., Jeribi A., *The class of order weakly demicompact operators*, Rev. R. Acad. Cienc. Exactas Fís. Nat. Ser. A Mat. RACSAM **114** (2) (2020), no. 2, Paper No. 76, 8 pages.

[6] Benkhaled H., Hajji M., Jeribi A., *L-weakly and M-weakly demicompact operators on Banach lattices*, Filomat **36** (2022), no. 13, 4319–4329.

[7] Benkhaled H., Hajji M., Jeribi A., *On the class of demi Dunford–Pettis operators*, Rend. Circ. Mat. Palermo Ser. (2) **72** (2022), no. 2, 901–911.

[8] Benkhaled H., Jeribi A., *On B-weakly demicompact operators on Banach lattices*, Vladikavkaz. Mat. Zh. **25** (2023), no. 4, 20–28.

[9] Guerre-Delabrière S., *Classical Sequences in Banach Spaces*, Monographs and Textbooks in Pure and Applied Mathematics, 166, Marcel Dekker, New York, 1992.

[10] Meyer-Nieberg P., *Banach Lattices*, Universitext, Springer, Berlin, 1991.

[11] Petryshyn W. V., *Construction of fixed points of demicompact mappings in Hilbert space*, J. Math. Anal. Appl. **14** (1966), 276–284.

[12] Wnuk W., *Banach lattice with weak Dunford–Pettis property*, Atti Sem. Mat. Fis. Univ. Modena **42** (1994), no. 1, 227–236.

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