Applications of Mathematics

Vít Dolejší; Michal Křížek; Jan Zeman

Special issue dedicated to Professor Ivo Babuška, the founder of the journal Applications of Mathematics

*Applications of Mathematics*, Vol. 69 (2024), No. 5, 541–544

Persistent URL: http://dml.cz/dmlcz/152628

**Terms of use:**

© Institute of Mathematics CAS, 2024

Institute of Mathematics of the Czech Academy of Sciences provides access to digitized documents strictly for personal use. Each copy of any part of this document must contain these *Terms of use*.




This document has been digitized, optimized for electronic delivery and stamped with digital signature within the project *DML-CZ: The Czech Digital Mathematics Library* http://dml.cz

# SPECIAL ISSUE DEDICATED TO PROFESSOR IVO BABUŠKA, THE FOUNDER OF THE JOURNAL APPLICATIONS OF MATHEMATICS

## EDITORIAL

Vít Dolejší, Michal Křížek, Jan Zeman, Praha

This issue of Applications of Mathematics is dedicated to the memory of Prof. Ivo Babuška, the founder of this journal, see [2], [3]. He was an excellent mathematician and civil engineer highly recognized all over the world. He made numerous groundbreaking contributions to applied mathematics, numerical analysis, and computational mechanics. After a very fruitful professional carrier, when he supervised about 40 PhD students, Ivo Babuška died at the age of 97 on April 12, 2023, see [1]. To remind his memory, we asked several of his former PhD students Z. Zhang, T. Tsuchiya, T. Strouboulis, K. Segeth, L. Demkowicz, J. M. Melenk, and C. Schwab to contribute to this special issue of Applications of Mathematics.

During the development of the finite element method it has been found that the rate of convergence of finite element approximations at some exceptional points in the domain exceeds the optimal global rate if finite element partitions have some regular geometric structure. This phenomenon has come to be known as *superconvergence*. Ivo Babuška made a lot of significant contributions to this topic. There is a very rich literature on superconvergence phenomena in the finite element method for solving elliptic and parabolic problems. However, only a few papers deal with the spectral volume method. The first paper of this issue written by Xu Yin, Waixiang Cao, and Zhimin Zhang focuses on superconvergence analysis of this numerical method for solving evolution problems described by one-dimensional diffusion and third-order wave equations with periodic boundary conditions. This topic is new. The authors prove several nontrivial superconvergence error estimates. In particular, they show that for the spectral volume method the errors of numerical fluxes at nodes and the cell averages are superconvergent with orders of $\mathcal{O}(h^{2k+1})$ and $\mathcal{O}(h^{2k})$, respectively.

DOI: 10.21136/AM.2024.0185-24 

© Institute of Mathematics, Czech Academy of Sciences 2024.

In the second paper, Kenta Kobayashi and Takuya Tsuchiya deal with finite element solution of the Poisson equation with homogeneous Dirichlet boundary conditions on bounded polygonal or polyhedral domains. Assuming the $H^2$-regularity of the true solution, they derive optimal error estimates on meshes that contain thin elements. For linear simplicial elements they prove linear convergence rate in the $H^1$-norm provided some elements degenerate. More precisely, they require that all "bad simplices" can be virtually covered by good simplices. The proof of the main theorem is highly non-trivial and generalizes the well-known result on the maximum angle condition by Babuška and Aziz from SIAM J. Numer. Anal. 1976. Numerical results for the Poisson equation with the 4th degree polynomial solution on the unit square nicely illustrate the theoretical findings.

In the third paper, Theofanis Strouboulis and Delin Wang derive guaranteed a posteriori error estimation for fully discrete solutions of parabolic problems based on their elliptic reconstructions. The authors deal with finite element solution of a nonstationary heat conduction problem with mixed boundary conditions on a bounded polygonal domain. They recover a fully discrete solution of this parabolic problem by means of elliptic reconstructions on each discrete time level. They prove that reconstructed solutions are superconvergent, i.e., they produce a better approximation order of the exact solution. This superconvergence property is illustrated by a large amount of numerical tests and extends former joint results on superconvergence by Babuška and Strouboulis.

In the fourth paper, Karel Segeth is concerned with the measurement of scalar physical quantities at nodes on a sphere in the 3D Euclidean space and the spherical radial basis function interpolation of the data acquired. A multiquadric is employed as the radial basis function and the corresponding trend is a polynomial of degree 2 considered in Cartesian coordinates. Recently, problems formulated on spheres have brought an increasing interest of researchers through the development of measurements on the Earth surface. Satellites are usually the means of acquiring such data. This paper is devoted to some ways of approximating and mapping measured physical quantities exhibiting anisotropy that can be expressed by means of a second-order tensor. A typical example employed is the measurement of magnetic susceptibility of rock having extensive applications in geosciences. The existence of an interpolation formula of the type considered is shown, and attention is paid to the choice of geodesic functions that are used in these formulas and define the distance between two nodes on a sphere. The advantages and drawbacks of the formulas chosen are demonstrated, in particular the strong dependence of interpolation results on the condition number of the matrix of the system considered and on round-off errors in general.

In the fifth paper, Leszek Demkowicz, Jay Gopalakrishnan, and Norbert Heuer investigate a model two-dimensional acoustic waveguide with lateral impedance bound-

ary conditions (and outgoing boundary conditions at the waveguide outlet). The governing operator is proved to be bounded below with a stability constant inversely proportional to the length of the waveguide. The presence of impedance boundary conditions leads to a non self-adjoint operator which considerably complicates the analysis. The main goal is to elucidate these complications and find tools that are applicable, as simply as possible. This paper is a continuation of two 2023 papers by L. Demkowicz, J. M. Melenk et al. on electromagnetic waveguides, and provides a stepping stone to the analysis of open circular waveguides, which serve as a more realistic model of bent optical fibers.

In the sixth paper, Jens Markus Melenk and Claudio Rojik study regularity of the solution of the Poisson equation $-\Delta u = f$ in generic polygonal domains with several types of boundary conditions. The work is motivated by shift theorems in Sobolev spaces, which state that, for the homogeneous Dirichlet boundary conditions, $f \in H^{-1+s}$ implies $u \in H^{1+s}$ for $s \in [0, s_0)$, with $s_0$ depending on the interior angles of the domain. The authors consider the limit case $s = s_0$ and prove a local shift theorem near a corner for Dirichlet, Neumann, and mixed boundary conditions in standard Besov spaces. Their proof is based on the expansion of the solution in corner domains in terms of a (known) singularity function, a smooth cut-off function, and a regular remainder.

In the seventh paper, Joost A. A. Opschoor and Christoph Schwab explore the approximation properties of $\mathrm{ReLU}^k$ Deep Neural Networks (DNN) and their relationship to the *hp* finite element spaces introduced by Guo and Babuška in the inaugural volume of Comp. Mech. in 1986. In particular, the authors prove DNN's exponential emulation rates for Gervey-regular functions with isolated point singularities in bounded, polygonal or polyhedral domains within suitable weighted Sobolev spaces (related to countably normed spaces by Guo and Babuška from Proc. Roy. Soc. Edinburgh Sec. A 1997). Moreover, for regular simplicial partitions of polyhedral domains, the authors prove that networks combining ReLU and $\mathrm{ReLU}^2$ activations exhibit the same approximation properties as the *p*-version of the finite element method for elliptic boundary value problems. They also establish that for shape-regular partitions, the numbers of neurons and non-zero parameters are proportional to the number of degrees of freedom in the *hp*-method.

*References*


[1] *J. Chleboun, M. Křížek, K. Segeth, T. Vejchodský*: Professor Ivo Babuška, founder of Applications of Mathematics, passed away. Appl. Math., Praha *68* (2023), 377–386. zbl MR doi

[2] *J. Chleboun, K. Segeth, T. Vejchodský* (eds.): Programs and Algorithms of Numerical Mathematics 13: Dedicated to the 80th birthday of Professor Ivo Babuška. Institute of Mathematics AS CR, Prague, 2006.

[3] *K. Segeth*: Professor Ivo Babuška. Appl. Math. *60* (2015), 469–472. zbl MR doi

*Authors' addresses*: *Vít Dolejší*, Faculty of Mathematics and Physics, Charles University, Sokolovská 83, 186 75 Praha 8, Czech Republic, e-mail: `vit.dolejsi@mff.cuni.cz`; *Michal Křížek*, Institute of Mathematics of the Czech Academy of Sciences, Žitná 25, 115 67 Praha 1, Czech Republic, e-mail: `krizek@math.cas.cz`; *Jan Zeman*, Faculty of Civil Engineering, Czech Technical University, Thákurova 7, 160 00 Praha 6, Czech Republic, e-mail: `jan.zeman@cvut.cz`.