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A STOCHASTIC VERSION OF VIDYASAGAR THEOREM ON THE STABILIZATION OF INTERCONNECTED SYSTEMS

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The purpose of this paper is to provide sufficient conditions for the feedback asymptotic stabilization in probability for a class of affine in the control nonlinear stochastic differential systems. In fact, under the assumptions stated in this paper we prove the existence of a control Lyapunov function that according to the stochastic version of Artstein's theorem guarantees the asymptotic stability in probability by means of a state feedback law that is smooth except eventually at the equilibrium. This result generalizes the well-known theorem of Vidyasagar concerning the feedback stabilization problem for interconnected control systems.

Keywords: asymptotic stability in probability, control Lyapunov function, smooth state feedback law

Classification: 60H10, 93C10, 93D05, 93D15, 93E15

1. INTRODUCTION

The asymptotic stabilization in probability of nonlinear stochastic differential systems by means of state feedback laws is an important task in control theory. The stabilizability of various types of nonlinear stochastic differential systems has been studied by many authors in the past decades (see, for instance Gao and Ahmed [13], Florchinger [8] – [11], Deng, Krstić and Williams [6], Daumail and Florchinger [5] or Abedi, Leong and Chaharborj [2]) by making use of the stochastic version of the Lyapunov theorem proved by Khasminskii in [15].

Necessary and sufficient Lyapunov type conditions for the asymptotic feedback stabilization in probability of stochastic differential systems are given by Florchinger in [8] – [10]. The stabilizers obtained in these papers are smooth except possibly at the equilibrium and their design relies on the knowledge of an appropriate control Lyapunov function.

The aim of this paper is to provide sufficient conditions for the existence of control Lyapunov functions for a class of interconnected stochastic systems that according to the stochastic version of Artstein's theorem [7] (see also [9] or [10]) guarantees asymptotic stabilization in probability by means of state feedback laws that are smooth except

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possibly at the equilibrium. In particular, recall the stochastic Artstein's theorem [7] (see also [9] or [10]) provides an explicit formula for the stabilizing state feedback law. The result proved in this paper extends to the stochastic context Vidyasgar's theorem [24] on asymptotic stabilization for large–scale systems and relies on the proof of Vidyasagar's theorem given by Tsinias in [20]. In addition, this result can be applied in order to design stochastic observers for interconnected stochastic systems with an output. This fact is the object of an ongoing line of research on stochastic observers for stochastic differential systems and will be discussed in a forthcoming paper. It is also fair to note that a stochastic version of Vidyasagar's theorem based on the work of Tsinias [22] has been proved by Boulanger in [4] for a more restrictive class of stochastic differential systems. Nevertheless it appears that the result established in [4] does not apply to design stochastic observers. Other techniques in order to design stabilizers for stochastic interconnected systems have been developed by different authors in the past years (see, for instance Abedi and Leong [1], Oumoun [17] or Himmi and Oumoun [14]). Note also that the stabilization of composite stochastic systems via stochastic Luenberger observers has been investigated by Florchinger in [12] whereas input–to–state stability of nonlinear interconnected systems has been studied by Silva, McFadyen and Ford in [18].

This paper is divided into three sections and is organized as follows. In section one, we recall some basic results on the Lyapunov asymptotic stability in probability of the equilibrium solution of stochastic differential equations proved by Khasminskii in [15]. In section two, we introduce the class of stochastic differential systems affine in the control we are dealing with in this paper and we recall the stochastic version of Artstein's theorem proved in [7] which provides a universal formula for a state feedback stabilizer for the asymptotic stabilization in probability of the equilibrium solution of such stochastic differential systems when a control Lyapunov function is known. In section three, we prove the main theorem of the paper which extends to the stochastic context Vidyasagar's theorem on local stabilization proved in [24] to the class of stochastic differential systems we are dealing with in this paper. The main tools used in this paper are the stochastic Lyapunov machinery developed by Khasminskii [15], the converse Lyapunov theorem proved by Kushner in [16] and the stochastic version of Artstein's theorem proved in [7].

2. ASYMPTOTIC STABILITY IN PROBABILITY

For this paper to be self contained, we summarize in this section basic results related to the Lyapunov asymptotic stability in probability of the equilibrium solution of a stochastic differential equation introduced by Khasminskii in [15].

On a complete filtered probability space $\left(\Omega, \mathcal{F}, (\mathcal{F}_t)_{t\geq 0}, P\right)$, consider the stochastic process solution $x_t \in \mathbb{R}^n$ of the stochastic differential equation written in the sense of Itô,

$$x_t = x_0 + \int_0^t f(x_s)\,\mathrm{d}s + \int_0^t g(x_s)\,\mathrm{d}w_s \tag{1}$$

where

1. x_0 is given in $\mathbb{R}^n$,

2. $(w_t)_{t\geq 0}$ is a standard Wiener process with values in $\mathbb{R}^m$ defined on the probability space $\left(\Omega, \mathcal{F}, (\mathcal{F}_t)_{t\geq 0}, P\right)$,

3. f and g are Lipschitz functions mapping $\mathbb{R}^n$ into $\mathbb{R}^n$ and $\mathbb{R}^{n\times m}$, respectively, vanishing in the origin and with less than linear growth; i. e. there exists a positive constant K such that for any $x, y \in \mathbb{R}^n$,

$$|f(x) - f(y)| + ||g(x) - g(y)|| \leq K|x - y|$$

and

$$|f(x)|^2 + ||g(x)||^2 \leq K(1 + |x|^2).$$

If for any $s \geq 0$ and $x \in \mathbb{R}^n$, $x_t^{s,x}$, $s \leq t$, denotes the solution at time t of the stochastic differential equation (1) starting from the state x at time s, the notion of asymptotic stability in probability for the equilibrium solution of the stochastic differential equation (1) is defined as follows.

Definition 2.1.

1) The equilibrium solution $x_t \equiv 0$ of the stochastic differential equation (1) is stable in probability if for any $s \geq 0$ and $\epsilon > 0$,

$$\lim_{x\to 0} P\left(\sup_{s\leq t} |x_t^{s,x}| > \epsilon\right) = 0.$$

2) The equilibrium solution $x_t \equiv 0$ of the stochastic differential equation (1) is locally asymptotically stable in probability if it is stable in probability and for any $s \geq 0$ and x in a neighborhood of the origin in $\mathbb{R}^n$,

$$P\left(\lim_{t\to+\infty} |x_t^{s,x}| = 0\right) = 1.$$

Then, if L denotes the infinitesimal generator of the stochastic process solution of the stochastic differential equation (1); that is the second order differential operator defined for any function φ in $C^2(\mathbb{R}^n; \mathbb{R})$ by

$$L\varphi(x) = \nabla\varphi(x) f(x) + \frac{1}{2}\mathrm{Tr}\left(g(x)g(x)^\tau \nabla^2\varphi(x)\right)$$

the following stochastic Lyapunov theorem has been proved by means of martingale theory arguments (see theorem 5.4.4 in [15]).

Theorem 2.2. Assume that there exists a Lyapunov function V defined on a neighborhood N of the origin in $\mathbb{R}^n$ (i. e. a positive definite function V in $C^2(N; \mathbb{R})$) such that

$$LV(x) < 0$$

for any $x \in N \setminus \{0\}$. Then, the equilibrium solution $x_t \equiv 0$ of the stochastic differential equation (1) is locally asymptotically stable in probability.

3. PROBLEM SETTING

In this section, we introduce the class of stochastic differential systems affine in the control we are dealing with in this paper.

On a complete filtered probability space $\left(\Omega, \mathcal{F}, (\mathcal{F}_t)_{t\geq 0}, P\right)$, consider the stochastic process $(x_{1,t}, x_{2,t})_{t\geq 0}$ with values in $\mathbb{R}^{n_1} \times \mathbb{R}^{n_2}$ solution of stochastic differential system written in the sense of Itô,

$$x_{1,t} = x_{1,0} + \int_0^t f_1(x_{1,s}, x_{2,s})\, \mathrm{d}s + \int_0^t g_1(x_{1,s}, x_{2,s})\, \mathrm{d}w_s \tag{2}$$

$$x_{2,t} = x_{2,0} + \int_0^t f_2(x_{1,s}, x_{2,s})\, \mathrm{d}s + \int_0^t h(x_{1,s}, x_{2,s})\, u\, \mathrm{d}s + \int_0^t g_2(x_{1,s}, x_{2,s})\, \mathrm{d}w_s) \tag{3}$$

where

1. $x_{1,0}$ and $x_{2,0}$ are given in $\mathbb{R}^{n_1}$ and R^{n_2}, respectively,
2. $(w_t)_{t\geq 0}$ is a standard Wiener processes defined on the probability space $\left(\Omega, \mathcal{F}, (\mathcal{F}_t)_{t\geq 0}, P\right)$ with values in $\mathbb{R}^d$,
3. f_1 and g_1 are Lipschitz functions mapping $\mathbb{R}^{n_1} \times \mathbb{R}^{n_2}$ into $\mathbb{R}^{n_1}$ and $\mathbb{R}^{n_1\times d}$, respectively, vanishing in the origin and with less than linear growth,
4. f_2, g_2 and h are Lipschitz functions mapping $\mathbb{R}^{n_1} \times \mathbb{R}^{n_2}$ into $\mathbb{R}^{n_2}$, $\mathbb{R}^{n_2\times d}$ and $\mathbb{R}^{n_2\times m}$, respectively, vanishing in the origin and with less than linear growth,
5. u is a measurable control law with values in $\mathbb{R}^m$.

Then, one can introduce the notion of stabilizing state feedback law for the stochastic differential system (2) – (3) as follows.

Definition 3.1. A measurable function u mapping $\mathbb{R}^{n_1} \times \mathbb{R}^{n_2}$ into $\mathbb{R}^m$, vanishing in the origin, is said to be a stabilizing state feedback law for the stochastic differential system (2) – (3) if the equilibrium solution of the closed–loop system

$$x_{1,t} = x_{1,0} + \int_0^t f_1(x_{1,s}, x_{2,s})\, \mathrm{d}s + \int_0^t g_1(x_{1,s}, x_{2,s})\, \mathrm{d}w_s$$

$$x_{2,t} = x_{2,0} + \int_0^t f_2(x_{1,s}, x_{2,s})\, \mathrm{d}s + \int_0^t h(x_{1,s}, x_{2,s})\, u(x_{1,s}, x_{2,s})\, \mathrm{d}s + \int_0^t g_2(x_{1,s}, x_{2,s})\, \mathrm{d}w_s$$

is locally asymptotically stable in probability.

Denoting by L the infinitesimal generator of the stochastic process solution of the unforced stochastic differential system (2) – (3); that is the second order differential operator defined for any function φ in $C^2(\mathbb{R}^{n_1} \times \mathbb{R}^{n_2}; \mathbb{R})$ by

$$L\varphi(x_1, x_2) = \nabla\varphi(x_1, x_2)\, F(x_1, x_2) + \frac{1}{2}\mathrm{Tr}\left(G(x_1, x_2)\, G(x_1, x_2)^\tau\, \nabla^2\varphi(x_1, x_2)\right)$$

where $F(x_1, x_2) = \begin{pmatrix} f_1(x_1, x_2) \\ f_2(x_1, x_2) \end{pmatrix}$ and $G(x_1, x_2) = \begin{pmatrix} g_1(x_1, x_2) \\ g_2(x_1, x_2) \end{pmatrix}$, one can introduce the concept of control Lyapunov function as follows.

Definition 3.2. A Lyapunov function V defined on a neighborhood N of the origin in $\mathbb{R}^{n_1} \times \mathbb{R}^{n_2}$ is called a control Lyapunov function for the stochastic differential system (2) – (3) if for any $x \in N \setminus \{0\}$ the following condition holds

$$(\nabla_{x_2} V(x) h(x) = 0) \quad \Rightarrow \quad (LV(x) < 0)\,.$$

Then, the following extension of Artstein's theorem [3] (see also Sontag [19]) for the feedback asymptotic stabilization of stochastic differential systems affine in the control has been established in [7] and [10].

Theorem 3.3. Assume that V is a smooth control Lyapunov function defined on a neighborhood N of the origin in $\mathbb{R}^{n_1} \times \mathbb{R}^{n_2}$ for the stochastic differential system (2) – (3). Then, the state feedback law u defined on $\mathbb{R}^{n_1} \times \mathbb{R}^{n_2}$ by

$$u(x) = \rho\left(a(x), ||B(x)||^2\right) B(x)^\tau \tag{4}$$

where $a(x) = LV(x)$, $B(x) = \nabla_{x_2} V(x) h(x)$ and

$$\rho(a, b) = \begin{cases} -\dfrac{a + \sqrt{a^2 + b^2}}{b\left(1 + \sqrt{1 + b}\right)} & \text{if } b > 0 \\ 0 & \text{if } b = 0 \end{cases} \tag{5}$$

renders the stochastic differential system (2) – (3) locally asymptotically stable in probability.

4. THE MAIN THEOREM

In this section, we prove an extension of Vidyasagar's theorem on local stabilization stated in [24] (see also Tsinias [20]–[22]). This result also improves a previous work published in [9].

With this aim in view, assume there exist neighborhoods of the origin $N_1 \subset \mathbb{R}^{n_1}$ and $N_2 \subset \mathbb{R}^{n_2}$, a continuous function ϕ mapping N_1 into $\mathbb{R}^{n_2}$, vanishing in the origin, and a function W in $C^{2,2}(N_1 \times N_2; \mathbb{R}_+)$ satisfying the following assumptions:

(A1) The equilibrium solution $x_{1,t} \equiv 0$ of the stochastic differential system

$$x_{1,t} = x_{1,0} + \int_0^t f_1(x_{1,s}, \phi(x_{1,s}))\, \mathrm{d}s + \int_0^t g_1(x_{1,s}, \phi(x_{1,s}))\, \mathrm{d}w_s \tag{6}$$

is asymptotically stable in probability.

(A2) The function W is such that $W(x) = 0$ if, and only if, $x \in M_\phi = \{x = (x_1, x_2) \in N_1 \times N_2 \ / \ x_2 = \phi(x_1)\}$.

(A3) There exists a closed subset S of $(N_1 \times N_2) \setminus M_\phi$ such that

$$(S \cup M_\phi) \setminus \{0\} = \{x = (x_1, x_2) \in (N_1 \times N_2) \setminus \{0\} \ / \ \nabla_{x_2} W(x) h(x) = 0\}$$

and

$$(x \in S \setminus \{0\}) \quad \Rightarrow \quad (LW(x) < 0).$$

Then, we can prove the following result on the asymptotic stabilization in probability for the class of stochastic differential systems considered in this paper which extends Vidyasagar's theorem [24] (see also Tsinias [20]) to the stochastic context.

Theorem 4.1. Assume that assumptions **(A1)** to **(A3)** are satisfied. Then, there exists a control Lyapunov function Φ for the stochastic differential system (2)–(3) and therefore this stochastic differential system is locally asymptotically stabilizable in probability.

Proof. The equilibrium solution $x_{1,t} \equiv 0$ of the stochastic differential system (6) being, according to assumption **(A1)**, asymptotically stable in probability, the converse Lyapunov theorem proved by Kushner [16] asserts that there exists a Lyapunov function V_1 defined on N_1 such that for any $x_1 \in N_1 \setminus \{0\}$,

$$\nabla_{x_1} V_1(x_1) f_1(x_1, \phi(x_1)) + \frac{1}{2} \mathrm{Tr}\left(g_1(x_1, \phi(x_1)) g_1(x_1, \phi(x_1))^\tau \nabla^2_{x_1} V_1(x_1)\right) < 0.$$

Since the sets $\{0\} \times N_2$ and $M_\phi \setminus \{0\}$ are disjoint there exists a closed subset S_1 of $N_1 \times N_2$ so that its interior contains the set $\{0\} \times N_2 \setminus \{0\}$ and $(S_1 \cap M_\phi) \setminus \{0\} = \emptyset$. Then, by making use of partition of unity arguments similar to those employed in [3] or [23] for example, one can deduce from assumption **(A3)**, as in [20], that there exists a continuous function p mapping N_1 into $\mathbb{R}_+$, vanishing in the origin, which is positive definite and such that

$$(S_1 \cup S) \setminus \{0\} \cap \{x = (x_1, x_2) \in N_1 \times N_2 \ / \ W(x) < p(x_1)\} = \emptyset. \tag{7}$$

Now, let b be a strictly increasing continuous function mapping $\mathbb{R}_+$ into $\mathbb{R}_+$, vanishing in the origin, such that for any $x_1 \in N_1$,

$$b(|x_1|) \leq p(x_1)$$

and let also a be a strictly increasing continuous function mapping $\mathbb{R}_+$ into $\mathbb{R}_+$ such that for any $x_1 \in N_1$,

$$V_1(x_1) \leq a(|x_1|).$$

Then, the function V mapping N_1 into $\mathbb{R}_+$ defined for any $x_1 \in N_1$ by

$$V(x_1) = \int_0^{V_1(x_1)} \int_0^s b\left(a^{-1}(v)\right) \mathrm{d}v\mathrm{d}s$$

is obviously twice continuously differentiable on N_1 and such that for any $x_1 \in N_1$,

$$V(x_1) \leq V_1(x_1) \int_0^{V_1(x_1)} b\left(a^{-1}(v)\right) \mathrm{d}v \leq (V_1(x_1))^2 \, b(|x_1|). \tag{8}$$

Furthermore, consider a smooth function θ mapping $\mathbb{R}$ into $[0,1]$ such that $\theta(v) = 1$ if $|v| < \frac{1}{2}$ and $\theta(v) = 0$ if $|v| \geq 1$ and denote by Ψ the function mapping $N_1 \times N_2$ into $[0,1]$ defined for any $x \in N_1 \times N_2$ by

$$\Psi(x) = \begin{cases} \theta\left(\sigma \frac{W(x)}{V(x_1)}\right) & \text{si} \quad x_1 \neq 0 \\ \\ 0 & \text{si} \quad x_1 = 0 \end{cases}$$

where $\sigma = \max\left\{(V_1(x_1))^2, \ x_1 \in N_1\right\}$.

Then, with the above definition, one can prove that for any $x \in M_\phi \setminus \{0\}$, one has $\Psi(x) = 1$, $\nabla\Psi(x) = 0$ and $\nabla^2\Psi(x) = 0$. Indeed, if $x \in M_\phi \setminus \{0\}$, one has according with assumption **(A2)**, $W(x) = 0$ which implies that $\Psi(x) = \theta(0) = 1$, the last two assertions being obtained by means of the same arguments.

Moreover, for any $x = (x_1, x_2) \in S_1 \cup S$, one has $\Psi(x) = 0$, $\nabla\Psi(x) = 0$ and $\nabla^2\Psi(x) = 0$. Indeed, if $x_1 = 0$ the result is a direct consequence of the definition of the function Ψ and if $x_1 \neq 0$, according with (7), one has $p(x_1) \leq W(x)$ which implies that $b(|x_1|) \leq W(x)$ and according with (8), since $x_1 \neq 0$, one gets $\sigma \dfrac{W(x)}{V(x_1)} \geq 1$ and consequently, $\Psi(x) = 0$, the last two assertions being obtained by means of the same arguments.

In addition, the same discussion shows that the function Ψ is twice continuously differentiable on $(N_1 \times N_2) \setminus \{0\}$ and uniformly bounded on $N_1 \times N_2$.

Now, let k be a positive constant such that

$$k > \sigma \max\{|\theta'(s)|,\ s \in \mathbb{R}\}$$

and denote by Φ the function mapping $N_1 \times N_2$ into $\mathbb{R}_+$ defined for any $x = (x_1, x_2) \in N_1 \times N_2$, by

$$\Phi(x) = \Psi(x)V(x_1) + kW(x). \tag{9}$$

Then, the function Φ defined above is positive definite, twice continuously differentiable on $(N_1 \times N_2) \setminus \{0\}$ and continuous at the origin.

Indeed, obviously, $\Phi(0) = 0$ and conversely, if $\Phi(x) = 0$ one has $W(x) = 0$ and $\Psi(x)V(x_1) = 0$ and hence, taking assumption **(A2)** into account, one gets $x \in M_\phi$ and $\Psi(x)V(x_1) = 0$ which imply that $x = 0$. In fact, assuming that $x_1 \neq 0$ one gets from the above analysis that $\Psi(x) = 1$ and consequently that $V(x_1) = 0$ and it yields, according with the definition of the function V, that $V_1(x_1) = 0$ which contradicts the fact that V_1 is a Lyapunov function.

Therefore, the function Φ is positive definite and further, the function Φ is clearly twice continuously differentiable on $(N_1 \times N_2) \setminus \{0\}$ and since the function Ψ is bounded and

$$\lim_{x \to 0} W(x) = \lim_{x_1 \to 0} V(x_1) = 0$$

it is obvious that the function Φ is continuous at the origin.

In the sequel, we prove that the function Φ defined above is a control Lyapunov function for the stochastic differential system (2) – (3).

With this aim, first note that for any $x = (x_1, x_2) \in N_1 \times N_2 \setminus \{0\}$, one has

$$\nabla_{x_2}\Phi(x)h(x) = \left(\sigma\theta'\left(\sigma\frac{W(x)}{V(x_1)}\right)\nabla_{x_2}W(x) + k\nabla_{x_2}W(x)\right)h(x)$$

and hence, taking into account the definition of constant k, it yields that $\nabla_{x_2}\Phi(x)h(x) = 0$ if, and only if, $\nabla_{x_2}W(x)h(x) = 0$ and by assumption **(A3)** one has

$$(S \cup M_\phi) \setminus \{0\} = \{x = (x_1, x_2) \in (N_1 \times N_2) \setminus \{0\},\ \nabla_{x_2}\Phi(x)h(x) = 0\}.$$

Now, we prove that for any $x \in S \cup M_\phi \setminus \{0\}$, on has $L\Phi(x) < 0$.

If $x \in M_\phi \setminus \{0\}$, as discussed above, one has $\Psi(x) = 1$, $\nabla\Psi(x) = 0$ and $\nabla^2\Psi(x) = 0$ and according with assumption **(A2)**, $\nabla W(x) = 0$ and $\nabla^2 W(x) = 0$, which implies, since $x_2 = \phi(x_1)$, that

$$\begin{aligned} L\Phi(x)|_{x \in M_\phi \setminus \{0\}} &= \nabla_{x_1}V_1(x_1)f_1(x_1, \phi(x_1)) \\ &\quad + \frac{1}{2}\mathrm{Tr}\left(g_1(x_1, \phi(x_1))\, g_1(x_1, \phi(x_1))^\tau \nabla^2_{x_1}V_1(x_1)\right) \end{aligned}$$

and, taking into account the definition of the Lyapunov function V_1, it yields

$$L\Phi(x)|_{x\in M_\phi\setminus\{0\}} < 0.$$

If $x \in S\setminus\{0\}$, as discussed above, one has $\Psi(x) = 0$, $\nabla\Psi(x) = 0$ and $\nabla^2\Psi(x) = 0$ and consequently,

$$L\Phi(x)|_{x\in S\setminus\{0\}} = kLW(x)$$

and, taking into account assumption **(A3)**, it yields

$$L\Phi(x)|_{x\in S\setminus\{0\}} < 0.$$

Therefore, the function Φ given by (9) is a control Lyapunov function for the stochastic differential system (2) − (3) and by applying the stochastic version of Artstein's theorem (theorem 3.3), the stochastic differential system (2) − (3) si asymptotically stabilizable in probability by means of the state feedback u defined for any $x \in \mathbb{R}^{n_1} \times \mathbb{R}^{n_2}$ by

$$u(x) = \rho\left(a(x), ||B(x)||^2\right) B(x)^\tau$$

where the function ρ is defined in (5) with $a(x) = L\Phi(x)$, $B(x) = \nabla_{x_2}V(x)h(x)$.

□

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