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Siddhartha Vadapalli; Venkata Vamsi Cittiboyna; Venkata Satya Sujeeth Chintha; Aditya Subham; Sravanth Kumar Bongu; Badrinath Bezawada

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A BACKSTEPPING APPROACH BASED DIVING CONTROL OF AUTONOMOUS UNDERWATER VEHICLE USING H_∞ CONTROL WITH EXTERNAL DISTURBANCES

Vadapalli Siddhartha, Chittiboyina Venkata Vamsi Chintha Venkata, Satya Sujith, Aditya Subham, Bongu Sravanth Kumar, and Bezawada Badrinath

A nonlinear H_∞ control using the linear matrix inequality (LMI) for an autonomous underwater vehicle (AUV) that is equipped with parametric uncertainties and external disturbances is discussed. The proposed controller is intended to attain the stability of the AUV system and performance in the occurrence of disturbances and uncertainties. Initially, the asymptotic stability of the nominal AUV system is shown by using the Lyapunov stability function and the LMI condition is attained. In the second scenario, the external disturbance is considered as an input, and accordingly, the LMI condition for a perturbed AUV system, i. e., the nominal AUV system, is achieved in the presence of external disturbances by substituting the state matrix in the obtained LMI. The improvement of the proposed controller is further discussed in the third scenario by determining the asymptotic stability condition of the perturbed uncertain AUV system, i. e., uncertainties are added to the perturbed AUV system. The perturbation is considered a signal that varies with time. The proposed control algorithm exhibits the effective tracking of the desired depth in all three scenarios. Among all the scenarios, it is observed that the depth tracking by AUV is more effective for the perturbed uncertain AUV system when compared with the remaining two scenarios. Most of the existing AUV controllers mainly focus on disturbance rejection, where the parametric uncertainties are not considered, and also lack stability analysis when the uncertainty levels are increased. The proposed control approach presents the novelty in addressing the parametric uncertainties and time-varying disturbances in contrast to the existing methods that do not consider uncertainties or lack stability under varying conditions. A Lyapunov-based LMI approach is adapted to ensure asymptotic stability for the nominal, perturbed, and perturbed uncertain AUV systems demonstrating superior depth tracking performance, notably in the formidable scenario with disturbances as well as uncertainties. The YALMIP tool is used to develop the proposed control strategy within the MATLAB/Simulink framework. In the first scenario, i. e., for a nominal AUV system, the settling time in tracking the desired depth is found to be 58 seconds. For the perturbed AUV system and the perturbed uncertain AUV system, the settling time is found to be 51 seconds and 37 seconds, respectively. It is observed that the settling time for the third scenario is much less than that of the other two scenarios. On the other hand, the steady state error (SSE) for the nominal AUV system and the perturbed uncertain AUV system is found to be 0.83% and 0.67%, respectively. There is a decrease in the SSE in tracking the depth of the AUV. It can be

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inferred that the AUV system with perturbations and uncertainty has a superior performance when compared with the other scenarios. In addition, the proposed H_∞ control algorithm is compared with a fractional order fast terminal sliding mode controller to show the efficacy of the proposed control approach.

Keywords: H_∞ control, linear matrix inequality, external disturbance, robust control, uncertainties

1. INTRODUCTION

The perception of exploiting marine vehicles is accelerating day by day and led to the development of several robotic submersible vehicles that can dive deep into the ocean, considered a research hotspot [48]. One such robotic submersible vehicle is the AUV. AUVs navigate the complex underwater environment by accomplishing diverse missions autonomously. Alongside underwater exploration, AUVs are extensively used in industrial tasks and military operations.

1.1. Related work

The authors in [1, 21] adapted an integral sliding mode control (SMC), Euler-Lagrange based SMC to address the disturbances and uncertainties in AUV. The authors in [41] adapted a high-order SMC to control the depth. However, the high order SMC approach need to consider the effect of disturbances and uncertainties. Besides, the authors in [23, 24, 25] discussed the adaptive fuzzy proportional integrative SMC, adaptive neuro-fuzzy SMC, adaptive integral sliding mode control (AISMC) to eliminate the chattering phenomenon. However, there is a necessity to consider the effect of disturbances/uncertainties and the stability approach. The authors in [3, 16, 26, 27] proposed several fuzzy controllers such as neural network robust adaptive control, memorized sample data controller, type-2 fuzzy logic controller to study the depth tracking of AUV. However, there is a need to focus on the presence of disturbances/uncertainties.

The depth control of AUV is studied in [39, 40] by adapting a robust H_2 optimal control and robust H_∞ control. Besides, memory-based delay dependent H_∞ robust control algorithm and a proportional control based H_∞ control is elaborated in [9, 17] to track the desired depth of AUV. Furthermore, a nonlinear H_∞ control (NHC) is explained in [28, 29, 30, 31, 32, 33] for an AUV in diving and steering planes. The authors developed the NHC controller through nonlinear matrix inequalities and Hamilton–Jacobi–Issacs inequality based L_2 gain approach. A linear quadratic regulator (LQR) based double loop depth tracking control and L_1 adaptive autopilot method is presented in [13, 38] to study the depth tracking behavior of AUV when subjected to uncertainties/disturbances. Besides, the authors proposed an adaptive line-of-sight guidance (ALOS) law and an estimated state observer to estimate dynamic disturbances. The diving motion of AUV is addressed in [10] by adapting a resonant plus proportional control law to study the tracking control problem by imparting external disturbances.

The authors in [2, 36, 47] proposed some disturbance techniques through particle swarm optimization (PSO) algorithm, active disturbance rejection control algorithm, novel hybrid PSO using a PID controller to study the depth tracking of AUV by considering uncertainties and disturbances along with variations in hydrodynamic parameters.

In addition, a backstepping technique based on torque effect and a deep reinforcement learning algorithm is explored in [18, 43] to explain the depth tracking of AUV in the existence of disturbances and uncertainties. An underactuated AUV in the depth plane is elaborated in [45] by deriving an identification model for pitch dynamics by using the weighted least squares algorithm. However, the proposed control algorithm should consider the effect of uncertainties along with external disturbances. A technique using immersion and invariance is elaborated in [8] to implement a control law that tracks the depth position of the AUV in the presence of disturbances and uncertainties. The trajectory tracking of unmanned underwater vehicles (UUVs) is explored in [7] by using an MPC approach by adapting the incremental sparse Gaussian process (ISGP). The depth tracking challenge in the diving motion of a torpedo-shaped AUV is elaborated in [11] by adapting the immersion and invariance (I&I) technique.

Referring to Aerospace applications, the authors in [15, 20, 22, 46] developed the robust controllers that addressed the presence of uncertainties/disturbances. Besides, the authors in [34, 35], and [44] developed the backstepping-based fractional order SMC, fault-tolerant control with fixed time convergence, and a sampled-data robust control through a quasi-linear parameter varying (LPV) model respectively to address the existence of uncertainties/disturbances.

A quick summary of research gaps for the above literatures are summarized here. Several SMC controllers, such as integral SMC [21], high-order SMC [1], adaptive fuzzy–PI SMC, adaptive integral SMC [25], adaptive neuro fuzzy SMC [23, 24] need to consider the presence of external disturbances/uncertainties. Besides, some of the fuzzy/neural controllers in [16, 27], and [3] did not consider the effect of uncertainties/external disturbances. In addition, a robust extended Kalman filter, as discussed in [40], ignored the presence of external disturbances. Even some of the H_∞ controllers explained in [32] and [17] did not address the uncertainties. In contrast, the authors of this manuscript have addressed the limitations by considering a H_∞ controller within a LMI framework adapted from [5]. The synthesis of the control algorithm has a systematic and convex optimization approach by adapting the LMI-based formulation. A guaranteed performance in terms of disturbance attenuation and robustness for AVUs operating in unpredictable underwater environments is discussed. In the present manuscript, the authors discussed the diving control of AUV in three different scenarios, i. e., the nominal AUV system, where the vehicle operates under ideal modeled conditions, the perturbed AUV system, where certain parameters deviate from their nominal values, and perturbed uncertain AUV system which includes both uncertainties and external disturbances. The simulation results demonstrate that the proposed controller tracks the desired depth in all the three cases illustrating the robustness and adaptibility of the approach in the real world condtions where the uncertainties cannot be avoided. In addition, a Lyapunov stability analysis is elaborated to prove the stability of the closed loop system mathematically and also, the tracking errors converge asymptotically.

The proposed control approach is elaborated in terms of the development of nonlinear Lyapunov based H_∞ control, adapting the LMIs. The proposed control algorithm ensures asymptotic stability and superior tracking performance of an AUV under the existence of time-varying disturbances and uncertainties. Most importantly, the external disturbance is considered as a time-varying signal rather than a constant or bounded disturbance. Besides, stability conditions for all three scenarios through rigorous mathematical analysis are also discussed.

1.2. Key contributions

- The depth control of AUV is observed by proposing a robust control algorithm employing LMIs that offers a convex optimization framework.
- Based on several existing studies, the proposed control algorithm tracks the depth of AUV in the existence of external disturbances and uncertainties, which enhances robustness in real underwater applications.
- Three scenarios are considered that demonstrate the effectiveness of the proposed controller by showing the simulation results that tracks the desired depth in all the scenarios.
- The backstepping approach proposed in the manuscript is adapted from [32].
- To show the efficacy of the proposed control approach, a comparison is made with a hybrid fractional order fast terminal sliding mode controller (FOFTSMC) [6].

1.3. Outline of the manuscript

Initially the AUV modeling is elaborated in Section 2. The problem statement along with the control approach is discussed in Section 3. The design of the control algorithm by studying the various LMIs along with the stability analysis is explained in Section 4. The simulation results to track the desired depth for all the three cases i. e., nominal AUV system, perturbed nominal AUV system, perturbed uncertain AUV system are depicted in Section 5. The final analysis of the manuscript in short is described in Section 6.

S.No	Notation used	Name of the notation
1	f_i, g_i	Nonlinear state space of AUV
2	w_i, q_i, z_i, θ_i	States of AUV in diving plane
3	C_x	Hydrodynamic parameters
4	δ_s	Stern angle
5	z_E	Depth error
6	z_D	Desired depth
7	θ_E	Pitch orientation error
8	θ_D	Desired pitch angle
9	x	State vector
10	y	Output vector
11	u_n	Nonlinear control input
12	ω	Disturbance vector
13	D_i, D_o	Input and output disturbance matrices
14	ΔA_i	Uncertainty and disturbance

Continued on next page

S.No	Notation used	Name of the notation
15	K	Gain matrix
16	β	Positive real scalar
17	P, Q	Matrices of known dimensions
18	$\mathcal{G}_{yu}$	H_∞ norm
19	θ_I	Approaching angle
20	$\mathcal{K}_i$	Positive backstepping gain
21	ξ_i	Lyapunov function used in backstepping approach
22	α	An unknown positive parameter
23	$\mathcal{V}(x)$	Lyapunov function used in LMIs
24	$\mathtt{X}, \mathtt{W}$	Symmetric positive matrices
Subscripts used		
25	D	Desired values
26	i	For diving notations
27	E	Error values

Tab. 1: Nomenclature table showing the meaning of notations.

2. MODELING OF AUV

Designing effective control algorithms requires the accurate modeling of an AUV through which the real-time behavior can be governed. A mathematical framework that represents the AUV motion by focusing on its behavior in the diving plane is elaborated. The AUV movement can be understood by involving two key aspects, i. e., kinematics and dynamics. Kinematics deals with the geometry of motion, where the forces are not considered. On the other hand, dynamics relate the forces and moments acting on the AUV. When the AUV is operating in the diving plane, the primary concern lies with three degrees of freedom, i. e., surge, heave, and pitch motions. Modeling of AUV helps to capture the important characteristics related to the response of control inputs and environmental disturbances. Besides, the design of the controllers is also enabled, through which the depth of the AUV can be regulated. The stern angle (δ_s) of the AUV is responsible for having the desired heave and pitch behaviors. The equations that describe the AUV dynamics are represented below. The structure of the AUV for the reference frame is shown in Figure 1.

The problem set-up and control objectives are explained below.

- A stern plane is used to regulate the depth and pitch angle of the AUV.
- In practice, the constant surge velocity is considered in many operational modes. Hence, a constant surge velocity is maintained.
- When the AUV is operating in the vertical plane, yaw or sway motions are excluded from the AUV modeling.

The nonlinear state space model of the AUV is represented in Equation 1 considering the heave and pitch motions.

$$\dot{x} = f_i(x) + g_i(x)u, \tag{1}$$

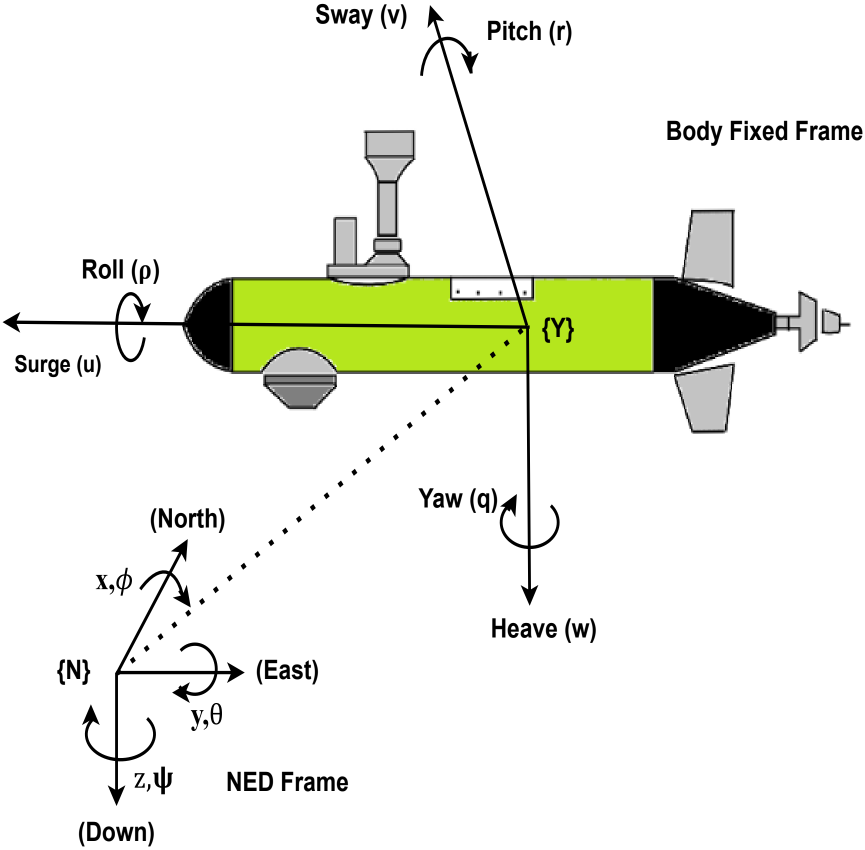


Fig. 1: Structure of AUV with reference frames.

where x indicates a column vector containing $\dot{w}$, $\dot{q}$, and $\dot{\theta}$, the input (u) represents the stern angle (δ_s) of AUV in the diving plane.

The equations of motion involving the heave and pitch dynamics are shown in Equations (2), (3) and (4), (5).

$$m(\dot{w}_i - u_i q_i) = (W - B)\cos(\theta_i) + C_{Z_w} u_i w_i + C_{Z_q} u_i q_i + C_{z_{\delta_s}} u_{i^2} \delta_s + C_{Z_{\dot{w}}} w_{\dot{i}} \quad (2)$$

$$\dot{z}_i = -u_i \sin(\theta_i) + w_i \cos(\theta_i) \quad (3)$$

$$I_y \dot{q}_i = z_B B \sin(\theta_i) + C_{M_w} u_i w_i + C_{M_q} u_i q_i + C_{M_{\delta_s}} {u_i}^2 \delta_s + C_{M_{\dot{q}}} \dot{q}_i \quad (4)$$

$$\dot{\theta}_i = q_i. \quad (5)$$

2.1. Model assumptions

Assumption 1. The simulation of the control algorithm will consider a constant surge velocity.

Remark 1. The control algorithm that is designed is not dependent on time and hence a constant surge velocity is considered.

Assumption 2. Sway, Yaw and Roll motions are ignored.

Remark 2. Sway and yaw motions are not considered because the diving plane dynamics are not dependent on the lateral motion. Furthermore, the parameters that are used by the AUV in the simulation purpose are similar to that of flat fish type AUV [42] for which the roll motion is undesirable.

Assumption 3. Neutral or slightly positive buoyancy

Remark 3. The values of weight (W) and buoyancy (B) of AUV are almost equal but will have a slight difference that allows the AUV towards natural diving in practical point of view.

Assumption 4. Linearized hydrodynamic coefficients

Remark 4. The values of the hydrodynamic coefficients are assumed to known and constant that works on experimental data as explained in [42]

3. PROBLEM FORMULATION

The equations of motion shown above are considered to explain the problem. The assumptions discussed in the previous section are considered to design the control algorithm.

3.1. Problem statement

A robust algorithm will be considered to track the desired depth. The depth error presented in Equation (6) has to be minimized so that AUV tracks the desired depth.

$$\lim_{t\to\infty} z_E(t) = 0, \tag{6}$$

where $z_E = z_i$ - Z_D. Here, z_i represent the actual depth and Z_D represent the desired depth.

The minimizing of the depth error will achieve a desired pitch angle θ_D for which the pitch orientation error is reduced to zero as shown in Equation (7).

$$\lim_{t\to\infty} \theta_E(t) = 0, \tag{7}$$

where $\theta_E = \theta_i$ - θ_D. Here, θ_i and θ_D represent the actual pitch angle and the desired pitch angle.

3.2. Control approach

The following nonlinear AUV system is considered that is added with uncertainties and excited with disturbance.

$$\left.\begin{aligned} \dot{x} &= f(x) + (A_i + \Delta A_i)x + B_i u_n + D_i \omega \\ y &= C_i x + D_o \omega \end{aligned}\right\}, \tag{8}$$

where the AUV system is defined by real-valued state vectors (x), output vector (y) that has dimensions n and m, a nonlinear control input (u_n), and a disturbance vector (ω). Besides, f(x) is a nonlinear model of AUV system. Furthermore, the state matrix is represented as A_i and followed by input matrix and output matrix represented by B_i, and C_i respectively. Besides, D_i and D_o are the input and output disturbance known matrices. The AUV system excited with uncertainty along with disturbance is indicated as ΔA_i where $\Delta A_i = \mathcal{M}\Delta\mathcal{N}$ where $\mathcal{M} \in \mathbb{R}^n \times n$ and $\mathcal{N} \in \mathbb{R}^n \times n$. The matrix Δ contain uncertain parameters and $\|\Delta^T \Delta\|$ is bounded by I. [19].

A nonlinear state feedback control law shown in the Equations 9 and 10 is explained here.

$$u_n = u - B_i^{-1} f(x) \tag{9}$$

$$u = Kx. \tag{10}$$

Assumption 5. Function $f(x)$ is assumed to be continuously differentiable.

Remark 5. If the input matrix is found to be a non-square matrix or rank deficit matrix, then pseudo inverse will be used instead of normal inverse.

Remark 6. The existence of a positive real scalar β guarantees the validity of an inequality between matrices P and Q, where P and Q are of known dimensions [14, 19].

$$P^T Q + Q^T P \leq \beta P^T P + \beta Q^T Q, \ \ \beta > 0. \tag{11}$$

The above assumptions are essential in designing a nonlinear state feedback control.

4. CONTROL DESIGN

A control design for the three different scenarios is explalined here by considering the design algorithms. Initially, a nominal state space system without disturbances is considered and shown in Equation 12.

$$\begin{aligned} \dot{x} &= A_i x + B_i u \\ y &= C_i x. \end{aligned} \tag{12}$$

where

$$A_i = \begin{bmatrix} \frac{C_{Z_w} u_i}{m - C_{Z_{\dot{w}}}} & \frac{(mu_i + C_{Z_q} u_i)}{m - C_{Z_{\dot{w}}}} & \frac{(W-B)}{m - C_{Z_{\dot{w}}}} \\ \frac{C_{M_w} u_i}{I_y - C_{M_{\dot{q}}}} & \frac{C_{M_q} u_i}{I_y - C_{M_{\dot{q}}}} & \frac{Z_B B}{I_y - C_{M_{\dot{q}}}} \\ 0 & 1 & 0 \end{bmatrix}, \; B_i = \begin{bmatrix} \frac{C_{Z_{\delta_s}} {u_i}^2}{(m - C_{Z_{\dot{w}}})} \\ \frac{C_{M_{\delta_s}} {u_i}^2}{(I_y - C_{M_{\dot{q}}})} \\ 0 \end{bmatrix}$$

If the transfer function of control input to output is considered shown in Equation 13 considered from [25], then H_∞ norm of the transfer function i. e., $\|\mathcal{G}_{yu}(s)\|_\infty$ has to be minimized to attenuate the effect of inputs on the outputs.

$$\mathcal{G}_{yu} = C(SI - A)^{-1} B \tag{13}$$

$$\|y\|_2 \leq \|\mathcal{G}_{yu}(s)\|_\infty \|u\|_2. \tag{14}$$

The above inequality in Equation (14) taken from [12] explains the relation between the boundedness of the output energy considered by the product of H_∞ norm of transfer function and the input energy. Figure 2 illustrates the control structure of the proposed algorithm.

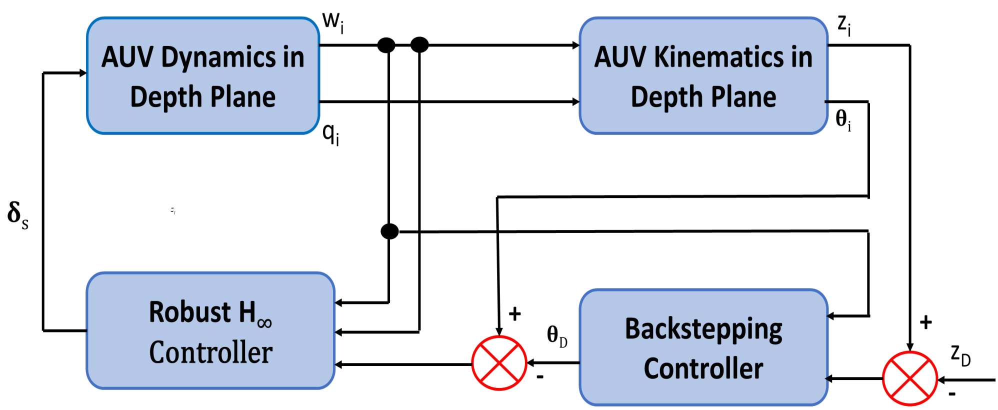


Fig. 2: Robust H_∞ control configuration in diving plane.

4.1. Backstepping approach – Desired pitch angle

A backstepping technique is employed to track the desired depth.θ_D represents the desired pitch angle. The theorem shown in 1 is essential for understanding the backstepping control of a diving plane.

Theorem 1. A desired pitch angle is obtained by considering the Equation (3) as shown below.

$$\theta_D = \theta_I \tanh(\mathcal{K}_i z_E) + \sigma_I, \tag{15}$$

where θ_I acts as a constraint, defining the maximum permitted approaching angle, $\mathcal{K}_i$ is a positive gain, z_E indicates the depth error, and $\sigma_I = \tan^{-1}(w_i/u_i)$.

Proof. A Lyapunov function $\xi_i = 1/2(z_E{}^2)$ exists for any possible z_E that needs to satisfy the condition $\xi_i \leq 0$. The derivation of the Lyapunov function will further lead to

$$\dot{\xi}_i = z_E \dot{z}_i = -u_i z_E \sin\theta_i + w_i z_E \cos\theta_i. \tag{16}$$

On simplifying the above Equation (16),

$$\begin{aligned} \dot{\xi}_i &= -w_i(\sin\theta_i \cos\sigma_I - \cos\theta_i \sin\sigma_I) \\ \dot{\xi}_i &= -w_i \sin(\theta_i - \sigma_I), \end{aligned} \tag{17}$$

where $w_i = \sqrt{u_i{}^2 + w_i{}^2}$. Hence, for $\theta_i - \sigma_I = \theta_I \tanh(\mathcal{K}_{ib} z_E)$ the derivative of ξ_i denoted as $\dot{\xi}_i$ will always be non-positive, regardless of the value of z_E. □

4.2. Robust study and stability analysis

All the three different scenarios i. e., nominal AUV system, perturbed AUV system, and the uncertain perturbed AUV system are elaborated in this section. Each scenario is studied using an individual theorem through LMIs along with the asymptotic stability condition.

Theorem 2. If $\mathcal{G}_{yu}$ shown in Equation (13) is provided and assuming that $\|\mathcal{G}_{yu}\|_\infty \leq \alpha$ where α denotes an unknown parameter constrained to be positive. then a positive symmetric matrix P exists such that the following LMI hold

$$\begin{pmatrix} A_i{}^T P + P A_i & P B_i & C_i{}^T \\ B_i{}^T P & -\alpha I & 0 \\ C_i & 0 & -\alpha I \end{pmatrix} < 0. \tag{18}$$

The nominal system shown in Equation (12) is said to be asymptotically stable on satisfying the above inequality.

Proof. Refer Appendix A. □

The detailed flow for the proof of nominal AUV system is shown in the Figure 3. Besides, a pseudo code based on algorithm is shown in Algorithm 1.

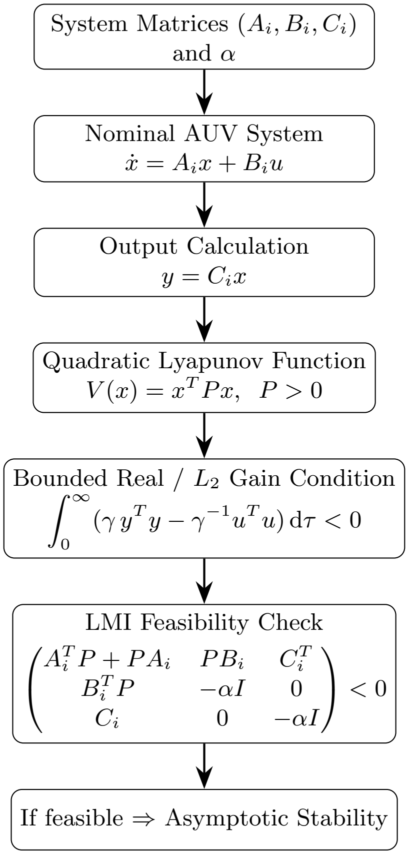


Fig. 3: Flow Representation for a Nominal AUV System.

Algorithm 1 Stability check for a nominal AUV system

Require: System matrices A_i, B_i, C_i and $\alpha > 0$

Define system dynamics: $\dot{x} = A_i x + B_i u$
Compute output: $y = C_i x$
Define Lyapunov function: $V(x) = x^T P x$, $P = P^T > 0$
Apply bounded-real L_2 gain condition: $\int_0^\infty (\gamma y^T y - \gamma^{-1} u^T u)\, \mathrm{d}\tau < 0$
Derive LMI constraint:

$$\begin{pmatrix} A_i^T P + P A_i & P B_i & C_i^T \\ B_i^T P & -\alpha I & 0 \\ C_i & 0 & -\alpha I \end{pmatrix} < 0$$

Solve LMI for P
if LMI is feasible **then**
  return System is asymptotically stable
else
  return Stability not guaranteed
end if

The second scenario deals with the addition of disturbance to the nominal AUV system where the nonlinear control law shown in Equation (9) is substituted in Equation (8). If that substitution is made, then the state feedback system with disturbances is obtained as shown in Equation (19).

$$\begin{cases} \dot{x} = (A_i + B_i K)x + D_i\omega \\ y = C_i x + D_o\omega. \end{cases} \tag{19}$$

The new transfer function considering the disturbance matrices is shown in Equation (20)

$$G_{y\omega} = C(SI - A_i + B_i K)^{-1} D_i + D_o. \tag{20}$$

Theorem 3. If α is a positive integer and assuming that $\|G_{y\omega}\|_\infty \leq \alpha$ and also provided that the symmetric positive matrices $\mathtt{X}$ and $\mathtt{W}$ exist then the following LMI is satisfied:

$$\begin{pmatrix} (A_i\mathtt{X} + B_i\mathtt{W})^T + A_i\mathtt{X} + B_i\mathtt{W} & D_i & (C_i\mathtt{X})^T \\ {D_i}^T & -\alpha I & {D_0}^T \\ C_i\mathtt{X} & D_0 & -\alpha I \end{pmatrix} < 0. \tag{21}$$

On satisfying the above LMI, the nominal AUV system with disturbances shown in Equation (19) is asymptotically stable.

Proof. Refer Appendix B. □

The detailed flow for the proof of a perturbed AUV system is shown in the Figure 4. Besides, a pseudo code based on algorithm is shown in Algorithm 2.

Now moving onto the next scenario i. e., perturbed uncertain AUV system where the uncertain AUV system with disturbances shown in Equation (8) is considered. Referring to [25], the transfer function is presented below.

$$G_{y\omega} = C_i(SI - A_i - \Delta A_i - B_i K)^{-1} D_i + D_0 = \begin{pmatrix} A_i + \Delta A_i + B_i K & D_i \\ C_i & D_0 \end{pmatrix}. \tag{22}$$

Referring to [12], from the input and output relationships the Equation 23 is expressed below.

$$\|y\|_2 \leq \|G_{y\omega}(s)\|_\infty \|\omega\|_2. \tag{23}$$

The impact of perturbations and uncertainties on the AUV system shown in Equation (8) will be minimized using a robust H_∞ controller, thereby ensuring asymptotic stability.

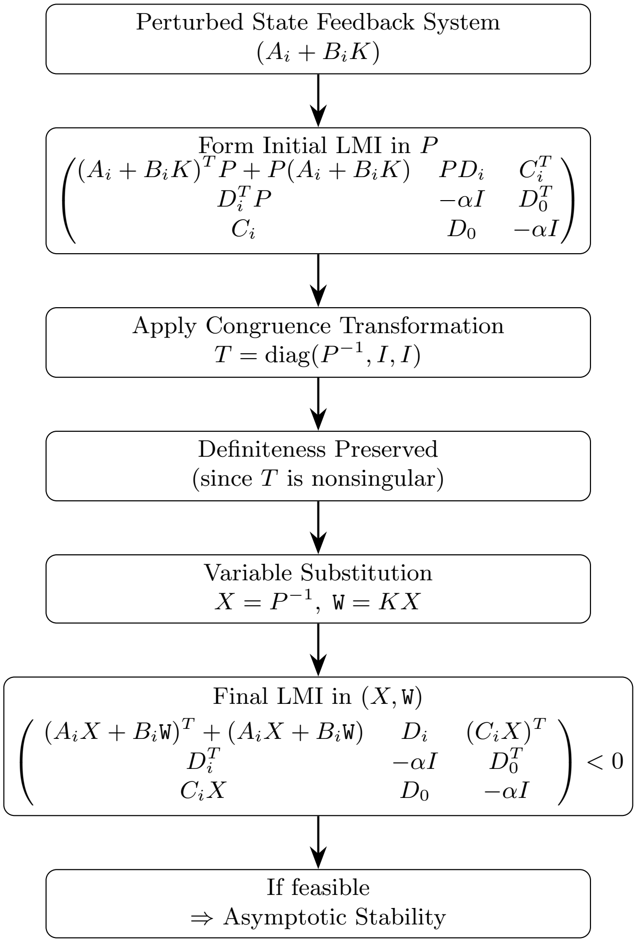


Fig. 4: Flow Representation for a Perturbed AUV System.

Theorem 4. Considering the $G_{y\omega}$ and assuming $\|G_{y\omega}\|_\infty \leq \alpha$ where α is a positive number. If a scalar β along with $\mathtt{X}$ and $\mathtt{W}$ exists where $\mathtt{X}$ is a positive symmetric matrix then the following inequality is satisfied.

$$\begin{pmatrix} \sigma(\mathtt{X},\mathtt{W}) & D_i & (C_i\mathtt{X})^T & (\mathcal{N}\mathtt{X})^T \\ {D_i}^T & -\alpha I & {D_0}^T & 0 \\ C_i\mathtt{X} & D_0 & -\alpha I & 0 \\ \mathcal{N}\mathtt{X} & 0 & 0 & -\beta I \end{pmatrix} < 0, \tag{24}$$

where $\sigma(\mathtt{X},\mathtt{W}) = (A_i\mathtt{X} + B_i\mathtt{W}) + (A_i\mathtt{X} + B_i\mathtt{W})^T + \beta\mathcal{M}\mathcal{M}^T$ The H_∞ control law is given as shown in Equation (25)

$$u = \mathcal{W}\mathtt{X}^{-1}x. \tag{25}$$

Algorithm 2 Stability check for a perturbed AUV system

Require: System matrices A_i, B_i, C_i, D_i, D_0, and $\alpha > 0$
Ensure: Feasibility of final LMI for asymptotic stability

Construct perturbed state feedback system:

$$\dot{x} = (A_i + B_i K)x + D_i \omega$$

Form initial LMI in $P = P^T > 0$:

$$\begin{pmatrix} (A_i + B_i K)^T P + P(A_i + B_i K) & PD_i & C_i^T \\ D_i^T P & -\alpha I & D_0^T \\ C_i & D_0 & -\alpha I \end{pmatrix} < 0$$

Apply congruence transformation:

$$T = \mathrm{diag}(P^{-1}, I, I)$$

Since T is nonsingular, matrix definiteness is preserved
Perform variable substitution:

$$X = P^{-1}, \qquad \mathtt{W} = KX$$

Reformulate the LMI in terms of $(X, \mathtt{W})$:

$$\begin{pmatrix} (A_i X + B_i \mathtt{W})^T + (A_i X + B_i \mathtt{W}) & D_i & (C_i X)^T \\ D_i^T & -\alpha I & D_0^T \\ C_i X & D_0 & -\alpha I \end{pmatrix} < 0$$

if LMI is feasible **then**
  return System is asymptotically stable
else
  return Stability not guaranteed
end if

It follows that the nominal AUV system, when perturbed and subject to uncertainty, is asymptotically stable.

The detailed flow for the proof of a perturbed uncertain AUV system is shown in the Figure 5. Besides, a pseudo code based on algorithm is shown in Algorithm 3.

P r o o f. Refer Appendix C. □

5. RESULTS AND DISCUSSION

The three LMIs discussed above are analyzed accordingly to elaborate the simulation results. In the first LMI a nominal AUV system is considered. The second and third LMIs deal with perturbed AUV system and perturbed uncertain AUV system respectively. For the linear AUV system Equation (12), the initial states are set to

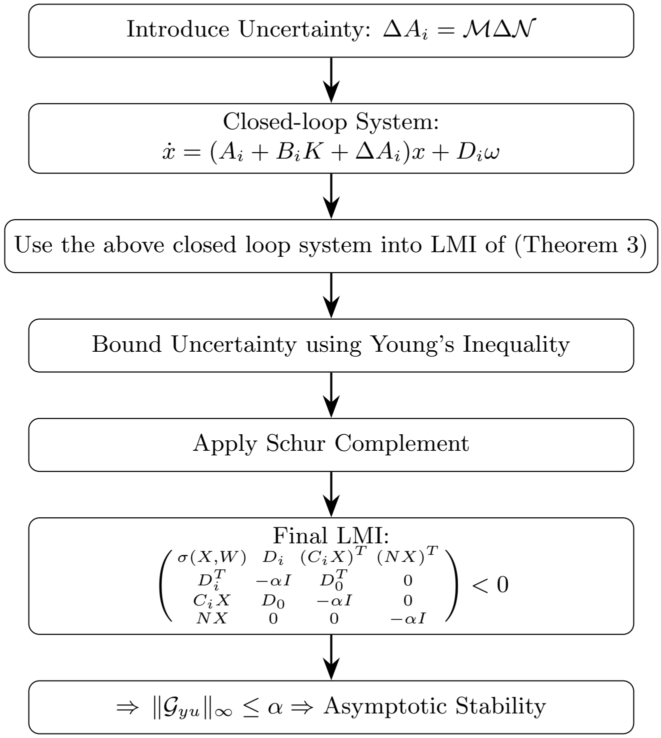


Fig. 5: Flow Representation for a perturbed uncertain AUV System.

$[w_i, q_i, \theta_i] = [0.1, 0.1, 0.1]$. Accordingly, the state matrix, input matrix, and output matrix are obtained as shown below.

$$A_i = \begin{pmatrix} -0.8761 & 1.4166 & -0.0003 \\ 1.2451 & -3.7165 & -0.2633 \\ 0 & 1.0000 & 0 \end{pmatrix}, \; B_i = \begin{pmatrix} -0.4027 \\ -0.9331 \\ 0 \end{pmatrix}, \; C_i = \begin{pmatrix} 0 & 0 & 1 \end{pmatrix}.$$

The developed control algorithm is simulated at a constant surge velocity of 2m/s. Besides the backstepping gain is considered as 0.4. In all the cases the desired depth is considered as a variable step signal at different time instants i. e., follows a step-like pattern over time. It maintains a value of 2 meters for the initial 200 time units, then jumps to 8 meters for the subsequent 200 time units, and finally settles at 6 meters for the last 200 time units.

5.1. Numerical analysis for all the three scenarios

A numerical analysis for all the three scenarios that depict the values of α, $\mathcal{W}$, $\mathtt{X}$, and K are discussed in this subsection.

Algorithm 3 Stability check for a perturbed uncertain AUV system

Require: System matrices A_i, B_i, C_i, D_i, D_0, uncertainty matrices $\mathcal{M}, \mathcal{N}$, and $\alpha > 0$
Ensure: Verify $\|\mathcal{G}_{yu}\|_\infty \leq \alpha$ and stability

Introduce parametric uncertainty: $\Delta A_i = \mathcal{M}\Delta\mathcal{N}$
Form closed-loop system: $\dot{x} = (A_i + B_i K + \Delta A_i)x + D_i\omega$
Substitute into disturbance attenuation LMI (from Theorem 3)
Bound uncertainty using Young's inequality:

$$M\Delta NX + (M\Delta NX)^T \leq \beta\mathcal{M}\mathcal{M}^T + \beta^{-1}(NX)^T(NX)$$

Obtain robustified LMI in (X, W)
Apply Schur complement to include (NX) term
Final LMI:

$$\begin{pmatrix} \sigma(X,W) & D_i & (C_i X)^T & (NX)^T \\ D_i^T & -\alpha I & D_0^T & 0 \\ C_i X & D_0 & -\alpha I & 0 \\ NX & 0 & 0 & -\alpha I \end{pmatrix} < 0.$$

Solve LMI for $X > 0$, W, $\beta > 0$
if solution exists **then**
  return $\|\mathcal{G}_{yu}\|_\infty \leq \alpha$ and asymptotic stability guaranteed
else
  return Stability not guaranteed
end if

Case 1: Nominal AUV system If the LMI shown in Equation (18) is solved using the MATLAB/Simulink and YALMIP tool, then the values of α, $\mathcal{W}$, $\mathtt{X}$, and K are obtained as shown below.

$$\begin{aligned} \alpha &= -0.5454 \\ \mathtt{W} &= \begin{pmatrix} 0.2793 & 0.3258 & 0.1304 \end{pmatrix} \\ \mathtt{X} &= \begin{pmatrix} 0.4721 & 0.1341 & -0.2514 \\ 0.1341 & 0.0297 & -0.3016 \\ -0.2514 & -0.3016 & 3.3489 \end{pmatrix} \\ K &= \begin{pmatrix} 3.2742 & -10.7198 & -0.6806 \end{pmatrix}. \end{aligned}$$

Case 2: Perturbed AUV system A desired depth tracking for a perturbed nominal AUV system is explained, where the disturbance is considered as $\omega = sin(t)$. Besides, the the sensor noise coefficient is also considered that is denoted by ε. Furthermore, the input disturbance vector and output disturbance vector are considered as shown below.

$$D\_i = \begin{pmatrix} {}^{g_1}/_{(m - C_{Z_{\dot{w}}})} & 0 & 0 \\ {}^{g_2}/_{(m - C_{Z_{\dot{w}}})} & \varepsilon & 0 \\ 0 & 0 & \varepsilon \end{pmatrix}, D\_0 = \begin{pmatrix} 0 & W_u & 0 \end{pmatrix},$$

where the internal disturbance coefficients correlated with heave dynamics and pitch dynamics are g_1 and g_2 respectively and W_u disturbance associated with control input.

If the LMI shown in Equation (21) is solved using the MATLAB/Simulink and YALMIP tool, then the values of α, $\mathcal{W}$, $\mathtt{X}$, and K are obtained as shown below.

$$\begin{aligned}
\alpha &= 1.7463 \\
\mathtt{W} &= \begin{pmatrix} 1.0193 & 0.7950 & 1.1254 \end{pmatrix} \\
\mathtt{X} &= \begin{pmatrix} 1.5530 & 0.3826 & -0.2125 \\ 0.3826 & 0.2915 & -0.3193 \\ -0.2125 & -0.3193 & 0.5209 \end{pmatrix} \\
K &= \begin{pmatrix} -3.0375 & 23.5101 & 15.3330 \end{pmatrix}.
\end{aligned}$$

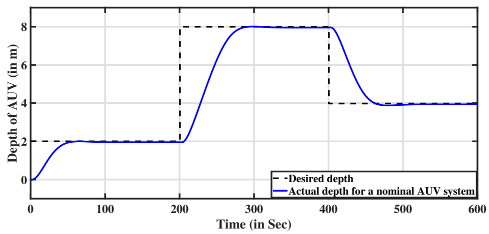
Case 3: Perturbed uncertain AUV system The disturbance considered in the previous subsection will be added to the AUV system along with an uncertainty of 10% to the state matrix. As mentioned $\mathcal{M}$ and $\mathcal{N}$ are the known matrices of appropriate dimensions, they are shown below.
$\mathcal{N} = \begin{pmatrix} 0.0020 & 0.0003 & 0.0074 \\ 0.1005 & 0.0048 & -0.3685 \\ 0.0061 & 0.0062 & 0.0086 \end{pmatrix}$ and $\mathcal{M} = eye(3)$ On solving the LMI shown in Equation (25) using the MATLAB/Simulink and YALMIP tool, the values of α, $\mathcal{W}$, $\mathtt{X}$, and K are obtained as shown below.

$$\begin{aligned}
\alpha &= 167.7772 \\
\mathtt{W} &= \begin{pmatrix} 116.9249 & 68.7232 & 174.6523 \end{pmatrix} \\
\mathtt{X} &= \begin{pmatrix} 326.9132 & 110.7360 & -79.0572 \\ 110.7360 & 91.7693 & -61.4909 \\ -79.0572 & -61.4909 & 57.0603 \end{pmatrix} \\
K &= \begin{pmatrix} 0.5297 & 9.5440 & 14.0789 \end{pmatrix}.
\end{aligned}$$

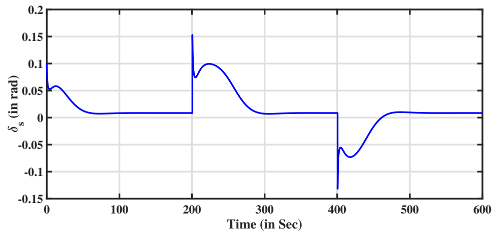
5.2. Ablation study for the proposed controller

The main focus of this study is to minimize the effect of perturbation and uncertainty. This section discusses the analysis of the proposed H_∞ controller alone under nominal, perturbed, and perturbed-uncertain scenarios.

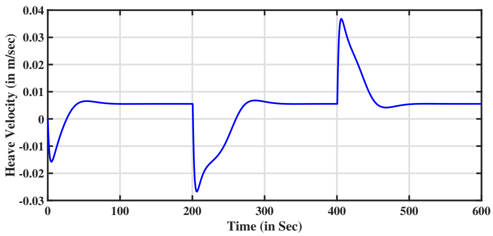
Case 1: Nominal AUV system The depth tracking for a nominal AUV system is shown in the Figure 6. AUV tracks the desired depth depicted in Figure 6a and settles at 58 seconds. However, a small steady state error (SSE) about 0.83% exists that is usually neglected and a very small peak overshoot of 0.2% exists. Referring to Figure 6b, the AUV has the control input ranging from 0.15 radians to -0.12 radians indicating minimal energy consumption. Similarly referring to Figures 6c and 6d, the proposed H_∞ control achieves accurate tracking demonstrating smaller heave velocity and less pitch angle variations.



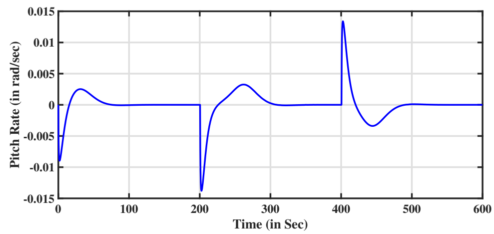
(a) Depth tracking description for AUV



(b) Variation of control input

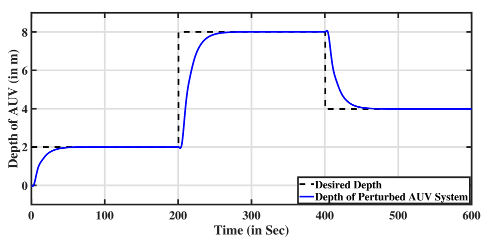


(c) Progress of heave velocity

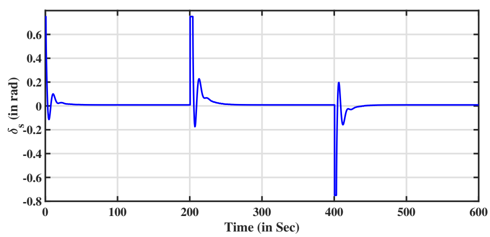


(d) Progress of pitch rate

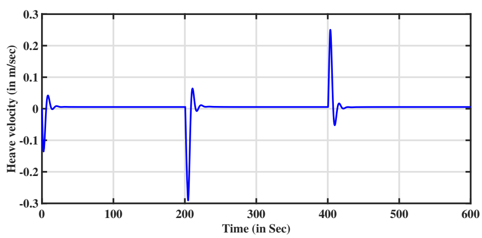
Fig. 6: Reference depth tracking control for a nominal AUV system.



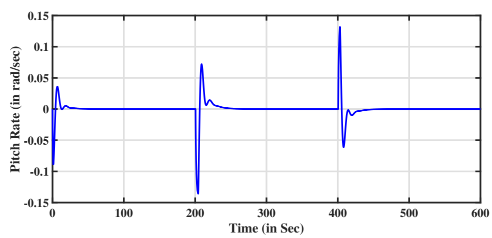
(a) Depth tracking description for a AUV



(b) Variation of control input

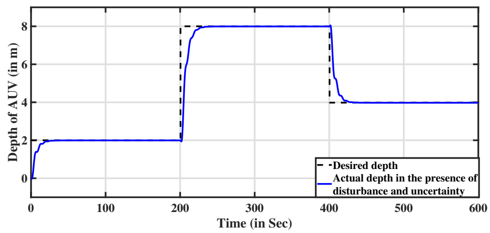


(c) Progress of heave velocity

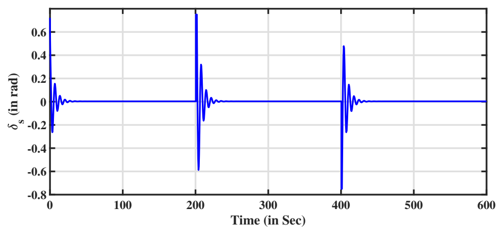


(d) Progress of pitch rate

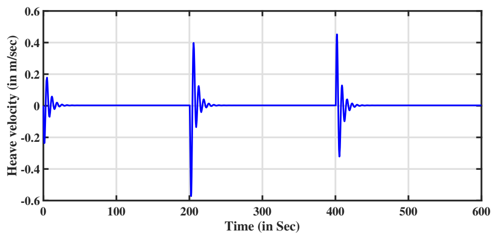
Fig. 7: Reference depth tracking control for a perturbed AUV system



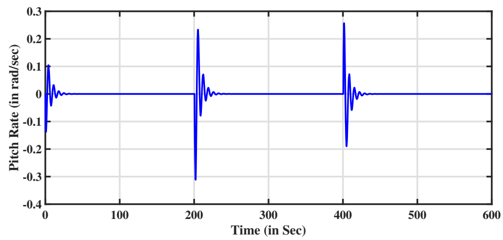
(a) Depth tracking description for a AUV



(b) Variation of control input



(c) Progress of heave velocity



(d) Progress of pitch rate

Fig. 8: Reference depth tracking control for a uncertain perturbed AUV system.

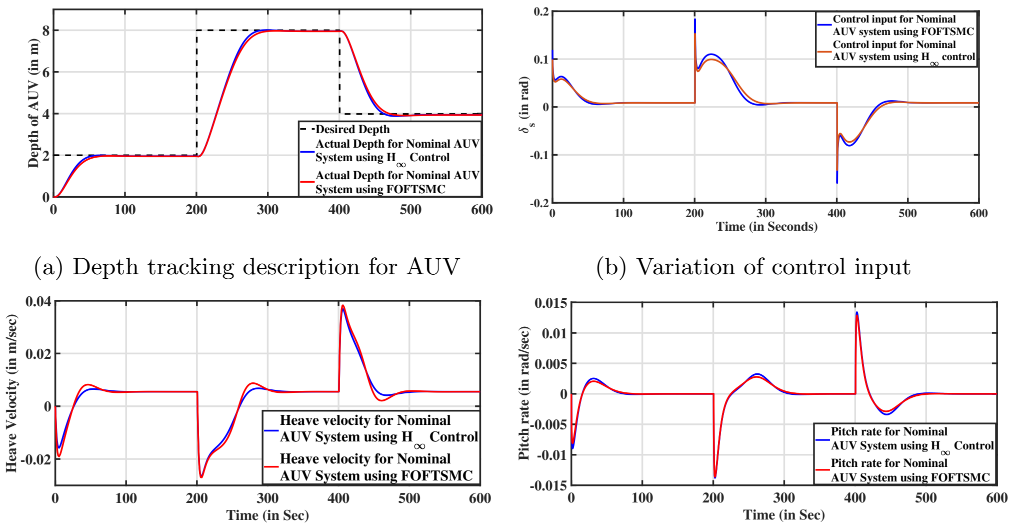
Case 2: Perturbed AUV system The Figure 7 depicts the control of depth for an AUV system when it is subjected to only disturbances without uncertainty. Despite the presence of disturbance it is observed that the AUV tracks the desired depth depicted in Figure 7a. However, in terms of settling time, the proposed H_∞ control has a settling time of about 51 seconds. Considering the effect of peak overshoot and SSE, it is found that the control algorithm has no peak overshoot and SSE. The control input is portrayed in Figure. 7b. It is observed that the proposed H_∞ control seems to be smoother in the control input leading to low energy consumption. Referring to Figures 7c and 7d, it is observed that the proposed control algorithm tend to oscillate to settle to zero initially in terms of heave velocity and pitch rate. It is observed that the proposed control approach settles at a faster rate for heave velocity and pitch rate. Besides, on the return to zero pitch, the proposed H_∞ control has more faster diminished oscillations.

Case 3: Perturbed uncertain AUV system In the third scenario, AUV is subjected to disturbances and uncertaintes simultaneously. The depth tracking of AUV is illustrated in Figure 8a. It is observed that the AUV tracks the desired depth with a very small settling time about 37 seconds. The control input exerted by the AUV to track the desired depth is depicted in Figure 8b. It is observed that the control input has very few oscillations that consumes very minimal energy. Similar kind of explanation can be provided for the heave velocity and pitch rate shown in the Figures 8c and 8d respectively.

5.3. Comparative study with FOFTSMC

A comparative study is made with FOFTSMC because it is a recent nonlinear robust controller that provides faster time convergence and strong disturbance/uncertainty rejection in the underwater applications. As both FOFTSMC and H_∞ consider the similar assumptions, they can be used for fair comparisons under identical conditions.

Case 1: Nominal AUV system The ability of the AUV to follow a desired depth in a perfect environment, without disturbances or uncertainties is investigated.



(a) Depth tracking description for AUV

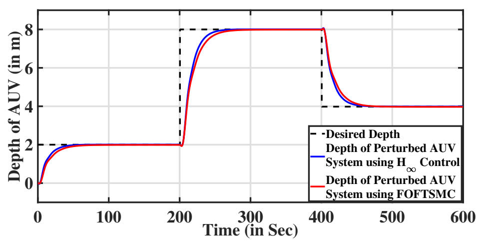
(b) Variation of control input

(c) Progress of heave velocity

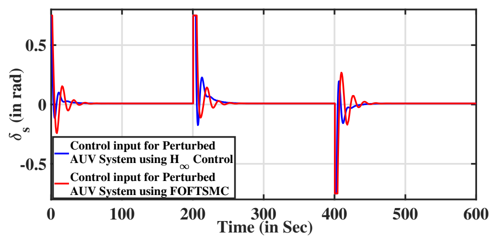
(d) Progress of pitch rate

Fig. 9: Compared reference depth tracking control for a nominal AUV system.

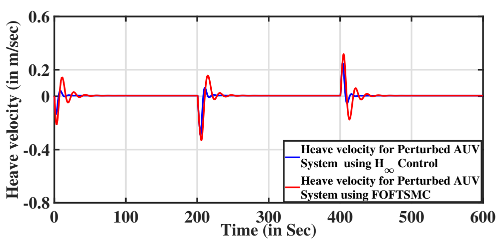
Figure 9 presents the depth tracking performance of the nominal AUV system using the proposed H_∞ control approach and the FOFTSMC method. From the figure it can be deduced that both controllers are able to track the desired depth profile in the absence of parametric uncertainties. However, the performance of the proposed control with FOFTSMC is enhanced in terms of quantitative inspection. The proposed controller yields a faster settling time of approximately 58 seconds, whereas FOFTSMC requires about 69 seconds to reach steady state. This reflects an improvement of nearly 16%. In addition, the proposed controller exhibits a noticeably lower overshoot (0.2%) compared to FOFTSMC (4.5%), which indicates a better damped transient response. Both controllers achieve a small steady-state tracking error (0.83%), which is acceptable for depth regulation tasks. The associated control effort is illustrated in Figure 9b. Here, the proposed controller generates smoother and lower-amplitude control inputs, whereas FOFTSMC exhibits high-frequency switching behavior. This characteristic is known to increase actuator workload and induce higher mechanical stress in underwater thrusters. Practically, smoother control inputs are desirable as they improve energy utilization and reduce wear on actuation hardware. A similar analysis is observed in Figures. 9c and 9d. The FOFTSMC method produces larger variations in heave velocity, indicating more aggressive vertical motion. In contrast, the proposed H_∞ control achieves comparable tracking accuracy with smaller velocity variations. Furthermore, both controllers regulate the pitch rate effectively, confirming that the nominal vehicle remains dynamically stable under each control strategy. Overall, these observations demonstrate that although both controllers achieve depth tracking, the proposed H_∞ control approach offers superior transient performance (reduced overshoot and settling time) while simultaneously demonstrating smoother actuation and lower control input, which are considered for energy-constrained autonomous marine platforms.



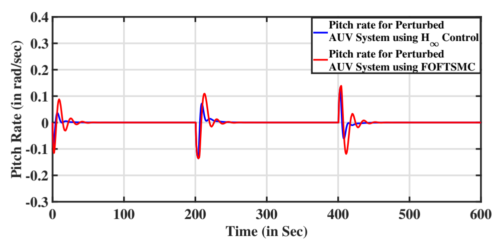
(a) Depth tracking description for AUV



(b) Variation of control input



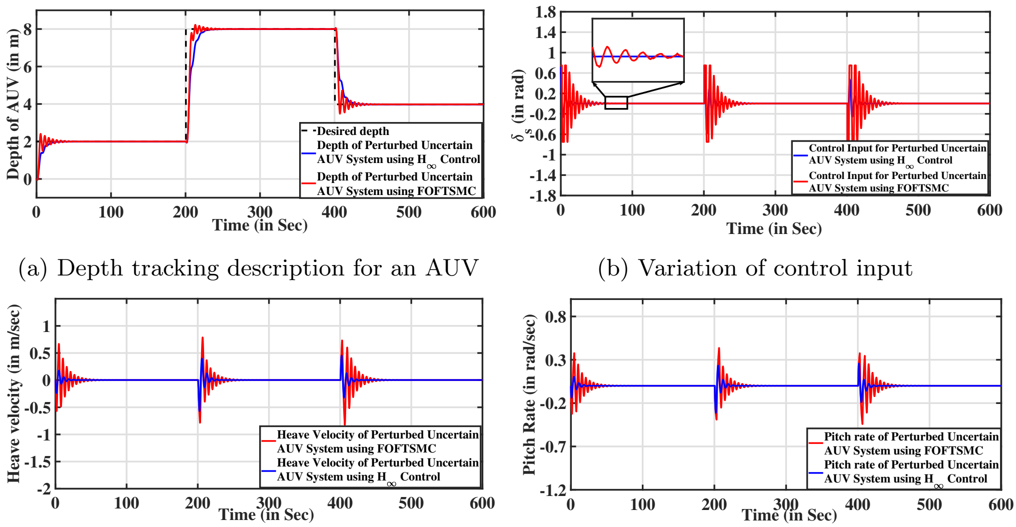
(c) Progress of heave velocity



(d) Progress of pitch rate

Fig. 10: Compared reference depth tracking control for a perturbed AUV system.

Case 2: Perturbed AUV system Figure 10 presents the depth tracking results of the AUV when the system is subjected to external disturbances but does not include model uncertainties. The results indicate that both control approaches are capable of maintaining accurate reference tracking under disturbance, confirming that the nominal dynamics and actuator configuration possess sufficient disturbance rejection characteristics. However, clear performance differences can be observed in the transient response. The proposed H_∞ controller achieves a settling time of approximately 51 seconds, whereas the FOFTSMC method requires about 62 seconds, which corresponds to an improvement of nearly 18% in convergence speed. Moreover, neither controller exhibits noticeable overshoot or steady-state depth deviation, indicating that disturbance effects are successfully attenuated at steady operating conditions. The associated control inputs, shown in Figure 10b, further highlight important practical distinctions. The FOFTSMC strategy generates abrupt control variations and persistent switching behavior, which are characteristic of sliding-mode methods and are known to increase actuator wear and energy demand. In contrast, the proposed H_∞ controller produces smoother control actions with reduced amplitude fluctuations and reduced control effort. Similarly, referring to Figures. 10c and 10d, both controllers exhibit a transient phase for a very small duration and then enters into steady state phase. Normally, the vehicle rejects the initial disturbance inputs. However, the FOFTSMC method demonstrates larger initial oscillations and a longer settling period in both heave velocity and pitch rate, whereas the proposed H_∞ control suppresses these oscillations more rapidly. Faster decay of heave and pitch oscillations is desirable for missions involving sensor payloads.



(a) Depth tracking description for an AUV (b) Variation of control input

(c) Progress of heave velocity (d) Progress of pitch rate

Fig. 11: Compared reference depth tracking control for an uncertain perturbed AUV system.

Case 3: Perturbed Uncertain AUV System The analysis for depth tracking of an AUV system where the AUV is exposed to uncertainties and disturbances is elaborated. The value of β on solving the LMI is obtained as 77.084. Here β is a positive scalar value that ensures the robustness and feasibility of LMI in the presence of uncertainty. The larger value of β is required in order to maintain LMI feasibility. Furthermore, the large value of β prevents the nonlinear part of the system to make it much less stable.

To assess the robustness of the proposed H_∞ control algorithm, depth tracking performance was evaluated for an AUV operating under simultaneous system uncertainty and external disturbances. The tracking responses in Figure 11a indicate that both controllers are capable of stabilizing the system; however, the proposed strategy substantially converges at a faster rate. The settling time for the H_∞ controller is approximately 37 seconds, while FOFTSMC requires about 66 seconds, corresponding to a reduction of nearly 44% in transient duration. Both controllers maintain steady-state tracking

Scenario	Controller	Settling Time (s)	Overshoot (%)	SSE (%)	Remarks
Nominal	H_∞	58	0.2	0.83	Smooth input
Nominal	FOFTSMC	69	4.5	0.83	Higher overshoot
Perturbed	H_∞	51	0	0	Fast & Smooth
Perturbed	FOFTSMC	62	0	0	More settling time
Uncertain+ Perturbed	H_∞	37	0	0.67	Stable under uncertainty and perturbations
Uncertain+ Perturbed	FOFTSMC	66	0	0	More settling time

Tab. 2: Aggregate performance comparison of the AUV for all three scenarios.

accuracy, although the proposed approach exhibits a very small residual steady-state error of roughly 0.67%, which is practically negligible in depth regulation tasks. Additional distinctions are observed in the control input profiles shown in Figure 11b. The FOFTSMC method generates high-frequency oscillations, which can increase control energy and impose greater mechanical stress on actuator thrusters. In contrast, the proposed H_∞ controller produces smoother control signals with markedly reduced oscillations, indicating an energy-efficient and actuator-friendly control effort. The vehicle motion states shown in Figure 11c and Figure 11d further explains the ability of the proposed controller. Under uncertainty and disturbances, FOFTSMC induces larger heave and pitch fluctuations with longer decay times, whereas the H_∞ approach suppresses oscillatory components more effectively. Reduced heave and pitch excursions indicate the vertical motion to be smooth, improved hydrodynamic efficiency, and less convergence to disturbances. Overall, the proposed H_∞ controller achieves superior transient performance, smoother actuator behavior, and better disturbance attenuation.

6. CONCLUSION AND FUTURE SCOPE

6.1. Conclusion

The study of depth tracking of AUV is elaborated in this manuscript under the existence of perturbations and uncertainties. The design of a nonlinear H_∞ control algorithm that employs the LMI approach is adapted for tracking the AUV depth. The performance of the proposed control algorithm is compared in three different scenarios, i. e., nominal AUV system, perturbed AUV system, and perturbed uncertain AUV system. The simulation results depict that in all three scenarios, AUV tracks the desired depth except with a slight SSE in the first scenario. In addition, the efficiency of the proposed control algorithm is explained further by doing the comparison with FOFTSMC. On comparing the proposed control algorithm with FOFTSMC, it is observed that, for a nominal AUV system, the settling time and peak overshoot for FOFTSMC are found to be more when compared with the proposed control algorithm. In the second scenario, where perturbations are added to the AUV system, the settling time is found to be longer for FOFTSMC than the proposed control algorithm. However, the peak overshoot and SSE are found to be zero in both controllers. Besides, the heave velocity and pitch rate seem to consume more oscillations and a bit more time to settle for FOFTSMC. On the other hand, the proposed control approach has more faster diminishing oscillations. The third scenario depicts the tracking of desired depth by adding disturbance and 10% uncertainty to the AUV system. The settling time is found to be 29 seconds less for the FOFTSMC than the proposed control algorithm. However, in terms of SSE, the proposed control algorithm has 0.67%, which can be considered a very small value. From the above analysis, it can be concluded that the H_∞ control performs better than the FOFTSMC in terms of SSE and settling time. Besides, more control energy is consumed for the FOFTSMC than the H_∞ control that may lead to hydrodynamic noise. The simulation of the control algorithm is done through the MATLAB/Simulink environment.

6.2. Future scope

The control algorithm has not considered the effect of actuator faults. Usually, the presence of an actuator fault can predominantly degrade the performance of an AUV in terms of reduced manoeuvrability, deviation from the original depth tracking, increased energy consumption, incomplete missions, etc. Thus, considering the future approach, the authors would like to impart the actuator faults into the AUV system and see that the depth tracking is still achieved.

APPENDIX A

Referring to [5], for a controllable system, $\|\mathcal{G}_{yu}\|_\infty \leq \gamma^{-1}$ exists where γ is a unknown positive parameter, indicating L_2 gain from u to y is bounded by γ^{-1}, i. e.,

$$\sup \frac{\|y\|_2}{\|u\|_2} \implies \int_0^\infty (\gamma y^T(\tau)y(\tau) - \gamma^{-1}u^T(\tau)u(\tau)\,\mathrm{d}\tau) < 0. \tag{A.1}$$

The robust condition using an LMI is achieved by considering the Lyapunov function shown in Equation (A.2).

$$\mathcal{V}(x) = x^T P x, \quad P > 0. \tag{A.2}$$

To satisfy Equation (A.1), it is necessary to ensure Equation (A.3).

$$\dot{\mathcal{V}}(x) + (\gamma y^T(\tau)y(\tau) - \gamma^{-1}u^T(\tau)u(\tau)) < 0. \tag{A.3}$$

On integrating Equation (A.3) and by considering $x(0) = 0$ then $V(0) = x^T P x = 0$, it yields

$$\mathcal{V}(\infty) + \int_0^\infty [\gamma y^T(\tau)y(\tau) - \gamma^{-1}u^T(\tau)u(\tau)]\,\mathrm{d}\tau < 0. \tag{A.4}$$

If $\mathcal{V}(x) > 0$ then to get the asymptotic stability condition $\dot{\mathcal{V}}(x) < 0$. From the Equations (12) and (A.2) it yields

$$\dot{\mathcal{V}}(x) = \dot{x}^T Px + x^T P\dot{x} = (A_i x + B_i u)^T Px + x^T P(A_i x + B_i u). \tag{A.5}$$

Substituting into Equation (A.3), it yields

$$\begin{aligned} &x^T(A_i^T P + PA_i + \gamma C_i^T C_i)x + u^T(-\gamma^{-1} I)u \\ &+x^T(PB_i + \gamma C_i^T)u + u^T(B_i^T P + \gamma C_i)x < 0 \end{aligned} \tag{A.6}$$

The matrix form representation for the Equation (A.6) is presented below in Equation (A.7).

$$\begin{pmatrix} x \\ u \end{pmatrix}^T \begin{pmatrix} {A_i}^T P + PA_i + \gamma {C_i}^T C_i & PB_i + \gamma {C_i}^T \\ {B_i}^T P + \gamma C_i & -\gamma^{-1} I \end{pmatrix} \begin{pmatrix} x \\ u \end{pmatrix} < 0. \tag{A.7}$$

Let $\begin{pmatrix} {A_i}^T P + PA_i + \gamma {C_i}^T C_i & PB_i + \gamma {C_i}^T \\ {B_i}^T P + \gamma C_i & -\gamma^{-1} I \end{pmatrix} = \mathcal{H}$. Now to establish the relations shown in Equations (A.1) and (A.3), the matrix $\mathcal{H}$ is considered to be as negative definite. If an LMI representation of $\mathcal{H}$ yields a positive definite matrix P, then the condition $\|\mathcal{G}_{yu}\|_\infty \leq \gamma^{-1}$ holds. Applying the Schur's complement from [4] to the matrix $\mathcal{H}$ it results in

$$\text{LMI}(P) : \begin{cases} \begin{pmatrix} A_i^T P + PA_i & PB_i & C_i^T \\ B_i^T P & -\gamma^{-1} I & 0 \\ C_i & 0 & -\gamma^{-1} I \end{pmatrix}, \\ P > 0. \end{cases} \tag{A.8}$$

Defining $\alpha = \gamma^{-1}$, the new LMI can be formed as shown in Equation (A.9)

$$\begin{cases} \begin{pmatrix} {A_i}^T P + PA_i & PB_i & {C_i}^T \\ {B_i}^T P & -\alpha I & 0 \\ C_i & 0 & -\alpha I \end{pmatrix} < 0 \\ P > 0 \\ \alpha > 0 \end{cases} \tag{A.9}$$

Hence, if the LMIs in Equations (A.8) and (A.9) exist then the nominal system shown in Equation (12) is asymptotically stable.

APPENDIX B

From the Equations (19) and (18), the following LMI is obtained [5].

$$\begin{pmatrix} (A_i + B_i K)^T P + P(A_i + B_i K) & PD_i & {C_i}^T \\ {D_i}^T P & -\alpha I & {D_0}^T \\ C_i & D_0 & -\alpha I \end{pmatrix}. \tag{B.1}$$

Applying the congruence transformation ($T = diag(P^{-1}, I, I)$) to Equation (B.1) yields

$$\begin{pmatrix} P^{-1}{A_i}^T + P^{-1}K^T{B_i}^T + A_iP^{-1} + B_iKP^{-1} & D_i & P^{-1}{C_i}^T \\ {D_i}^T & -\alpha I & {D_0}^T \\ C_iP^{-1} & D_0 & -\alpha I \end{pmatrix}. \tag{B.2}$$

On applying the congruence transformation, the definiteness of the LMI is maintained because of the nonsingulariy of the T matrix. Furthermore, the terms in the LMI can be further simplified by considering $P^{-1} = X$ and $KX = \mathtt{W}$. Then

$$(A_i + B_iK)X = A_ix + B_iW.$$

Then the LMI shown in Equation (B.2) will be rewritten as shown below.

$$\begin{pmatrix} (A_i\mathtt{X} + B_iK\mathtt{X})^T + (A_i\mathtt{X} + B_iK\mathtt{X}) & D_i & (C_i\mathtt{X})^T \\ {D_i}^T & -\alpha I & {D_0}^T \\ C_i\mathtt{X} & D_0 & -\alpha I \end{pmatrix} < 0, \tag{B.3}$$

where $K = \mathtt{W}\mathtt{X}^{-1}$ If the two matrices $\mathtt{W}$ and $\mathtt{X}$ exist then the above LMI in Equation (B.3) is similar to that of the LMI shown in Equation (21) and hence asymptotically stable.

APPENDIX C

Introducing an uncertainty (ΔA_i) into the perturbed AUV system where $\Delta A_i = \mathcal{M}\Delta\mathcal{N}$ [5]. Considering the equations (8) and (9), the following closed loop system is attained.

$$\begin{cases} \dot{x} = (A_i + \Delta A_i + B_iK)x + D_i\omega \\ y = C_ix + D_0\omega. \end{cases} \tag{C.1}$$

To ensure disturbance attenuation $\|\mathcal{G}_{yu}\|_\infty \leq \alpha$, substitute the above state matrices (C.1) in the theorem 3, the following LMI is obtained.

$$\begin{pmatrix} ((A_i + \Delta A_i)\mathtt{X} + B_i\mathtt{W}) + ((A_i + \Delta A_i)\mathtt{X} + B_i\mathtt{W})^T & D_i & (C_i\mathtt{X})^T \\ {D_i}^T & -\alpha I & {D_0}^T \\ C_i\mathtt{X} & D_0 & -\alpha I \end{pmatrix} < 0. \tag{C.2}$$

The uncertainty term $\Delta A_i\mathtt{X}$ can be expressed as shown in Equation (C.3) adapted from [19].

$$\Delta A_i\mathtt{X} = \mathcal{M}\Delta\mathcal{N}\mathtt{X}. \tag{C.3}$$

Using the Equation (C.3) and from the property of Young's inequality, ($M\Delta NX + (M\Delta NX)^T \leq \beta\mathcal{M}\mathcal{M}^T + \beta^{-1}(\mathcal{N}X)^T(\mathcal{N}X)$), where $\beta > 0$. The above LMI can be rewritten as

$$\begin{pmatrix} (A_i\mathtt{X} + B_i\mathtt{W}) + (A_i\mathtt{X} + B_i\mathtt{W})^T + \beta MM^T + \beta^{-1}\mathtt{X}N^TN\mathtt{X} & D_i & (C_i\mathtt{X})^T \\ {D_i}^T & -\alpha I & {D_0}^T \\ C_i\mathtt{X} & D_0 & -\alpha I \end{pmatrix} < 0. \tag{C.4}$$

Applying the Schur complement to the above LMI [4], it leads to

$$\begin{pmatrix} \sigma(\mathtt{X},\mathtt{W}) & D_i & (C_i\mathtt{X})^T & (\mathcal{N}\mathtt{X})^T \\ {D_i}^T & -\alpha I & {D_0}^T & 0 \\ C_i\mathtt{X} & D_0 & -\alpha I & 0 \\ \mathcal{N}\mathtt{X} & 0 & 0 & -\alpha I \end{pmatrix} < 0, \tag{C.5}$$

where $\sigma(\mathtt{X},\mathtt{W}) = (A_i\mathtt{X}+B_i\mathtt{W})+(A_i\mathtt{X}+B_i\mathtt{W})^T+\beta\mathcal{M}\mathcal{M}^T$. From the control law $u = \mathtt{W}\mathtt{X}^-1x$, where $\mathtt{X} > 0$ is feasible and $\mathtt{W}$, $\beta > 0$ ensure equation (C.5), that implies $\|\mathcal{G}_{yu}\|_\infty \leq \alpha$ and asymptotic stability of the uncertain perturbed AUV system.

DECLARATIONS

The AI tools ChatGPT and Gemini were used to check for any grammatical mistakes. All authors declare that they have no conflicts of interest.

CODE AVAILABILITY

The basic MATLAB simulation code regarding the LMI solving is available at
`https://github.com/aaronmqs/LMI/tree/main/Class%205%20Polytopic%20Uncertainties`

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Siddhartha Vadapalli, Corresponding author. Vishnu Institute of Technology, Department of Electronics and Communication Engineering, Vishnupur, Kovvada Rd, Kovvada, 534202. Andhra Pradesh.
e-mail: vadapalli.siddhartha@gmail.com, siddhartha.v@vishnu.edu.in

Venkata Vamsi Cittiboyna, Vishnu Institute of Technology, Department of Electronics and Communication Engineering, Vishnupur, Kovvada Rd, Kovvada, 534202. Andhra Pradesh.
e-mail: 21pa1a0438@vishnu.edu.in

Venkata Satya Sujeeth Chintha, Vishnu Institute of Technology, Department of Electronics and Communication Engineering, Vishnupur, Kovvada Rd, Kovvada, 534202. Andhra Pradesh.
e-mail: 21pa1a0435@vishnu.edu.in

Aditya Subham, *Vishnu Institute of Technology, Department of Electronics and Communication Engineering, Vishnupur, Kovvada Rd, Kovvada, 534202. Andhra Pradesh.*
e-mail: 21pa1a0406@vishnu.edu.in

Sravanth Kumar Bongu, *Vishnu Institute of Technology, Department of Electronics and Communication Engineering, Vishnupur, Kovvada Rd, Kovvada, 534202. Andhra Pradesh.*
e-mail: 21pa1a0425@vishnu.edu.in

Badrinath Bezawada, *Vishnu Institute of Technology, Department of Electronics and Communication Engineering, Vishnupur, Kovvada Rd, Kovvada, 534202. Andhra Pradesh.*
e-mail: 21pa1a0418@vishnu.edu.in