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Asymptotic behaviour of solutions of linear discrete equations

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Abstract. Asymptotic behaviour of a particular solutions of the linear discrete nonhomogeneous equation

$$\Delta u(k) = A(k)u(k) + g(k), \;\; k \in N(a)$$

is considered, where $\Delta u(k) = u(k+1) - u(k)$, $N(a) = \{a, a+1, \dots\}$, $a \in \mathbb{N}$ is fixed, $\mathbb{N} = \{0, 1, \dots\}$ and $A, g : N(a) \to \mathbb{R}$.

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Let us consider the linear discrete nonhomogeneous equation

$$\Delta u(k) = A(k)u(k) + g(k), \;\; k \in N(a) \tag{1}$$

where $\Delta u(k) = u(k+1) - u(k)$, $N(a) = \{a, a+1, \dots\}$, $a \in \mathbb{N}$ is fixed, $\mathbb{N} = \{0, 1, \dots\}$ and $A, g : N(a) \to \mathbb{R}$. Suppose $A(k) \neq 0$ for every $k \in N(a)$.

This is the preliminary version of the paper.

Let us construct a formal series which satisfies equation (1). Define a sequence of functions

$$f_0(k), f_1(k), \dots, f_n(k), \dots, \quad k \in N(a),$$

as follows:

$$f_0(k) = -\frac{g(k)}{A(k)}, \quad f_p(k) = \frac{\Delta f_{p-1}(k)}{A(k)}, \quad k \in N(a)$$

where $p = 1, 2, \dots$. Obviously, this sequence is well defined for every $k \in N(a)$. Define a *formal series*

$$\mathcal{FS}(k) := f_0(k) + f_1(k) + \dots + f_n(k) + \cdots . \tag{2}$$

Lemma 1. *Suppose $A(k) \neq 0$ for every $k \in N(a)$. Then the formal series $\mathcal{FS}(k)$ defined by relation (2) is a* formal solution *of equation (1).*

Theorem 2. [1] *Let us suppose that for every $k \in N(a)$ and a fixed $p \in \{0\} \cup \mathbb{N}$:*

1) $A(k) \neq 0$.
2) $f_{p+1}(k) < 0$, $\Delta f_p(k) < 0$ *and* $\Delta f_{p+1}(k) > 0$.

Then there exists a particular solution $u^{part} = u^{part}(k)$, $k \in N(a)$ of the discrete linear nonhomogeneous equation (1) such that the inequalities

$$\sum_{s=0}^{p+1} f_s(k) < u^{part}(k) < \sum_{s=0}^{p} f_s(k)$$

hold for every $k \in N(a)$.

Theorem 3. [1] *Let us suppose that for every $k \in N(a)$ and a fixed $p \in \{0\} \cup \mathbb{N}$:*

1) $A(k) \neq 0$.
2) $f_{p+1}(k) > 0$, $\Delta f_p(k) > 0$ *and* $\Delta f_{p+1}(k) < 0$.

Then there exists a particular solution $u^{part} = u^{part}(k)$, $k \in N(a)$ of the discrete linear nonhomogeneous equation (1) such that the inequalities

$$\sum_{s=0}^{p} f_s(k) < u^{part}(k) < \sum_{s=0}^{p+1} f_s(k)$$

hold for every $k \in N(a)$.

Example 4. Let us consider a linear discrete equation

$$\Delta u(k) = k^5 u(k) - k^6. \tag{3}$$

In accordance with Theorem 3 ($p = 0$) there exists a particular solution $u^{part} = u^{part}(k)$, $k \in N(1)$ of the equation (3) such that the inequalities

$$k < u^{part}(k) < k + \frac{1}{k^5}$$

hold for every $k \in N(1)$.

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