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SOME APPLICATION OF MARTINGALES IN BANACH SPACES

by

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This is an outline of the paper "Martingales with values in uniformly convex spaces" by Giles Pisier, that has come out in Israel J. Math.

We say a Banach space is q-convex ($2 \leq q < +\infty$) if there is an equivalent norm on X whose modulus of convexity fulfils: $\forall \varepsilon > 0$: $\delta(\varepsilon) \geq C \varepsilon^q$.

Let $(\Omega, (A_n)_{n \geq 0}, P)$ be the probability space, where $\Omega = \{-1,1\}^N$ with its Borel σ-algebra and the usual probability P. A_0 will be the trivial σ-algebra $\{\emptyset, \Omega\}$ on Ω and for $n \geq 1$ A_n will be the σ-algebra generated by the first n coordinates on Ω. A martingale relative to $(\Omega, (A_n)_{n \geq 0}, P)$ is called Walsh-Paley martingale.

If $(X_n)_{n \geq 0}$ is a martingale with values in a Banach space X, we denote by $(dX_n)_{n \geq 0}$ the sequence $dX_n = X_n - X_{n-1}$, $dX_0 = X_0$.

By $\|X\|_\infty$ we denote the essential supremum of $X(t)$.

Theorem 1. A Banach space X is super-reflexive iff for every $\alpha \in (1,+\infty)$ there is a constant C and $r > 1$ such that for all X-valued martingales $(X_n)_{n=0}$ satisfy

$$\sup \|X_n\|_\alpha \leq C\left(\sum_{n=0} \|dX_n\|_\alpha^r\right)^{\frac{1}{r}} .$$

Theorem 2. Let $1 \leq q < \infty$ and let X be a Banach space. Assume that there is a constant C for which all X-valued Walsh-Paley martingales $(X_n)_{n \geq 0}$ satisfy:

$$E\|X_0\|^q + \sum_{n \geq 1} E\|dX_n\|^q \leq C^q \sup_n E\|X_n\|^q$$

then X is q-convex.

Lemma. Let r be a number in $(1,2\rangle$ and X be a Banach space. Assume that - for some constant D - all the X-valued martingales $(X_m)_{m \geq 0}$ satisfy

$$\|X_m\|_2 \leq D(n+1)^{\frac{1}{r}} \sup_{0 \leq k \leq n} \|dX_k\|_\infty$$

Then for all $p < r$ there is a constant C_p for which all X-valued Walsh-Paley martingales $(X_m)_{m \geq 0}$ fulfil

$$\sup_n E\|X_n\|^p \leq C_p(E\|X_0\|^p + \sum_{n \geq 1} E\|dX_n\|^p).$$

Therefore, by Th. 2, X is p-convex.

From the foregoing theorems we get

Theorem 3 (Enflo, Pisier). Every super-reflexive space is p-convex for some $p > 1$.