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## FOURTH WINTER SCHOOL (1976)

## SOME APPLICATION OF MARTINGALES IN BANACH SPACES

by

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This is an outline of the paper "Martingales with values in uniformly convex spaces" by Giles Pisier, that has come out in Israel J. Math.

We say a Banach space is  $q$ -convex ( $2 \leq q < +\infty$ ) if there is an equivalent norm on  $X$  whose modulus of convexity fulfills:  $\forall \varepsilon > 0: \sigma(\varepsilon) \geq C \varepsilon^q$ .

Let  $(\Omega, (A_n)_{n \geq 0}, P)$  be the probability space, where  $\Omega = \{-1, 1\}^{\mathbb{N}}$  with its Borel  $\sigma$ -algebra and the usual probability  $P$ .  $A_0$  will be the trivial  $\sigma$ -algebra  $\{\emptyset, \Omega\}$  on  $\Omega$  and for  $n \geq 1$   $A_n$  will be the  $\sigma$ -algebra generated by the first  $n$  coordinates on  $\Omega$ . A martingale relative to  $(\Omega, (A_n)_{n \geq 0}, P)$  is called Walsh-Paley martingale.

If  $(X_n)_{n \geq 0}$  is a martingale with values in a Banach space  $X$ , we denote by  $(dX_n)_{n \geq 0}$  the sequence  $dX_n = X_n - X_{n-1}$ ,  $dX_0 = X_0$ .

By  $\|X\|_{\infty}$  we denote the essential supremum of  $X(t)$ .

Theorem 1. A Banach space  $X$  is super-reflexive iff for every  $\alpha \in (1, +\infty)$  there is a constant  $C$  and  $r > 1$  such that for all  $X$ -valued martingales  $(X_n)_{n=0}$  satisfy

$$\sup \|X_n\|_{\infty} \leq C \left( \sum_{n=0}^{\infty} \|dX_n\|_{\infty}^r \right)^{\frac{1}{r}}.$$

**Theorem 2.** Let  $1 \leq q < \infty$  and let  $X$  be a Banach space. Assume that there is a constant  $C$  for which all  $X$ -valued Walsh-Paley martingales  $(X_n)_{n \geq 0}$  satisfy:

$$E \|X_0\|^q + \sum_{n \geq 1} E \|dX_n\|^q \leq C^q \sup_n E \|X_n\|^q$$

then  $X$  is  $q$ -convex.

**Lemma.** Let  $r$  be a number in  $(1, 2)$  and  $X$  be a Banach space. Assume that - for some constant  $D$  - all the  $X$ -valued martingales  $(X_m)_{m \geq 0}$  satisfy

$$\|X_m\|_2 \leq D(n+1)^{\frac{1}{r}} \sup_{0 \leq k \leq n} \|dX_k\|_\infty$$

Then for all  $p < r$  there is a constant  $C_p$  for which all  $X$ -valued Walsh-Paley martingales  $(X_m)_{m \geq 0}$  fulfil

$$\sup_n E \|X_n\|^p \leq C_p (E \|X_0\|^p + \sum_{n \geq 1} E \|dX_n\|^p).$$

Therefore, by Th. 2,  $X$  is  $p$ -convex.

From the foregoing theorems we get

**Theorem 3 (Enflo, Pisier).** Every super-reflexive space is  $p$ -convex for some  $p > 1$ .