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ON CONCRETE FUNCTORS IN UNIFORM SPACES

by Jiří Vilímovský

We work in the category $\mathbf{U}$ of Hausdorff uniform spaces and uniformly continuous mappings. A functor $F : \mathbf{U} \to \mathbf{U}$ is called concrete, if $\Box F = F$, where $\Box$ is the forgetful functor into sets. F will be called a concrete reflector, if F is idempotent and FX is coarser than X for all X, dually F is a concrete coreflector, if it is idempotent and FX is finer than X for all X.

Recall that concrete reflectors correspond one to one with the classes of uniform spaces which are productive, hereditary and contain a compact interval. Concrete coreflectors correspond to the classes which are closed under sums, quotients and contain a nonvoid space.

The special role will play embedding preserving functors, i.e. those, for which FX is a subspace of FY provided that X is a subspace of Y. Observe that in the case of concrete coreflectors F is embedding preserving iff the corresponding class is hereditary.

<u>Theorem</u>: a) If $(\mathcal{R},F)$ is a concrete reflection (that means F is a concrete reflector and $\mathcal{R}$ is the corresponding reflective class), then there is the largest embedding preserving concrete reflection $(\overline{\mathcal{R}},\overline{F})$ contained in $(\mathcal{R},F)$. (See [4]).

b) If $(\mathcal{C},F)$ is a concrete coreflection, there is the smallest embedding preserving coreflection $(\overline{\mathcal{C}},\overline{F})$ containing $(\mathcal{C},F)$. (See [3]).

Our aim is to study the behavior of such functors on the compact interval I, the hedgehog $H(\omega)$ (the cone over ω with uniformly discrete uniformity), moreover we give some extremal coreflective conditions for "noncontaining" these spaces.

<u>Theorem</u> ([4]): If F is a concrete reflector in $\mathbf{U}$, then either $FH(\omega) = H(\omega)$, or $FH(\omega) = pH(\omega)$, where p stands for the pre-

compact reflector.

Moreover it can be proved that:

Theorem ([4]): If X is a distal space (i.e. a space having a base of finite-dimensional covers), F a concrete embedding preserving reflector, then there is some cardinal reflection p^m such that $FX = p^m X$.

Theorem ([2]): If F is a coreflector in U, then either $FI = I$, or all finite partitions of I into Baire sets are uniform in FI.

Theorem ([5]): If F is a coreflector in U, then either $FH(\omega) = H(\omega)$, or all finite cozero covers of $H(\omega)$ are uniform in $FH(\omega)$.

Theorem ([2]): There exists the largest coreflective subclass $\mathcal{E}_I$ of U not containing I. The following properties of a space X are equivalent:

a) $X \in \mathcal{E}_I$.

b) Each finite Baire partition of X is a uniform cover.

c) Each Baire-measurable $f : X \to I$ is uniformly continuous.

d) If $\{A_n\}_{n=0}^{\infty}$ is a family of subsets of X such that for $n \geq 1$ the set A_n is far from $A \setminus A_n$, where $A = \bigcup\{A_n;\ n=0,1,\dots\}$, then A_0 is far from $A \setminus A_0$ in X.

e) If f is a pointwise limit of uniformly continuous functions $f_h : X \to I$, then f is uniformly continuous.

The properties b), c), e) were studied previously by A.Hager and Z.Frolík. Note that from the theorem follows that for any space X having a nonuniform finite Baire partition one can inductively generate I from X.

Theorem ([5]): There exists the largest coreflective subclass $\mathcal{E}_H$ of U not containing $H(\omega)$. The following properties of a space X are equivalent:

a) $X \in \mathcal{E}_H$.

b) Each countable uniformly discrete union of boundedly finite uniformly discrete families is uniformly discrete.

c) If $f : X \to H(\omega)$ is uniformly continuous, then the f-preimage of each finite open cover of $H(\omega)$ is uniform in X.

d) If $f_n : X \to I$ is a sequence of uniformly continuous functions such that the family $\{\mathrm{coz}\ f_n;\ n \in \omega\}$ is uniformly discrete, then the mapping $\sum f_n$ is uniformly continuous.

e) For each subspace Y of X, $f : Y \to R$ a uniformly continuous real valued function, the preimage of each finite open cover of R is uniform in Y.

We finish with the following surprising result:

Theorem ([5]): (Assuming the nonexistence of a uniformly sequential cardinal.)

There exists the largest nontrivial hereditary coreflective subclass of $\mathcal{U}$, namely the class $\mathcal{E}_H$.

References

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