## Jan Reiterman; Vojtěch Rödl On non-zero dimensional atoms

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## ON NON-ZERO DIMENSIONAL ATOMS J. Reiterman, V. Rödl

We consider the lattice of uniformities on a countable set, say on  $\omega$  (w.r.t. the order " $a \prec \pm$  iff a is finer than  $\Xi$ "). A uniformity a is an atom in this lattice iff the only uniformity strictly finer than a is uniformly discrete one. The investigation of non-O-dimensional atoms was initiated by the fact that first constructions  $[PR]_{,}[S]$  led to O-dimensional atoms (a uniformity is O-dimensional if it admits a base consisting of partitions). E.g. in [PR] a complete description of atoms inducing non-discrete proximities is given; all these atoms are O-dimensional. The existence of non-zero dimensional atoms was established in [RR] under the CH.

If an atom & induces the discrete proximity then there exists an ultrafilter F with  $a < u_F$  where  $U_F$  is the uniformity consisting of all covers C with  $C \cap F \neq \emptyset$ . [PR]. The problem to describe all non-zero dimensional atoms or at least to characterize those ultrafilters F with  $a < u_F$ for some non-zero dimensional atom a seems to be difficult. By [S], each of these Fs is non-rare, that is, it admits a finiteto-one map  $q: \omega \rightarrow \omega$  such that g = qF is non-equivalent to F. So we considered an easier problem: characterize those Q for which there exists  $q_1 \notin A$  as above. The solution is given by the theorem below which solves also the problem of the existence of two uniformities  $u_1, u_2$  with dist  $u_4 \wedge$  $\wedge$  dist  $M_2 \neq$  dist  $(u_4 \wedge u_6)$ ; here dist  $M_4$ , the distality of  $u_3$ is the uniformity generated by M-covers of finite order. <u>Theorem</u>: For every ultrafilter  $\mathcal{G}$  on  $\omega$  there exist a finite-to-one map  $q: \omega \rightarrow \omega$  an ultrafilter  $\mathcal{F}$  on  $\omega$  with  $q\mathcal{F}=q$  and an uncountable family  $\{a_i\}_i$  of non-zero dimensional atoms such that  $a_i < \omega_{\mathcal{F}}$  for every i. The atoms  $a_i$  have the same distality. Thus, dist  $a_{ij} \wedge dist a_{ij} \neq$  $\neq$  dist  $(a_{ij} \wedge a_{ij})$  for any two of them.

## References :

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  - [S] P. Simon: Uniform Atoms on w, Seminar Uniform Spaces 1975 - 1976, MÚ ČSV Prague 1976, p. 7 - 35