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ON THE SINGLEVALUEDNESS AND DIFFERENTIATION OF METRIC PROJECTIONS

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Let X be a real Banach space and $M \subset X$ a closed subset of X. For $x \in X$ denote by $d_M(x)$ the distance from the point x to the set M. The metric projection P_M of the space X on the set M is defined as the /possibly/ multivalued operator

$$P_M(x) = \{ y \in M \; ; \; \|x-y\| = d_M(x) \} .$$

Denote by A_M the set of the multivaluedness of P_M.
The sets A_M were investigated e.g. in [2], [5] and [3].

Definition. Let $o \neq v \in X$ and Z be a topological complement of $\mathrm{Lin}\{v\}$. Let f be a Lipschitz function defined on Z. Then the set $M = \{ z + f(z)\, v \; ; \; z \in Z \}$ is termed a Lipschitz hypersurface.

Theorem 6 If X is a separable strictly convex Banach space then A_M can be always covered by countably many of Lipschitz hypersurfaces.

In the following the Frechet differentiability of multivalued operators is consider in the natural generalized sense. By N_M we denote the set of all points at which P_M is not Frechet differentiable. The sets N_M were investigated e.g. in [4] and [1].

Theorem [7] There exists a compact convex set $M \subset R^2$ such that $R^2 - (M \cup N_M)$ is a set of the first category.

Theorem [7] If X is a two dimensional strictly convex Banach space then N_M is always a set of /Lebesgue/ measure zero.

Theorem [7] Let X be a finite dimensional space with a norm q which belongs to the class $C^2(X - \{o\})$ and for which $D^2 q(x)(h,h) > 0$ for any linearly independent $x \neq o, h \neq o$. Then N_M is always a set of /Lebesgue/ measure zero.

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