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On generalizations of Lašnev's theorem

J. Chaber

We investigate spaces X with the following property (*) for any Y and any closed mapping $f: X \rightarrow Y$ Y = = $Y_0 \cup \bigcup_{i=1}^{\infty} Y_i$, where $f^{-1}(y)$ is compact for $y \in Y_0$ and Y_i is closed and discrete in Y for $i \ge 1$.

It has been proved by Lašnev that metric spaces satisfy (*).

A list of generalizations of Lašnev's result with exact references may be found in a recent survey paper on closed mappings [B] (see also [D] and [W]).

We prove

Theorem 1. Regular d'-spaces satisfy (*) .

As a corollary we get

Corollary 1 [W]. Moore spaces satisfy (*) .

<u>Theorem 2</u>. If X is Čech complete, $f : X \rightarrow Y$ closed and $\partial f^{-1}(y)$ is compact for $y \in Y$, then Y has a decomposition as in (*).

<u>Corollary 2.1</u>. Metalindelöf Čech complete spaces satisfy (*) . <u>Corollary 2.2</u> [D]. Dieudonné complete Čech complete spaces satisfy (*) .

The following example illustrates Theorem 1. <u>Example 1</u>. One can construct three topologies on the unit square such that the projection f of the resulting spaces X_n n=1,2,3 onto the unit interval I is continuous and closed and 1. X_1 as Hausdor σ -space and f c c for $y \in J$,

 X_2 of compact but f (y) is not compact \in

. X is paracompact and has a closure rv ve y compact t but $f^{-1}(y)$ is not Li delöf for

In Th orem 2 one cannot r place C ch completenes by t p-space property.

Example 2. There exists a p-space X and a closed mapping f of X onto a locally compact space Y such that $2f^{-1}(y)$ is compact for $y \in Y$ and Y does not have any decom omition so in (*).

In view of Corollary 2,1 and a result of Veličko the following problem seems to be natural

<u>Probl m.</u> Do metalindelöf p space satisfy (+) ?

Th method of proof of Theorem 2 cannot be used to solv this problem because of Ex mple 3. There exists a perfect mapping of a met lind l⁷f

p-space onto a space that is not a p-space.

References

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