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On generalizations of Lešnev's theorem

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We investigate spaces X with the following property

(*) for any Y and any closed mapping $f: X \longrightarrow Y$ $Y =$ $= Y_0 \cup \bigcup_{i=1}^{\infty} Y_i$, where $f^{-1}(y)$ is compact for $y \in Y_0$ and Y_i is closed and discrete in Y for $i \geq 1$.

It has been proved by Lašnev that metric spaces satisfy (*) .

A list of generalizations of Lašnev's result with exact references may be found in a recent survey paper on closed mappings [B] (see also [D] and [W]).

We prove

Theorem 1. Regular σ-spaces satisfy (*) .

As a corollary we get

Corollary 1 [W]. Moore spaces satisfy (*) .

Theorem 2. If X is Čech complete, $f: X \longrightarrow Y$ closed and $\partial f^{-1}(y)$ is compact for $y \in Y$, then Y has a decomposition as in (*) .

Corollary 2.1. Metalindelöf Čech complete spaces satisfy (*) .

Corollary 2.2 [D]. Dieudonné complete Čech complete spaces satisfy (*) .

The following example illustrates Theorem 1.

Example 1. One can construct three topologies on the unit square such that the projection f of the resulting spaces X_n $n=1,2,3$ onto the unit interval I is continuous and closed and

1. X_1 e a Hausdor σ-space and f

c c for $y\in$,

X_2 σ compact but f (y) is not compact

$\in$

. X is paracompact and has a closure rv ve

y compact t but $f^{-1}(y)$ is not Li delöf for

In Th orem 2 one cannot r place Č ch completenes by t p-space property.

<u>Example 2.</u> There exists a p-space X and a closed mapping f of X onto a locally compact space Y such that $\partial f^{-1}(y)$ is compact for $y\in Y$ and Y does not have any decom osition as in (*) .

In view of Corollary 2.1 and a result of Veličko the following problem seems to be naturél

<u>Probl m.</u> Do metalindelöf p space satisfy (*) ?

Th method of proof of Theorem 2 cannot be used to solv this problem because of

<u>Ex mple 3.</u> There exists a perfect mapping of a met lind l"f p-space onto a space that is not a p-space.

References

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