

Ivan Kolář

Lie derivatives and natural operators

In: Zdeněk Frolík (ed.): Abstracta. 8th Winter School on Abstract Analysis. Czechoslovak Academy of Sciences, Praha, 1980. pp. 108--109.

Persistent URL: <http://dml.cz/dmlcz/701188>

Terms of use:

© Institute of Mathematics of the Academy of Sciences of the Czech Republic, 1980

Institute of Mathematics of the Academy of Sciences of the Czech Republic provides access to digitized documents strictly for personal use. Each copy of any part of this document must contain these *Terms of use*.



This paper has been digitized, optimized for electronic delivery and stamped with digital signature within the project *DML-CZ: The Czech Digital Mathematics Library* <http://project.dml.cz>

LIE DERIVATIVES AND NATURAL OPERATORS

Ivan KOLÁŘ, Brno

Given two manifolds M, N , a smooth map $f: M \rightarrow N$, a vector field ξ on M and a vector field η on N , one defines the Lie derivative of f with respect to ξ and η by

$$L_{(\xi, \eta)} f := Tf \circ \xi - \eta \circ f: M \rightarrow TN.$$

Let F, G be two prolongation functors of the category of smooth manifolds into the category of smooth fibered manifolds. Every vector field ξ on M is prolonged into a vector field $F\xi$ on FM and into another vector field $G\xi$ on GM , [1]. Having a base-preserving morphism $\varphi: FM \rightarrow GM$, we define its Lie derivative with respect to ξ by

$$L_{\xi} \varphi := L_{(F\xi, G\xi)} \varphi: FM \rightarrow VGM,$$

where V denotes the vertical tangent bundle. If the values of G lie in the subcategory of smooth vector bundles, then $L_{\xi} \varphi$ can be considered as a map $L_{\xi} \varphi: FM \rightarrow GM$.

Let \bar{F}, \bar{G} be two further prolongation functors. An operator A transforming any base-preserving morphism $\varphi: FM \rightarrow GM$ into a base-preserving morphism $A\varphi: \bar{F}M \rightarrow \bar{G}M$ is called natural, if it holds

$$A((Gf)^{-1} \circ \varphi \circ Ff) = (\bar{G}f)^{-1} \circ A\varphi \circ \bar{F}f$$

for every diffeomorphism f . Operator A is said to be regular, if it transforms any smoothly parametrized family of morphisms into a smoothly parametrized family.

Theorem. Let $F, G, \bar{F}, \bar{G}, \xi, \varphi$ and A be as above and let the values of G and \bar{G} lie in the category of smooth vector bundles.

If A is natural, \mathbb{R} -linear and regular, then A commutes with the Lie derivative, i.e.

$$L_{\xi}(A\varphi) = A(L_{\xi}\varphi)$$

for all φ and ξ .

REFERENCES

- [1] S. A. SALVIOLI, On the theory of geometric objects,
 J. Differential Geometry, 7(1972), 257-278