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Examples of Stable C_0-Semigroups

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An important problem in the theory of semigroups of operators is to decide whether

$$s(A; U) = \omega(U) \tag{1}$$

for the generator A of a C_0-semigroup $\{U(t)\}_{t \geqq 0}$ of bounded linear operators in a Banach space, where $s(A; U) = \sup\{\mathrm{Re}(\lambda); \lambda \in \sigma(A)\}$ is the *spectral bound* of A and $\omega(U) = \inf\{t^{-1} \log \|U(t)\|; t > 0\}$ is the *growth bound* of U; see, for example, [16; 17; 21; 22; 28] and the relevant references therein. Semigroups satisfying (1) will be called *stable*.

Whereas it is always the case that $s(A; U) \leqq \omega(U)$, [12; § 1], it is known that (1) is not valid in general, neither for semigroups in Hilbert, [30], nor for positive semigroups in Banach lattices, [16]. However, there are many classes of C_0-semigroups for which (1) does hold; see, for example, [12; 17; 25; 28; 29].

A well known criterion guaranteeing stability is the spectral mapping property

$$\sigma(U(t)) \setminus \{0\} = \{e^{t\lambda}; \lambda \in \sigma(A)\}, \quad t \geqq 0,$$

together with eventual norm continuity of the C_0-semigroup $\{U(t)\}_{t \geqq 0}$; see [22; pp. 87–88]. This includes all eventually compact semigroups, all eventually differentiable semigroups, all holomorphic semigroups and all uniformly continuous semigroups. It may be of interest to note that there are examples of infinite dimensional Banach spaces in which *every* C_0-semigroup is stable. Let X be a Grothendieck space with the Dunford-Pettis property (e.g. L^∞-spaces, $H^\infty(\mathbb{D})$, certain $C(\Omega)$ spaces; see [20], for example). In such spaces X every C_0-semigroup is stable, by the above comments, since such semigroups are always uniformly continuous, [20; Theorem 6].

We wish to suggest some specific (and rather special) examples of stable C_0-semigroups coming from (some) areas of analysis, which are stable for very simple reasons; see Fact 2 below. If X is a Banach space, then $L(X)$ denotes the space of all bounded linear operators from X into itself.

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Fact 1. *Let $T: \mathbb{R} \to L(X)$ be a 1-parameter group with generator* A. *Define semigroups U_1 and U_2 by $U_1(t) = T(t)$, for $t \geqq 0$, and $U_2(t) = T(-t)$, for $t \geqq 0$, respectively. Then the generator A_1 of U_1 is given by $A_1 = A$ with $D(A_1) = D(A)$ and the generator A_2 of U_2 is given by $A_2 = -A$ with $D(A_2) = D(A)$.*

A C_0-semigroup $\{U(t)\}_{t \geqq 0}$ is said to be of *polynomial growth* if there exists a real number $k \geqq 0$ such that

$$\|U(t)\| = O(t^k), \quad t \geqq 0 .$$

Fact 2. *Let $T: \mathbb{R} \to L(X)$ be a C_0-group of polynomial growth. Let iA be the generator of T, that is, formally $T(t) = e^{itA}$, $t \in \mathbb{R}$. Then*

(i) $\sigma(A) \neq \emptyset$, *and*

(ii) $\sigma(A) \subseteq i\mathbb{R}$.

In particular, the C_0-semigroups U_1 and U_2 of Fact 1 are stable.

Property (i) is well known; see [22; p. 91], for example. To establish Property (ii), let $k \geqq 0$ satisfy $\|T(t)\| = O(|t|^k)$, for $t \in \mathbb{R}$. It follows that $\omega(U_1) \leqq 0$. Hence, for any $\lambda \in \mathbb{C}$ with $\mathrm{Re}(\lambda) > 0$ the integral $\int_0^\infty e^{-\lambda t} U_1(t)\, dt$ exists (as a Bochner integral for the strong operator topology in $L(X)$) and, by semigroup theory, coincides with the resolvent operator $(iA - \lambda I)^{-1}$. This shows that $\{z \in \mathbb{C};\ \mathrm{Re}(z) > 0\}$ is contained in $\varrho(iA)$. By considering U_2 a similar argument shows that $\{z \in \mathbb{C};\ \mathrm{Re}(z) < 0\}$ is also contained in $\varrho(iA)$. This establishes (ii). The stability of U_1 and U_2 now follows from the fact that $\sigma(A_j) \subseteq i\mathbb{R}$ implies $s(A_j; U_j) = 0$, for $j = 1, 2$, that the polynomial growth of T implies $\omega(U_j) \leqq 0$, for $j = 1, 2$, and that $s(A_j; U_j) \leqq \omega(U_j)$ always holds. □

Remark 1. For groups with *bounded* generators Fact 2 is well known and is equivalent to several other conditions; [9; p. 160]. We list a few classes of operators covered by this criterion.

(i) Hermitian or Hermitian equivalent operators (that is, $\|e^{itA}\| = O(1)$, $t \in \mathbb{R}$). This includes all prescalar-type spectral operators with real spectrum.

(ii) Operators on a Hilbert space of the form $A = TRS$ with $R \geqq 0$ and ST self-adjoint. Typically for this class $\|e^{itA}\| = O(|t|)$, $t \in \mathbb{R}$.

(iii) Algebraic operators with real spectrum.

(iv) Well-bounded operators (i.e. for some bounded interval $[a, b]$ there is $K > 0$ such that $\|p(A)\| \leqq K(|p(b)| + \int_a^b p'(u)|\, du)$, for all polynomials p). Again, typically for this class $\|e^{itA}\| = O(|t|)$, $t \in \mathbb{R}$.

(v) Nilpotent operators. Indeed, if $A^k \neq 0$ but $A^{k+1} = 0$, then the series expansion of the exponential function shows that $\|e^{itA}\| = O(|t|^k)$, $t \in \mathbb{R}$.

(vi) Many Fourier multiplier operators generate C_0-groups of polynomial growth. For some simple examples consider the setting of $L^p(\mathbb{R})$, $1 < p < \infty$, and real-valued functions $m \in BV(\mathbb{R})$. In this case, if $A(m) = m(-i\, \mathrm{d}/\mathrm{d}x)$ denotes the corresponding multiplier operator and $|||\cdot|||_p$ the p-multiplier norm, then the inequalities

$$\|e^{itA(m)}\| = |||e^{itm}|||_p \leqq K_p \|e^{itm}\|_{BV} \leqq K_p(1 + 2|t|, \|m\|_{BV}) ,$$

valid for each $t \in \mathbb{R}$, show that (typically) we have $\|e^{itA(m)}\| = O(|t|)$, $t \in \mathbb{R}$. This holds for *all* $p \in (1, \infty)$. However, for a particular p such an estimate is surely not optimal (consider already $p = 2$). For, choose $p \in (1, 2)$, say, and let q satisfy $1 < q < p$. By interpolation, for fixed $t \in \mathbb{R}$, it follows that

$$\|e^{itA(m)}\|_{L^p} \leqq \|e^{itA(m)}\|_{L^2}^{1-\lambda} \cdot \|e^{itA(m)}\|_{L^q}^{\lambda} \leqq |C_q t|^{\lambda},$$

where $p^{-1} = q^{-1}\lambda + 2^{-1}(1 - \lambda)$, that is, $\lambda = q(2 - p)/p(2 - q)$. Since $\lambda < 1$, for all $1 < q < p$, we see (still crudely) that a better estimate is $\|e^{itA(m)}\|_{L^p} = O(|t|^{\lambda})$, $t \in \mathbb{R}$. Of course, another point is that such an estimate holds, for a given p, for *all* $m \in BV(\mathbb{R})$ with m being $\mathbb{R}$-valued. For a *particular* $\mathbb{R}$-valued $m \in BV(\mathbb{R})$ this estimate may be improved. For instance, if $m = \chi_{[0,\infty)}$, then

$$e^{itm(x)} = e^{it}\chi_{[0,\infty)}(x) = e^{it}m(x), \quad x \in \mathbb{R},$$

and so $\|e^{itA(m)}\| = |e^{it}| \cdot \|A(m)\| = O(1)$, $t \in \mathbb{R}$, for every $p \in (1, \infty)$. It is also easy to exhibit multipliers not in $BV(\mathbb{R})$ which also generate C_0-groups of polynomial growth. Fot instance, the Marcinkiewicz theorem implies that $m = \sum_{j=-\infty}^{\infty} (-1)^j \chi_{E(j)}$ is a p-multiplier, for every $p \in (1, \infty)$, where $E(j) = (2^j, 2^{j+1}]$, $j = 0, \pm 1, \pm 2, \ldots$ Since

$$e^{itm(x)} = e^{it}\chi_F(x) + e^{-it}\chi_G(x) + \chi_H(x), \quad x \in \mathbb{R},$$

where $F = m^{-1}(\{1\})$, $G = m^{-1}(\{-1\})$ and $H = m^{-1}(\{0\})$, it again follows from the Marcinkiewicz theorem that $\|e^{itA(m)}\| = O(1)$, $t \in \mathbb{R}$.

Remark 2. Property (i) of Fact 2 may fail if the group (necessarily with unbounded generator) is not of polynomial growth, [18; p. 664]. Examples of C_0-groups, with unbounded generators, which are not of polynomial growth arise typically as purely imaginary powers of certain closed operators; see [8; 13; 23; 24; 27], for example.

The point of Fact 2 is that it also covers stability for various classes of groups with possibly *unbounded* generators (as noted, groups with bounded generators are always stable). We begin with the simplest case, namely uniformly bounded groups. For example, if $T: \mathbb{R} \to L(X)$ is any strongly continuous group of *isometries* and A is its generator, then $-iA$ is a Hermitian operator (c.f. [11; Ch. 8, § 2], for example). This includes the translation groups in $L^p(\mathbb{R})$ and $L^p(\mathbb{T})$, $1 \leqq p < \infty$, where $\mathbb{T} = \{z \in \mathbb{C}; |z| = 1\}$ and $T(t)f : e^{iu} \mapsto f(e^{i(t+u)})$, $t \in \mathbb{R}$, $e^{iu} \in \mathbb{T}$ and $f \in L^p(\mathbb{T})$. Groups of isometries in certain Banach spaces are very special, even if their generator is bounded. For instance, in the spaces $C^1([0, 1])$, $Lip([0, 1])$, $AC([0, 1])$, $H^{\infty}(\mathbb{D})$ or lip_{α} (for $0 < \alpha < 1$), the only (bounded) Hermitian operators are multiples of the identity, [4]. Of course, in such spaces there may exist unbounded generators of isometric C_0-groups, [5]. In l^p, $1 \leqq p \leqq \infty$, the only C_0-groups of isometries are those with a generator of the form $f \mapsto fg$, for $f \in l^p$, for some $\mathbb{R}$-valued function g, [15]. For contraction semigroups in Hilbert space, see [14].

Let $T: \mathbb{R} \to L(X)$ be a *Stone group*. That is, there exists a spectral measure $P : B(\mathbb{R}) \to L(X)$, necessarily unique and defined on the σ-algebera $B(\mathbb{R})$ of Borel subsets of $\mathbb{R}$, such that

$$T(t) = \int_{\mathbb{R}} e^{its}\, dP(s), \quad t \in \mathbb{R}, \tag{2}$$

is its Fourier-Stieltjes transform, [2]. The theory of integration with respect to spectral measures implies that such groups are always of class C_0 and are uniformly bounded: they are natural analogous of unitary groups in Hilbert space.

There is a larger class of groups, introduced in [1] and [7] which, although *not* necessarily Stone groups may, nevertheless, be considered as analogues of Stone groups since they can still be represented via (2), [1; Theorem 4.20], where now P is no longer a spectral measure but a *spectral family* of projections $\{P(\lambda); \lambda \in \mathbb{R}\}$ in $L(X)$ and the „integral" in (2) is not with respect to a σ-additive measure but exists, in a well-defined sense, by taking limits of certain Riemann-Stieltjes sums (c.f. [1] for the definitions). More precisely, let us call a C_0-group $T: \mathbb{R} \to L(X)$ a *Stone-like group* if,

(i) for each $t \in \mathbb{R}$ we have $T(t) = e^{iA(t)}$, where $A(t)$ is a well-bounded operator of type (B) with $\sigma(A(t)) \subseteq [0, 2\pi]$, and

(ii) $\sup\{\|P_t(\lambda)\|;\ t, \lambda \in \mathbb{R}\} < \infty$, where $P_t(\cdot)$ is the spectral family of $A(t)$, for each $t \in \mathbb{R}$.

It turns out that a Stone-like group is necessarily uniformly bounded and so, its generator has spectrum in $i\mathbb{R}$, [7; pp. 157–158]. Any Stone group is necessarily a Stone-like group, but not conversely. Many important 1-parameter groups of operators from classical analysis, such as translations in $L^p(\mathbb{R})$ or $L^p(\mathbb{T})$, $1 < p < \infty$ (note that $p \neq 1$), [1; § 4], strongly continuous groups of isometries (with unbounded generator) in $H^p(\mathbb{D})$, $1 < p < \infty$, [3; 7], and C_0-groups of isometries in H^p of the torus, $1 \leqq p < \infty$, [6], for example, are Stone-like groups. There are classes of Banach spaces in which *every* uniformly bounded C_0-group is a Stone-like group; this is the case for any closed subspace X of a space $L^p(\nu)$, $1 < p < \infty$, with ν an arbitrary σ-additive measure, [31; Theorem 4.21].

Stable C_0-semigroups also arise as other integral transforms of spectral measures. For instance, let $P : B([0, \infty)) \to L(X)$ be a spectral measure and

$$U(t) = \int_0^\infty e^{-st}\,\mathrm{d}P(s)\,, \quad t \geqq 0\,. \tag{3}$$

Then U is a stable C_0-semigroup. That U is strongly continuous (and uniformly bounded) again follows from the theory of integration with respect to spectral measures. Moreover, the generator A, of U, is given by $(-A)\,x = \lim_{n\to\infty} \int_0^n s\,\mathrm{d}P(s)\,x$, for each $x \in D(A) = \{y \in X; \{\int_0^n s\,\mathrm{d}P(s)\,y\}_{n=1}^\infty \text{ converges}\}$. If S denotes the support of the measure P, necessarily a subset of $[0, \infty)$, and $\alpha = \inf\{w; w \in S\}$, in which case $\alpha \in S$, then $\sigma(A) = -S$ and hence, $s(A; U) = -\alpha$. Since

$$U(t) = e^{-t\alpha} \int_0^\infty e^{-t(s-\alpha)}\,\mathrm{d}P(s)\,, \quad t \geqq 0\,,$$

and the family of operators $\{\int_0^\infty e^{-t(s-\alpha)}\,\mathrm{d}P(s); t \geqq 0\}$ is uniformly bounded (as $S \subseteq [\alpha, \infty)$) it follows that there is $K > 0$ satisfying $\|U(t)\| \leqq Ke^{-t\alpha}$, $t \geqq 0$. Accordingly, $\omega(U) \leqq -\alpha$ and hence, $s(A; U) = \omega(U) = -\alpha$ follows.

Remark 1. If $\{U(t)\}_{t \geqq 0}$ is a uniformly bounded semigroup of positive selfadjoint operators in a Hilbert space, then U always has the form (3) for some spectral measure P. For semigroups of scalar-type spectral operators in Banach spaces see [2; 26].

Examples of groups, with unbounded generators, which are of polynomial growth (but not necessarily uniformly bounded) often occur in the theory of Fourier multipliers. We restrict ourselves to very simple examples, namely on the group $\mathbb{R}$ and for $1 < p \leqq 2$. The first „curiousity" is the production of unbounded functions $m : \mathbb{R} \to \mathbb{R}$ for which each function $x \mapsto e^{itm(x)}$, $x \in \mathbb{R}$ and $t \in \mathbb{R}$, is indeed a (bounded) p-multiplier. Of course, $m(x) = x$ gives rise to the translation group in $L^p(\mathbb{R})$. However, this is essentially the only *polynomial* m giving rise to bounded operators. Using the van der Corput lemma and arguments along the lines of the proof of Lemma 1.3 in [19] it can be shown, for $p \neq 2$, that the bounded function $x \mapsto e^{im(x)}$, $x \in \mathbb{R}$, is a p-multiplier if and only if $m(x) = \alpha + \beta x$, $x \in \mathbb{R}$, for some α, $\beta \in \mathbb{R}$. Similarly, if m is an $\mathbb{R}$-valued rational function, then it can be shown that $x \mapsto e^{im(x)}$, $x \in \mathbb{R}$, is a p-multiplier if and only if $m(x) = \alpha + \beta x + r(x)$ for some α, $\beta \in \mathbb{R}$ and $r : \mathbb{R} \to \mathbb{R}$ is a *bounded* rational function. So, to produce groups from „unbounded multipliers" it is necessary to get away from rational functions. The next simplest example to try might be $m(x) = |x|$. But, then the identities

$$e^{itm(x)} = e^{itx}\chi_{[0,\infty)}(x) + e^{-itx}\chi_{(-\infty,0)}(x), \quad x \in \mathbb{R},$$

valid for each $t \in \mathbb{R}$, show that $\|e^{itA(m)}\| = |||e^{itm}|||_p = O(1)$, for $t \in \mathbb{R}$. Or, if $m = \sum_{k=-\infty}^{\infty} k\chi_{E(k)}$ with $E(k) = (2^{k-1}, 2^k]$, $k = 0, \pm 1, \pm 2, \ldots$, then

$$e^{itm} = \chi_{(-\infty,0]} + \sum_{k=-\infty}^{\infty} e^{ikm}\chi_{E(k)},$$

for each $t \in \mathbb{R}$, and it follows from the Marcinkiewicz theorem that $\|e^{itA(m)}\| = |||e^{itm}|||_p = O(1)$, $t \in \mathbb{R}$.

However, if $m(x) = \ln |x|$, for $x \in \mathbb{R} \setminus \{0\}$, then the Mihlin theorem shows that

$$\|e^{itA(m)}\| = |||e^{itm}|||_p = O(|t|), \quad t \in \mathbb{R}. \tag{4}$$

The same discussion (via interpolation) as for the case of bounded multipliers m shows that the estimate (4) is not optimal. For example, given $p \in (1, 2)$ and any $q \in (1, p)$ it aagin follows that $\lambda = q(2 - p)/p(2 - q) < 1$ satisfies $\|e^{itA(m)}\| = O(|t|^\lambda)$. Of course, for $p = 2$ we have $\|e^{itA(m)}\| = 1$, $t \in \mathbb{R}$. However, for $p \neq 2$, it is known that

$$\sup\{|||e^{itm}|||_p; t \in \mathbb{R}\} = \infty.$$

For more precise information on the norms $|||e^{itm}|||_p$, $t \in \mathbb{R}$, we refer to [10].

Fourier multiplier operators also provide a direct means of producing examples of C_0-groups of polynomial growth to an arbitrary high order. For, suppose that $m_1 : \mathbb{R} \to \mathbb{R}$ and $m_2 : \mathbb{R} \to \mathbb{R}$ are measurable functions (not necessarily bounded) such that $A(m_1)$ and $A(m_2)$ generate C_0-groups in $L^p(\mathbb{R})$, for some $p \in (1, \infty)$ with $p \neq 2$, satisfying $|||e^{itm_j}|||_p = O(|t|)^{k_j})$ for some $k_j > 0$ with k_j minimal, $j \in \{1, 2\}$;

for example, $m_1(x) = m_2(x) = \ln|x|$, $x \in \mathbb{R} \setminus \{0\}$, suffices. Then

$$(x, y) \mapsto e^{it(m_1(x)+m_2(y))}, \quad (x, y) \in \mathbb{R}^2,$$

is a p-multiplier for the group $\mathbb{R}^2$, for each $t \in \mathbb{R}$, [19; Theorem 1.13]. For general p-multipliers T_1 and T_2 in $L^p(\mathbb{R}^2)$ it is the case that $\|T_1T_2\| \leqq \|T_1\| \, . \, \|T_2\|$. However, if T_1 corresponds to a p-multiplier fuhction φ_1 depending just on x and T_2 corresponds to a p-multiplier function φ_2 depending just on y, then actually

$$\|T_1T_2\|_{L^p(\mathbb{R}^2)} = \|T_1\|_{L^p(\mathbb{R}^2)} \, \|T_2\|_{L^p(\mathbb{R}^2)}$$

and so, by [19; Theorem 1.13] again,

$$\|T_1T_2\|_{L^p(\mathbb{R}^2)} = \|T_1\|_{L^p(\mathbb{R})} \, \|T_2\|_{L^p(\mathbb{R})} \, .$$

Using this observation it follows that

$$\|e^{it(A(m_1+m_2))}\|_{L^p(\mathbb{R}^2)} = \|e^{itA(m_1)}\|_{L^p(\mathbb{R})} \, \|e^{itA(m_2)}\|_{L^p(\mathbb{R})} = O(|t|^{k_1+k_2}) \, .$$

By repeating this procedure it is possible to produce C_0-groups in $L^p(\mathbb{R}^n)$, with n large enough, such that $\|e^{itA(m)}\|_{L^p(\mathbb{R}^n)} = O(|t|^k)$ with $k > 0$ as large as desired.

We now discuss two small points addressing the question of whether such groups $\{e^{itA(m)}\}_{t\in\mathbb{R}}$ of multiplier operators in $L^p(\mathbb{R})$ really are strongly continuous and whether the domain of the generator $iA(m)$, as given by semigroup theory, coincides with the natural domain given via multiplier theory, namely (for $1 < p \leqq 2$)

$$\{f \in L^p(\mathbb{R}); \, im\hat{f} = \hat{g} \quad \text{for some} \quad g \in L^p(\mathbb{R})\}$$

with $iA(m)f = g$; here $\hat{}$ denotes the Fourier transform. Since $\lim_{t\to 0} e^{itm(x)} = 1$, for a.e. $x \in \mathbb{R}$, and $\sup\{\|e^{itA(m)}\|; |t| \leqq 1\}$ is finite (under the hypothesis that the group is of polynomial growth) it follows from multiplier theory that $e^{itA(m)} \to I$, as $t \to 0$, for the weak operator topology. Then classical semigroup theory implies that $e^{itA(m)} \to I$, as $t \to 0$, for the strong operator topology. Accordingly, we do have a C_0-group. That the two (possibly) different descriptions of the domain of $iA(m)$ coincide can be argued as in the proof of Theorem 21.4.2 of [18].

We conclude with a brief discussion about matrix multiplication semigroups, [32]. Let (Ω, Σ, ν) be a σ-finite measure space and fix $1 \leqq p < \infty$. For an integer $n \geqq 1$ let $L^p(\Omega, \mathbb{C}^n)$ be the space of functions $f : \Omega \to \mathbb{C}^n$ such that $\|f\| = (\int_\Omega \|f(w)\|^p \, . \, d\nu(w))^{1/p}$ is finite, where $\mathbb{C}^n$ has its standard Hilbert space norm. The space of all $(n \times n)$-matrices with entries from $\mathbb{C}$ is denoted by $M_n(\mathbb{C})$. Given a Σ-measurable function $Q : \Omega \to M_n(\mathbb{C})$ we define a closed, densely defined operator $A_p(Q)$, with domain

$$D(A_p(Q)) = \{f \in L^p(\Omega, \mathbb{C}^n) \, ; \, Qf \in L^p(\Omega; \mathbb{C}^n)\} \, ,$$

by $A_p(Q)f = Qf$, for each $f \in D(A_p(Q))$. If the resolvent set of $A_p(Q)$ is non-empty, then

$$\sigma(A_p(Q)) = \bigcap_{R\in[Q]} \overline{\bigcup_{\omega\in\Omega} \sigma(R(\omega))} \, ,$$

where $\sigma(R(\omega))$ is the spectrum of $R(\omega) \in M_n(\mathbb{C})$ and $[Q]$ is the equivalence class of all matrix-valued functions coinciding ν-a.e. with Q. Then $A_p(Q)$ is the generator of a C_0-semigroup if and only if

$$\sup\{\text{ess}-\sup\{\|e^{tQ(\omega)}\|;\ \omega\in\Omega\};\ t\in[0,1]\}<\infty,$$

where $\|\cdot\|$ denotes the usual norm in $M_n(\mathbb{C})$. Under this condition the weak spectral mapping theorem

$$\sigma(e^{tA_p(Q)}) = \overline{\{e^{t\lambda};\ \lambda\in\sigma(A_p(Q))\}},\quad t\geqq 0,$$

holds, from which it follows that $\{e^{tA_p(Q)}\}_{t\geqq 0}$ is stable.

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