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More facts about conjugate Banach spaces with the Radon-Nikodym property

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More Facts about Conjugate Banach Spaces with the Radon-Nikodym Property

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Corrected proofs of the following results of [S0] are given: if X is an Asplund space (respectively, X is a subspace of a gsg space) and K is a Corson compact then any operator from X to $C(K)$ interpolates through a Banach space Y such that Y is both Asplund and hereditarily weakly compactly generated (respectively, Y is wcg). If K is a Corson compact that is the continuous image of a so called Radon-Nikodym compact then K is an Eberlein compact.

We use the same terminology and notation as in **[S0]**. We slightly rephrase the Lemma on page 48. The proof should be clear from the inequalities

$$\begin{aligned}&\|((f_1 \vee g_1) - h_1) - ((f_2 \vee g_2) - h_2)\| \\ &\quad \leqq \|(f_1 \vee g_1) - (f_2 \vee g_1)\| + \|(f_2 \vee g_1) - (f_2 \vee g_2)\| + \|h_1 - h_2\| \\ &\quad \leqq \|f_1 - f_2\| + \|g_1 - g_2\| + \|h_1 - h_2\| .\end{aligned}$$

Lemma. *Suppose that C_1, C_2 and C_3 are bounded and equimeasurable subsets of $C(K)$. Then*

$$\{(f \vee g) - h : f \in C_1, g \in C_2, h \in C_3\}$$

is equimeasurable.

We give a correct proof of the Lemma on pages 49–50 of [S0]

Lemma. *Let K be a compact Hausdorff space and C a subset of $C(K)$ that is equimeasurable and separates the points of K (in some circles K is called a Radon-Nikodym compact). Let* **F** *be any subset of $C(K)$ that is point countable. There exists a subset* **G** *of $C(K)$ that is both equimeasurable and point countable and the algebra A generated by* **G** *contains* **F**.

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Proof. We may assume that C is a convex and symmetrical subset of the unit ball with $1_K \in C$. It is easy to check that $C \cdot C$ is also equimeasurable and, by induction, C^n is equimeasurable. It follows that

$$E = \sum_n 2^{-n} C^n$$

is equimeasurable. ¿From the Stone-Weierstraß theorem we know that $\cup_n n \cdot E$ is uniformly dense in $C(K)$. For fixed positive integers m and n define

$$\mathbf{F}_{m,n} = \mathbf{F} \cap \left((n \cdot E) + B\left(0, \frac{1}{m}\right)\right).$$

Observe, that for a fixed m, $\cup_n \mathbf{F}_{m,n} = \mathbf{F}$. For each $f_{m,n,\alpha} \in \mathbf{F}_{m,n}$ choose $h_{m,n,\alpha} \in E$ so that $\|nh_{m,n,\alpha} - f_{m,n,\alpha}\| < \frac{1}{m}$. Define

$$u_{m,n,\alpha} = \left(nh_{m,n,\alpha} \vee \frac{1}{m}\right) - \frac{1}{m}$$

which is non negative and observe that

$$n^{-1}u_{m,n,\alpha} = \left(h_{m,n,\alpha} \vee \frac{1}{mn}\right) - \frac{1}{mn} \in \left(E \vee \frac{1}{mn}\right) - \frac{1}{mn}.$$

Thus, $\mathbf{G} = \{n^{-1}u_{m,n,\alpha} : m,\ n,\ \alpha\}$ is equimeasurable. Fix $k \in K$, m and n; if $n^{-1}u_{m,n,\alpha}(k) > 0$ then

$$nh_{m,n,\alpha}(k) > \frac{1}{m}$$

which implies that $|f_{m,n,\alpha}(k)| > 0$. Thus, $\{n^{-1}u_{m,n,\alpha} : \alpha\}$ is also point countable. Thus, we have that $\mathbf{G}$ is point countable because it is the countable union of point countable sets. Also, $\mathbf{F}$ is a subset of the closed algebra A generated by $\mathbf{G}$ and

$$\mathbf{G} \subseteq \cup_{m,n}\left(E \vee \frac{1}{mn} 1_K\right) - \frac{1}{mn} 1_K.$$

Thus, the state space of A is both a Corson compact and a Radon-Nikodym compact.

The remarks beginning in the last paragraph of page 52 and continuing on page 53 are, at best, incomplete, and should be ignored.

Bibliography

[S0] Stegall, C., More Facts about conjugate Banach Spaces with the Radon-Nikodym Property II, Acta Universitatis Carolinae-Math. et Phys. 32 (1991), 47–54.