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σ-discrete decomposability of completely additive families

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σ-discrete decomposability of completely additive families

Z. Frolík and P. Holický

The main result says (all topological spaces are assumed to be completely regular):

Lemma. Let f be a perfect mapping of a topological space X onto a complete metric space M. A disjoint family $\{X_a\}$ of Souslin sets in X is completely additive in the class of all Souslin sets (that means, the union of each subfamily is a Souslin set) if and only if the family $\{f[X_a]\}$ is σ-discretely decomposable in the metric space M.

As an immediate consequence we get:

Theorem. Let Y be a Souslin set in a space which admits a perfect mapping onto a complete metric space. Then a disjoint family $\{Y_a\}$ of Souslin sets in Y is completely additive in the class of all Souslin sets if and only if the family $\{Y_a\}$ is σ-discretely decomposable w.r.t. the topologically fine uniformity of Y.

Corollary. Assume that $Y = A \times S$ where A is an analytic space, and S is a metric space which is a Souslin set in its completion. Then the statement of Theorem holds.

It should be remarked that Lemma for f to be an identity is due to F. Hansell [H], and for f to be a constant mapping is a particular case of a result of the first author [F_1]. Our proof uses the procedure of F. Hansell's, but not his result, and just the following result from [F_1] without any proof technique:

Proposition 1. If K is compact (or more generally, analytic) then every disjoint completely additive family in Souslin sets is countable.

For a better understanding we note:

Proposition 2. Let f be a perfect mapping of X onto a metric space M. Then a disjoint family $\{X_a\}$ in X is σ-discrete decomposable w.r.t. the topological fine uniformity iff $\{f[X_a]\}$ is σ-discretely decomposable in the metric space M.

Although the result was proved during the early tennis season of 1975, and nothing has been added since then, we must state that the final version is not polished enough to be submitted for definite publication. Certainly one feels that much stronger results must be proved or disproved. The definite text will be submitted to Bull. Acad. Polon.

For the purpose of this seminar we indicate the basic applications extending the results of the first author, see [F_2], [F_3],[F_4] .

Since hyper-Baire sets in a uniform space are Souslin sets in the underlying topological space we receive immediately the following corollaries.

Theorem A. If a uniform space X admits a perfect mapping onto a complete metric space, then the hyper-Baire-fine coreflection of X has all σ-discrete partitions ranging in hyper-Baire sets for a basis of uniform covers.

Theorem B. If $X = M \times K$ with K compact, and M complete metric, then the measurable coreflection of X is Baire-fine (i.e. every Baire mapping from X is uniformly continuou s w.r.t. the measurable coreflection of X).

It should be remarked that M in Theorem B can be replaced by any product of complete metric spaces (via Tashiyan Lemma).

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F. Hansell

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