ON ONE ORDERED CONTINUUM

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In the present paper the construction of the ordered continuum $\mathfrak{P}_7$ is given whose power is $2^{\aleph_0}$ and whose separability is $\aleph_1$. There are three kinds of point-characters in $\mathfrak{P}_7$ viz. c_{00}, c_{01}, c_{10}. Some interesting properties of $\mathfrak{P}_7$ are studied. The article is closely connected with NOVÁK's paper: On some ordered continua of power $2^{\aleph_0}$ containing a dense subset of power $\aleph_1$.

In this paper a continuum $\mathfrak{P}_7$ of power $2^{\aleph_0}$ is constructed by the method of identification of points in certain intervals in certain ordered continuum Q such that $\aleph_1$ is the least power of subsets which are dense in $\mathfrak{P}_7$. The continuum $\mathfrak{P}_7$ contains only points with characters c_{00}, c_{01}, c_{10}. The continuum $\mathfrak{P}_7$ is a quasi-homogeneous continuum, i. e., in every interval $J \subset \mathfrak{P}_7$ there exists a subinterval I similar to $\mathfrak{P}_7$. The continuum $\mathfrak{P}_7$ possesses the property π: Any disjoint uncountable system of intervals in $\mathfrak{P}_7$ contains an uncountable subsystem of intervals whose left end-points form an increasing or a decreasing sequence of points in $\mathfrak{P}_7$. The construction of the continuum $\mathfrak{P}_7$ gives the solution of the problem of J. NOVÁK[1]) introduced in his paper: On some ordered continua of power $2^{\aleph_0}$ containing a dense subset of power $\aleph_1$, i. e. if there exists the ordered continuum $\mathfrak{P}_7$.

Let Q be a lexicographically ordered continuum whose elements x are transfinite sequences of zeros and ones $[x_\lambda]_{\lambda<\omega_1} = x_0 x_1 \dots x_\lambda \dots$ ($\lambda < \omega_1$) (where $x_\lambda = 0$ or $x_\lambda = 1$ and ω_1 is the least uncountable ordinal), whereby every two neighbouring sequences are identified. According to NOVÁK, we say that the point $x \in Q$ has the property (c), (d), (e), (f), or (g) with the least ordinal $\alpha < \omega_1$ if there exists its development, i. e., the transfinite sequence $[x_\lambda]_{\lambda<\omega_1}$ satisfying the corresponding property:

(c) there exist two ordinary increasing sequences of indices $\{\tau_n\}_{n=0}^{\omega}$ and $\{\eta_n\}_{n=0}^{\omega}$ converging to ordinal α and such that $x_{\tau_n} = 0$ and $x_{\eta_n} = 1$ for every n,

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[1]) *J. Novák*, On some ordered continua of power $2^{\aleph_0}$ containing a dense subset of power $\aleph_1$, Czechosl. math. Journ. **76** (1951), 63—79.

(d) there exists the least ordinal β such that $x_\lambda = 1$ for $\beta \leqq \lambda < \alpha = \beta + \omega$ whereby β is a limit ordinal or 0,

(e) there exists the least ordinal β such that $x_\lambda = 0$ for $\beta \leqq \lambda < \alpha = \beta + \omega$ whereby β is a limit ordinal or 0,

(f) there exists the least ordinal β such that $x_\lambda = 1$ for $\beta \leqq \lambda < \alpha = \beta + \omega$ whereby β is an isolated positive ordinal,

(g) there exists the least ordinal β such that $x_\lambda = 0$ for $\beta \leqq \lambda < \alpha = \beta + \omega$ whereby β is an isolated positive ordinal,

whereby α is, in all these cases, the least ordinal with the mentioned property.

Let $0 < \alpha < \omega_1$ and let $i_0 i_1 \ldots i_\lambda \ldots$ $(\lambda < \alpha)$ be a sequence whereby $i_\lambda = 0$ or $i_\lambda = 1$. All points $x \in Q$ with developments $[x_\lambda]_{\lambda<\omega_1}$ such that $x_\lambda = i_\lambda$ for $\lambda < \alpha$, form a closed interval $I = I_{i_0 i_1 \ldots i_\lambda \ldots}$ $(\lambda < \alpha) \subset Q$ of order α. We say that $I = I_{i_0 i_1 \ldots i_\lambda \ldots}$ $(\lambda < \alpha)$ has the property (c), (d), (e), (f), or (g) if all points of I have this property with respect to the ordinal α.

Lemma 1. *Let $\mathfrak{S}_7$ be a system of all intervals $I_{i_0 i_1 \ldots i_\lambda \ldots}$ $(\lambda < \alpha) \subset Q$ with the property (c) or (f) but such that no interval $I_{i_0 i_1 \ldots i_\lambda \ldots}$ $(\lambda < \alpha')$ where $\alpha' < \alpha$ has either property (c) or property (f). Then the system $\mathfrak{S}_7$ is a disjoint system of intervals in Q.*

Proof. Let $I_{i_0 i_1 \ldots i_\lambda \ldots}$ $(\lambda < \alpha) \in \mathfrak{S}_7$ and $I_{j_0 j_1 \ldots j_\lambda \ldots}$ $(\lambda < \beta) \in \mathfrak{S}_7$ be two different intervals in Q. Then there exists an ordinal $\delta < \min(\alpha, \beta)$ such that $i_\lambda = j_\lambda$ for $\lambda < \delta$ and $i_\delta + j_\delta = 1$. Let $x \in I_{i_0 i_1 \ldots i_\lambda \ldots}$ $(\lambda < \alpha) \cap I_{j_0 j_1 \ldots j_\lambda \ldots}$ $(\lambda < \beta)$. The point x is the point with two developments $[x_\lambda]_{\lambda<\omega_1}$ and $[y_\lambda]_{\lambda<\omega_1}$ whereby $x_\lambda = i_\lambda$ for $\lambda < \alpha$, $y_\lambda = j_\lambda$ for $\lambda < \beta$. From the last assertion it follows that $x_\lambda = y_\lambda$ for $\lambda < \delta$, $x_\delta + y_\delta = 1$ and $x_\lambda \neq x_\delta$, $y_\lambda \neq y_\delta$ for $\lambda > \delta$. Consequently $i_\lambda \neq i_\delta$ for $\delta < \lambda < \alpha$ and $j_\lambda \neq j_\delta$ for $\delta < \lambda < \beta$. Further $I_{i_0 i_1 \ldots i_\lambda \ldots}$ $(\lambda < \delta)$ and $I_{j_0 j_1 \ldots j_\lambda \ldots}$ $(\lambda < \delta)$ have neither property (c) nor (f) and it is for $\delta < \lambda < \alpha$: $i_\lambda = 0$ if $i_\delta = 1$ or $i_\lambda = 1$ if $i_\delta = 0$, and for $\delta < \lambda < \beta$: $j_\lambda = 1$ if $i_\delta = 1$ or $j_\lambda = 0$ if $i_\delta = 0$. From that it follows that the interval $I_{i_0 i_1 \ldots i_\lambda \ldots}$ $(\lambda < \alpha)$ fails to have property (c) or, in the case $i_\delta = 1$, property (f) and $I_{j_0 j_1 \ldots j_\lambda \ldots}$ $(\lambda < \beta)$ fails to have property (c) or, in the case $i_\delta = 0$, property (f). In any case one of these two intervals does not belong to the system $\mathfrak{S}_7$; that is a contradiction. Therefore the system $\mathfrak{S}_7$ is a disjoint system of intervals in Q.

Lemma 2. *Every interval $I_{i_0 i_1 \ldots i_\lambda \ldots}$ $(\lambda < \alpha) \in \mathfrak{S}_7$ has the character c_{00} in Q, if it has property (c), or the character c_{01} in Q, if it has property (f). Let $\mathbf{U}\mathfrak{S}_7$ be the set of all points of Q belonging to some intervals of $\mathfrak{S}_7$. Suppose that $x \in Q - \mathbf{U}\mathfrak{S}_7$ and that x is no end-point in Q. Then the character of the point x is c_{01} or c_{10} in Q.*

Proof. Let $I = I_{i_0 i_1 \ldots i_\lambda \ldots}$ $(\lambda < \alpha) \in \mathfrak{S}_7$. Let $x \in I$ be the left end-point with the development $i_0 i_1 \ldots i_\lambda \ldots 000 \ldots = [x_\lambda]_{\lambda<\omega_1}$ and let $y \in I$ be the right end-point with the development $i_0 i_1 \ldots i_\lambda \ldots 111 \ldots = [y_\lambda]_{\lambda<\omega_1}$. Let I have property (c); then there exists a sequence $\{{}^\xi x\}_{\xi=0}^{\alpha}$ of points in Q, the development of the point ${}^\xi x$ being $[{}^\xi x_\lambda]_{\lambda<\omega_1}$ where ${}^\xi x_\lambda = x_\lambda$ for $\lambda < \omega_1$ and $\lambda \neq \xi$, and ${}^\xi x_\xi = 0$. Since – according to the property (c) — there exists an infinite number of different indices such that $\eta_n \to \alpha$ and $i_{\eta_n} = 1$, it follows therefore that there exists an ordinary increasing sequence $\{{}^{\xi_n} x\}_{n=0}^{\omega}$ left converging to the point x in Q. The character of the point x is $c_{0\sigma}$ in Q. Likewise, it can be proved that the character of the point y is $c_{\varrho 0}$ in Q. The character of I is therefore c_{00} in Q. Let I have the property (f), i. e., there exists an isolated ordinal β such that $i_{\beta-1} = 0$ and $i_\lambda = 1$ for $\beta \leqq \lambda < \alpha = \beta + \omega$; then there exists an ordinary increasing sequence $\{{}^n x\}_{n=0}^{\omega}$ left converging to the point x in Q, the development of the point ${}^n x$ being $[{}^n x_\lambda]_{\lambda<\omega_1}$ where ${}^n x_\lambda = x_\lambda$ for $\lambda < \omega_1$ and $\lambda \neq \beta + n$, and ${}^n x_{\beta+n} = 0$. In this case, the point x has the character $c_{0\sigma}$ in Q. The point y has two developments in Q viz. $i_0 i_1 \ldots i_\lambda \ldots 111$ and $[j_\lambda]_{\lambda<\omega_1}$ where $j_\lambda = i_\lambda$ for $\lambda < \beta - 1$, $j_{\beta-1} = 1$ and $j_\lambda = 0$ for $\lambda \geqq \beta$. Therefore there exists in Q a decreasing sequence of points $\{y^\xi\}_{\xi=}^{\omega_1}$ right converging to y whereby the development of y^ξ is $[y^\xi_\lambda]_{\lambda<\omega_1}$ where $y^\xi_\lambda = j_\lambda$ for $\lambda < \omega_1$ and $\lambda \neq \beta + \xi$, and $y^\xi_{\beta+\xi} = 1$. Consequently the point y has the character $c_{\varrho 1}$ in Q and so the interval I has the character c_{01} in Q.

If $x \in Q - \mathbf{U}\mathfrak{S}_7$ with the development $[x_\lambda]_{\lambda<\omega_1}$ is not an end-point in Q, it has not property (c) and its development cannot contain uncountably many 0's and uncountably many 1's simultaneously. There must exist the least ordinal β such that $x_\lambda = 0$ for $\beta \leqq \lambda < \omega_1$ or $x_\lambda = 1$ for $\beta \leqq \lambda < \omega_1$. The ordinal β cannot be an isolated ordinal since (the first case) the point x would have two developments and x would be the right end-point of the interval $I_{i_0 i_1 \ldots i_\lambda \ldots}$ $(\lambda < \beta + \omega) \in \mathfrak{S}_7$ whereby $i_\lambda = x_\lambda$ for $\lambda < \beta - 1$, $i_{\beta-1} = 0$ and $i_\lambda = 1$ for $\beta \leqq \lambda < \beta + \omega$; or (the second case) the point x would belong to the interval $I_{i_0 i_1 \ldots i_\lambda \ldots}$ $(\lambda < \beta + \omega) \in \mathfrak{S}_7$ as the right end-point whereby $i_\lambda = x_\lambda$ for $\lambda < \beta + \omega$. Let us consider the first case. The development $[x_\lambda]_{\lambda<\omega}$ must contain at least one 1 because x is not the left end-point in Q. There must be an infinite number of indices $\lambda < \beta$ such that $x_\lambda = 1$; otherwise the ordinal β could not be the least ordinal with the prescribed property and the limit ordinal at the same time. Therefore, we can choose from the sequence $\{{}^\xi x\}_{\xi=0}^{\beta}$, whereby ${}^\xi x$ has the development $[{}^\xi x_\lambda]_{\lambda<\omega_1}$, ${}^\xi x_\lambda = x_\lambda$ for $\lambda < \omega_1$ and $\lambda \neq \xi$, and ${}^\xi x_\xi = 0$, an ordinary increasing sequence $\{{}^{\xi_n} x\}_{n=0}^{\omega}$ left converging to the point x in Q. The sequence $\{y^\xi\}_{\xi=0}^{\omega_1}$ of points of Q, whereby y^ξ has the development $[y^\xi_\lambda]_{\lambda<\omega_1}$, $y^\xi_\lambda = x_\lambda$ for $\lambda < \omega_1$ and $\lambda \neq \beta + \omega$, and $y^\xi_{\beta+\xi} = 1$, is an uncountable decreasing sequence right converging to the point x in Q. Therefore, the character of the point x is c_{01} in Q. In the second case, because the point x is not the right end-point in Q and the ordinal β is the least ordinal with the above mentioned property, the development of the point x must contain an infinite number of $x_\lambda = 0$, $\lambda < \beta$. The sequence $\{{}^\xi x\}_{\xi=0}^{\omega_1}$ of points of Q, whereby ${}^\xi x$ has the development $[{}^\xi x_\lambda]_{\lambda<\omega_1}$ and ${}^\xi x_\lambda = x_\lambda$ for $\lambda < \omega_1$ and $\lambda \neq \beta + \xi$, and ${}^\xi x_{\beta+\xi} = 0$, is an uncountable sequence left converging to the point x in Q. Further we can choose from the sequence $\{y^\xi\}_{\xi=0}^{\beta}$ of points of Q whereby y^ξ has the development $[y_\lambda^\xi]_{\lambda<\omega_1}$, $y_\lambda^\xi = x_\lambda$ for $\lambda < \omega_1$ and $\lambda \neq \xi$, and $y_\xi^\xi = 1$, an ordinary decreasing sequence of points right converging to the point x in Q. Therefore, in this case, the character of the point x is c_{10} in Q and the proof of Lemma is completed.

Let $\mathfrak{P}_7$ be a system containing all intervals $X \in \mathfrak{S}_7$ and all one-point sets (x) such that $x \in Q - \mathbf{U}\mathfrak{S}_7$. We are going to arrange the points of $\mathfrak{P}_7$ as follows: $X, Y \in \mathfrak{P}_7$ and $X < Y$ if and only if there is $x < y$ for all points $x \in X$ and for all points $y \in Y$ in Q.

Theorem I. *The set $\mathfrak{P}_7$ is a quasi-homogeneous ordered continuum of the pover $2^{\aleph_0}$ containing no countable dense subset. $\mathfrak{P}_7 = A_{00}^{(7)} \cup A_{01}^{(7)} \cup$ $\cup\, A_{10}^{(7)} \cup E^{(7)}$ where $A_{\varrho\sigma}^{(7)}$ are disjoint subsets dense in $\mathfrak{P}_7$ for $\varrho, \sigma = 0, 1$ and the set $A_{\rho\sigma}^{(7)}$ is the set of all points with the character $c_{\rho\sigma}$ in $\mathfrak{P}_7$. The sets $A_{01}^{(7)}$ and $A_{10}^{(7)}$ have the power $\aleph_1$ and $E^{(7)} = \{(a), (b)\}$ whereby a and b are the endpoints in Q.*

Proof. The sets $A_{00}^{(7)}$, $A_{01}^{(7)}$ and $A_{10}^{(7)}$ are not empty. As a matter of fact the points $I_{i_0 i_1 \dots i_n \dots}$ $(n < \omega)$, $I_{j_0 j_1 \dots j_n \dots}$ $(n < \omega)$ and x with the development $[x_\lambda]_{\lambda<\omega_1}$ whereby $i_{2k} = 0$ and $i_{2k+1} = 1$ for $k = 0, 1, 2, \dots, j_0 = 0$, $j_n = 1$ for $n = 1, 2, 3, \dots$, and $x_\lambda = 0$ for $\lambda < \omega$ and $x_\lambda = 1$ for $\omega \leqq \lambda < \omega_1$, belong to $\mathfrak{P}_7$. From this fact, according to Theorem 1 and Lemma 1 of the above cited paper of J. NOVÁK[2]) and according to our Lemma 2, it follows that $\mathfrak{P}_7$ is an ordered continuum with points of character c_{00}, c_{01}, c_{10}. In $\mathfrak{P}_7$ there cannot exist a countable dense subset, $\mathfrak{P}_7$ containing points with character c_{01} and c_{10}.

Let $J \subset \mathfrak{P}_7$ be any interval with the end-points $p < q$, $p \in \mathfrak{P}_7$ and $q \in \mathfrak{P}_7$. If p and q are common points in $\mathfrak{P}_7$, then let $[p_\lambda]_{\lambda<\omega_1}$ and $[q_\lambda]_{\lambda<\omega_1}$ be their developments; if one of them or both are the interval-points in $\mathfrak{P}_7$, then let $[p_\lambda]_{\lambda<\omega_1}$ be the development of the right end-point of the interval p in Q and $[q_\lambda]_{\lambda<\omega_1}$ the development of the left end-point of the interval q in Q. As $p < q$ there must exist an index δ such that $p_\lambda = q_\lambda$ for $\lambda < \delta$, $p_\delta = 0 < q_\delta = 1$ and the least index $\gamma > \delta$ such that there is $p_\gamma = 0$ or $q_\gamma = 1$. If $\gamma \geqq \delta + \omega$, the point p has the property (f). In this case $[p_\lambda]_{\lambda<\omega_1}$ is the development of the right end-point of the interval p in Q and consequently $p_\lambda = 1$ for $\lambda > \delta$ and $p_\gamma = 1$. If $p_\gamma = 0$, then evidently $\gamma < \delta + \omega$. If $p_\gamma = 0$ we put $e_\lambda = p_\lambda$ for $\lambda < \gamma$, $e_\gamma = e_{\gamma+2} = 1$ and $e_{\gamma+1} = 0$ and if $q_\gamma = 1$, we put $e_\lambda = q_\lambda$ for $\lambda < \gamma$ and $e_\gamma = e_{\gamma+1} = 0$,

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[2]) l. c. s.

$e_{\gamma+2} = 1$. If $I_{e_0 e_1 \dots e_\lambda \dots}$ $(\lambda < \delta)$ would have the property (c) or (f) it would be: $p, q \in I_{e_0 e_1 \dots e_\lambda \dots}$ $(\lambda < \delta) \in \mathfrak{S}_7$ and consequently $p = q$. Therefore with respect to the fact that $e_\lambda = 0$ or $e_\lambda = 1$ for $\delta \leqq \lambda < \gamma$ whereby in the last case $\gamma < \delta + \omega$ we can assert that the interval $I_{e_0 e_1 \dots e_\lambda \dots}$ $(\lambda < \gamma + 3)$ has neither the property (c) nor (f); consequently $[\bar{x}_\lambda]_{\lambda<\omega_1}$ and $[\bar{y}_\lambda]_{\lambda<\omega_1}$ where $\bar{x}_\lambda = \bar{y}_\lambda = e_\lambda$ for $\lambda < \gamma + 3$ and $\bar{x}_\lambda = \bar{y}_\lambda = 0$ for $\gamma + 3 \leqq \lambda < < \gamma + \omega$, $\bar{x}_\lambda = \bar{y}_\lambda = 1$ for $\gamma + \omega \leqq \lambda < \gamma + \omega \cdot 2$ and $\bar{x}_\lambda = 0$, $\bar{y}_\lambda = 1$ for $\gamma + \omega \cdot 2 \leqq \lambda$ are developments of the common points $\bar{x}$ and $\bar{y}$ and it is $p < \bar{x} < \bar{y} < q$ in $\mathfrak{P}_7$. Let I be the interval in $\mathfrak{P}_7$ with the end-points $\bar{x}$ and $\bar{y}$ in $\mathfrak{P}_7$. Then $I \subset J$.

Let $z \in \mathfrak{P}_7$, $z = I_{i_0 i_1 \dots i_\lambda \dots}$ $(\lambda < \alpha)$ be an interval-point. We put $z' = = f(z) = I_{j_0 j_1 \dots j_\lambda \dots}$ $(\lambda < \gamma + \omega \cdot 2 + \alpha)$ whereby $j_\lambda = \bar{x}_\lambda$ for $\lambda < \gamma + + \omega \cdot 2$ and $j_{\gamma+\omega.2+\lambda} = i_\lambda$ for $\lambda < \alpha$. If z has the property (c) or (f) then z' has the same property. Evidently $z' \in I \subset J$. Now, let z be a common point of $\mathfrak{P}_7$ with the development $[z_\lambda]_{\lambda<\omega_1}$. Then $z' = f(z)$ with the development $[z'_\lambda]_{\lambda<\omega_1}$ whereby $z'_\lambda = \bar{x}_\lambda$ for $\lambda < \gamma + \omega \cdot 2$ and $z'_{\gamma+\omega.2+\lambda} = = z_\lambda$ for $\lambda < \omega_1$, is a common point as well and $z' \in I$. It is easy to verify that the correspondence $z' = f(z)$ is a similarity. Therefore $\mathfrak{P}_7$ is a quasi-homogeneous continuum.

All sets $A_{00}^{(7)}$, $A_{01}^{(7)}$, $A_{10}^{(7)}$ are dense in $\mathfrak{P}_7$ because they are not empty and $\mathfrak{P}_7$ is a quasi-homogeneous continuum. Now, let us notice that the power of the system $\mathfrak{S}_7$ does not exceed the power of the set of all intervals in Q, viz. $2^{\aleph_0}$. From the property (c) it follows that any interval $I_{i_0 i_1 \dots i_\lambda \dots}$ $(\lambda < \omega)$ whereby $i_\lambda = 0$ for an infinite number of $\lambda < \omega$ and $i_{\lambda'} = = 1$ for an infinite number of $\lambda' < \omega$ belongs to $\mathfrak{S}_7$. Thus the power of $A_{00}^{(7)}$ must be $2^{\aleph_0}$. If $x \in \mathfrak{P}_7 - A_{00}^{(7)}$, then the development $[x_\lambda]_{\lambda<\omega_1}$ of x fails to have the property (c) and consequently there is a finite increasing sequence of indices $\alpha_0 < \beta_0 < \alpha_1 < \beta_1 < \dots < \alpha_\nu < \beta_\nu < \dots$, whereby $0 \leqq \alpha_\nu < \beta_\nu < \omega_1$ or $0 \leqq \alpha_\nu < \beta_\nu < \alpha$ such that $x_\lambda = 0$ for and only for $\alpha_\nu \leqq \lambda < \beta_\nu$ for every ν. Because we can attach one and at most one point $x \in \mathfrak{P}_7 - A_{00}^{(7)}$ to every sequence like this the power of the set $\mathfrak{P}_7 - - A_{00}^{(7)}$ cannot exceed the cardinal number $\aleph_1$ of all finite increasing sequences of ordinals $< \omega_1$. The power of $A_{01}^{(7)}$ and $A_{10}^{(7)}$ is $\aleph_1$, $A_{01}^{(7)}$ and $A_{10}^{(7)}$ being dense in $\mathfrak{P}_7$ and because there cannot exist a countable set which would be dense in $\mathfrak{P}_7$. It remains to verify that the one-point-sets (a) and (b) are the common points in $\mathfrak{P}_7$ where a and b are the end-points in Q. This follows from the fact, that neither the point a nor the point b has property (c) or (f).

According to J. Novák we denote by $\mathfrak{P}_3$ the ordered continuum which contains all intervals $I_{i_0 i_1 \dots i_\lambda \dots}$ $(\lambda < \alpha) \subset Q$ with the property (c) and all one-point-sets (x) whereby $x \in Q$ fails to have property (c). He has proved that $\mathfrak{P}_3$ has the property π.

Theorem 2. *There exists a subset P of $\mathfrak{P}_3$ which is similar to $\mathfrak{P}_7$. $\mathfrak{P}_7$ has the property π.*

Proof. Let $z = I_{i_0 i_1 \ldots i_\lambda \ldots}$ $(\lambda < \alpha) \in \mathfrak{P}_7$ be an interval-point with the property (c); then $z' = f(z) = z$ is an interval-point in $\mathfrak{P}_3$. If $z \in \mathfrak{P}_7$ is an interval-point with property (f), then $[z'_\lambda]_{\lambda < \omega_1}$ whereby $z'_\lambda = i_\lambda$ for $\lambda < \alpha$ and $z_\lambda = 0$ for $\lambda \geqq \alpha$ is a development of the point $z' = f(z) \in \mathfrak{P}_3$. Let z be a common point in $\mathfrak{P}_7$. Then the point $z' = f(z) = z$ is a common point in $\mathfrak{P}_3$, too. It is easy to verify that f is a similarity of $\mathfrak{P}_7$ on the set $P \subset \mathfrak{P}_3$ of all points $f(z)$. Now $\mathfrak{P}_7$ has the property π because $\mathfrak{P}_3$, and consequently P, has it as well.

J. Novák has constructed the continua $\mathfrak{P}_1, \ldots, \mathfrak{P}_6$ using the following properties: $\mathfrak{P}_3$: (c); $\mathfrak{P}_4$: (c) and (d); $\mathfrak{P}_5$: (c) and (e); $\mathfrak{P}_6$: (c), (d) and (e); $\mathfrak{P}_1$: (c), (d) and (f); $\mathfrak{P}_2$: (c), (e) and (g).

In this paper, I have constructed the continuum $\mathfrak{P}_7$ by using properties (c) and (f). All remaining combinations of the properties (d), (e), (f) and (g) with the property (c) are as follows:

(c) and (g); (c), (d) and (g); (c), (e) and (f); (c), (f) and (g); (c), (d), (e) and (f); (c), (d), (e) and (g); (c), (e), (f) and (g); (c), (d), (e) and (g); (c), (d), (e), (f) and (g).

It is easy to see that we get a continuum which is similar to $\mathfrak{P}_7$ reversed using the properties (c) and (g). Remaining combinations don't lead to any ordered continuum because the corresponding systems of intervals $\mathfrak{S}$ are not disjoint. For instance the intervals $I_{i_0 i_1 \ldots i_\lambda}$ $(\lambda < \omega \,.\, 2)$ and $I_{j_0 j_1 \ldots j_\lambda \ldots}$ $(\lambda < \omega)$, $i_0 = 0$, $i_\lambda = 1$ for $1 \leqq \lambda < \omega \,.\, 2$ and $j_0 = 1$, $j_\lambda = 0$ for $1 \leqq \lambda < \omega$ defined by means of the combination (c), (d) and (g) have a common point. The intervals $I_{i_0 i_1 \ldots i_\lambda \ldots}$ $(\lambda < \omega)$ and $I_{j_0 j_1 \ldots j_\lambda \ldots}$ $(\lambda < \omega \,.\, 2)$, $i_0 = 0$, $i_\lambda = 1$ for $1 \leqq \lambda < \omega$ and $j_0 = 1$ and $j_\lambda = 0$ for $1 \leqq \lambda < \omega \,.\, 2$, defined by (c), (e) and (f), have also a common point and $I_{i_0 i_1 \ldots i_\lambda \ldots}$ $(\lambda < \omega)$ and $I_{j_0 j_1 \ldots j_\lambda \ldots}$ $(\lambda < \omega)$, $i_0 = 1$, $j_0 = 0$, $i_\lambda = 0$ and $j_\lambda = 1$ for $1 \leqq \lambda < \omega$, defined by (c), (f) and (g), have a common point as well. It is easily seen, that further combinations define systems, which are not disjoint.