Frank Harary; Edgar M. Palmer The smallest graph whose group is cyclic

Czechoslovak Mathematical Journal, Vol. 16 (1966), No. 1, 70-71

Persistent URL: http://dml.cz/dmlcz/100711

Terms of use:

© Institute of Mathematics AS CR, 1966

Institute of Mathematics of the Czech Academy of Sciences provides access to digitized documents strictly for personal use. Each copy of any part of this document must contain these *Terms of use*.



This document has been digitized, optimized for electronic delivery and stamped with digital signature within the project *DML-CZ: The Czech Digital Mathematics Library* http://dml.cz

THE SMALLEST GRAPH WHOSE GROUP IS CYCLIC¹)

FRANK HARARY and ED PALMER, Michigan

(Received January 5, 1965)

Frucht was the first to solve the problem posed by KÖNIG [3]: which abstract finite groups F have the property that there exists a graph G such that the automorphism group of G is isomorphic with F? The answer may be said to be affirmative since FRUCHT [1] showed (by constructive methods) that every group F has this property. Frucht later showed [2] that given F, there is a cubic graph G (in which every point has degree 3) whose group $\Gamma(G) \cong F$ (is isomorphic with F). In particular

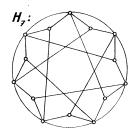


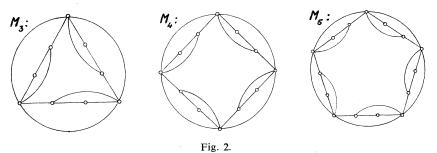
Fig. 1. Sabidussi's graph for C_7 .

he proved that for the cyclic group C_n of order n > 3 there is a graph G_n with p = 3n points and $q = (n^2 + 7n)/2$ lines such that $\Gamma(G_n) \cong C_n$. This suggests the problem of determining the smallest graph (with fewest points or lines) with given automorphism group, and as a special case, whose group is cyclic of order $n \ge 3$.

For n = 3, Frucht [2] constructed a graph of order 10 whose group is isomorphic with C_3 . For *n* a prime power ≥ 7 , SABIDUSSI [4] showed that the minimum number of points in any graph with group C_n is 2*n*, and he constructed graphs H_n for C_n with 2*n* points. In the three smallest cases, n = 3, 4, and 5, Sabidussi exhibited graphs H_n with group C_n such that each H_n has 3*n* points and 6*n* lines, a larger number of lines than in the graphs constructed by Frucht.

¹) This work was supported in part by US Air Force research grant AF-AFOSR-754-65.

The following three graphs M_n , n = 3, 4, 5 have the minimum number of lines as well as the other properties (group C_n and minimum order 3n). It can be verified by exhaustion that there are exactly two other graphs for C_3 with p = 9 and q = 15. For n > 5, n a prime power, the corresponding graphs M_n have more points and lines than Sabidussi's graphs H_n . The determination of all connected graphs with group C_n , $n \ge 4$, having minimum number of points or lines remains an unsolved problem.





References

- R. Frucht: Herstellung von Graphen mit vorgegebener abstrakter Gruppe. Compositio Math., 6 (1938), 239-250.
- [2] R. Frucht: Graphs of degree 3 with given abstract group. Canad. J. Math., 1 (1949), 365-378.
- [3] D. König: Theorie der endlichen und unendlichen Graphen. Leipzig, 1936; reprinted New York, 1950, p. 5.
- [4] G. Sabidussi: On the minimum order of graphs with given automorphism group. Monatshefte für Math., 63 (1959) 124-127.

Author's addresses: Department of Mathematics of the University of Michigan, Ann Arbor, USA

Резюме

НАИМЕНЬШИЙ ГРАФ С ЦИКЛИЧЕСКОЙ ГРУППОЙ

Ф. ХАРАРЫ (F. Harary), Э. ПАЛМЕР (E. Palmer), Мичиган

Авторы предлагают примеры трех графов, для которых группы автоморфизмов циклические третьего, четвертого соотв. пятого порядка и числа ребер минимальны.