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Czechoslovak Mathematical Journal, Vol. 35 (1985), No. 1, 43–51

Persistent URL: http://dml.cz/dmlcz/101995

Czechoslovak Mathematical Journal, 35 (110) 1985, Praha

REDUCTION OF BAIRE-MEASURABILITY TO UNIFORM CONTINUITY

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(Received January 3, 1983)

The main result is Main Lemma in § 1 which describes a construction of a Baire-equivalent and σ-dd-equivalent analytic space X_∞ to a given analytic space X such that the elements of a given σ-dd family of Baire sets in X become closed and open; some further properties are preserved. This result is applied in § 1 to a reduction of Baire measurable maps of analytic spaces into metric spaces to continuous maps. In § 2 the Main Lemma is applied to obtain a generalization of characterizations of point-ω-analytic spaces among analytic spaces to the general (=non-separable) case. For separable case the main references are [Fro$_1$], [Fre] and [T]. In § 3 the following basic problem is touched. Let $f: X \to Y$ be Baire measurable, and let f [open (X)] have a σ-discrete base; is it then true that Y is analytic (or Luzin) if X is analytic (or Luzin, and f is 1-1). It should be noted that Theorem 1 in § 1 is the proper generalization of the main result of [Fro$_1$] to the non-separable case. For the convenience of the reader we recall the basic definitions. The terminology is chosen so that the generalizations of the results from the separable theory have almost the same formulation.

By a cardinal we mean the corresponding initial ordinal. By a space we mean a uniform T_2 space. By a topological space we mean the corresponding topological fine uniformity, so "discrete" means that there exists a continuous pseudometric such that the family is metrically discrete.

If K is a cardinal then the collection of K-Baire sets is the smallest σ-algebra $\mathrm{Ba}_K(X)$ which is closed under the unions of discrete families of cardinal $\leqq K$, and such that each uniformly continuous function (=real valued) is measurable. So $\mathrm{Ba}_\omega(X)$ is the usual σ-algebra of "Baire sets" denoted usually by "$\mathrm{Ba}(X)$"; we denote by $\mathrm{Ba}(X)$ the union of $\{\mathrm{Ba}_K(X)\}$ and the elements of this algebra will be called *Baire sets in X*. In analytic spaces Baire sets are just bi-Suslin sets [F-H$_3$]; this is a consequence of the 1[st] Separation Principle for non-separable case. It should be noted that in the metric case Baire sets were introduced under the name hyper-Borel sets in [H$_1$], and in uniform spaces the Baire sets were introduced in [Fro$_2$] under the name hyper-Baire sets. For properties see [F-H$_3$].

A family $\{X_a \mid a \in A\}$ is called *σ-discretely decomposable* (abb. σ-dd) if there exists a family $\{X_{an} \mid a \in A,\ n \in \omega\}$ such that $X_a = \bigcup\{X_{an} \mid n \in \omega\}$ for each a in A, and $\{X_{an} \mid a \in A\}$ is discrete for each n in ω. For properties we refer to [F-H_2], [Fro_1], [Fro_3], [K-P]. Here we use just very elementary properties except for the deep result from [F-H_1] recalled in § 1. If X is a space we denote by σ-dd (X) the set of all σ-dd families of subsets of X.

By an usco-compact correspondence we mean an upper semicontinuous compact-valued correspondence $f: X \to Y$. Note that the actual domain $\{x \mid fx \neq \emptyset\}$ of f is a closed set in X. A correspondence f is σ-dd-preserving ([F-H_1], [F-H_2]) if for each σ-dd family $\{X_a\}$ in X the family $\{f[X_a]\}$ is σ-dd in Y.

A space X is called *analytic*, if there exists an usco-compact σ-dd-preserving correspondence f from a complete metric space M onto X; if f is disjoint (i.e. $m_1 \neq m_2 \Rightarrow fm_1 \cap fm_2 = \emptyset$) then X is called *Luzin*. If f is single-valued then we speak about *point-analytic* or *point-Luzin spaces*. If the weight of M is $\leqq K, K \geqq \omega_0$, then we speak about K-analytic spaces, etc.

For properties of analytic spaces I refer to [F-H_2]. For the proof of Main Lemma we shall use the fact from [F-H_2] that each analytic space is σ-dd-simple. Recall that X is called *σ-dd-simple* if for each discrete family $\{X_a \mid a \in A\}$ in X, and for each family $\{Y_{ab} \mid b \in B_a\}$, $a \in A$, of discrete families in X, the family $\{X_a \cap Y_{ab} \mid a \in A, b \in B_a\}$ is σ-dd or equivalently, if $\{X_a\}$ and all $\{Y_{ab}\}$ are σ-dd then so is $\{X_a \cap Y_{ab}\}$. In fact, every paracompact uniform space (particularly, each metrizable space) is σ-dd-simple, and every analytic space is paracompact; this will be used in § 1 on point-analytic spaces (see the comment following Remarks following the proof of Theorem 5). For the proof of Theorems 1 and 2 we need the following deep fact from [F-H_2], Theorem 1.

If $\{X_a \mid a \in A\}$ is a point-finite family of sets in X such that $\bigcup\{X_a \mid a \in B\} \in \mathrm{Ba}(X)$ for each $B \subset A$ (i.e. if $\{X_a\}$ is completely $\mathrm{Ba}(X)$-additive) then $\{X_a\}$ is σ-dd. Finally, we shall need the following characterization of point-analytic spaces among all analytic spaces (see [F-H_3], Theorem 3,1).

Theorem 0. *Each of the following two conditions is necessary and sufficient for an analytic space X to be point-analytic.*

(1) *There exists a 1-1 continuous mapping of X onto a metric space.*

(2) *There exists a 1-1 σ-dd-preserving continuous mapping of X onto a metric space.*

The only non-trivial part is that (2) is necessary.

1. Main Lemma. *Assume that A is an anylytic space, $\{\mathscr{B}_n \mid n \in \omega\}$ is a sequence of σ-dd partitions of A, and $\mathscr{B}_n \subset \mathrm{Ba}(A)$ for each n. Define by induction the spaces A_n as follows: $A_0 = A$, and*

$$A_{n+1} = \sum\{B \mid B \in \mathscr{B}_n\}$$

where each B is considered to be a subspace of A_n. Since each $\mathscr{B}_n$ is a partition we may and shall assume that the underlying sets of all A_n coincide with that of A

(of course, a cover $\mathscr{U}$ of A_{n+1} is uniform iff for each $B \in \mathscr{B}_n$ the trace of $\mathscr{U}$ on B is a uniform cover of the subspace B of A_n). Let A_∞ have the coarsest uniformity finer than each A_n. Then

(0) *A_∞ is analytic,*

(1) $\mathrm{Ba}(A_\infty) = \mathrm{Ba}(A)$,

(2) σ-dd $(A_\infty) = \sigma$-dd (A),

(3) *If A is point-analytic, Luzin or point-Luzin, then so is A_∞,*

(4) *If $\bigcup\{\mathscr{B}_n\}$ is a network for A, then the topology of A_∞ is metrizable,*

(5) *If $\bigcup\{\mathscr{B}_n\}$ distinguishes the points of A, then there exists a 1-1 continuous mapping of A_∞ onto a metric space,*

(6) *If $\bigcup\{\mathscr{B}_n\}$ distinguishes the points of A, and A is analytic or Luzin, then A_∞, and hence A, is point-analytic or point-Luzin, respectively,*

(7) *If $X \subset A$ is cut by no $B \in \bigcup\{\mathscr{B}_n\}$, then the topologies on X inherited from A and A_∞ coincide.*

Proof. For $k \in \omega$ we denote by (i_k) the statement (i), $i = 0, 1, 2, 3$ with A_∞ replaced by A_k, and prove by induction that $i(k)$ holds for each $k \in \omega$, $i = 1, 2, 3$. Assume (i_k) holds for each $i = 0, 1, 2, 3$, and check (i_{k+1}). The sum of analytic spaces is analytic, and hence (0_{k+1}) holds. Since $\mathscr{B}_k$ is σ-dd in both A_k and A_{k+1}, and $\mathscr{B}_k \subset \mathrm{Ba}\,(A_k)$ as well as $\mathscr{B}_k \subset \mathrm{Ba}\,(A_{k+1})$ we have in both spaces that C is a Baire set iff each $C \cap B$, $B \in \mathscr{B}_k$, is a Baire-set. Hence (1_{k+1}) holds.

Since A_k is analytic, and hence σ-dd-simple, it is clear that any discrete family in A_{k+1} is σ-dd in A_k, hence (2_{k+1}) holds. The respective classes in (3) are preserved by formations of sums, and if A_k has one of the properties then so have any Baire set in A_k, and hence (3_{k+1}) holds. Now let $f\colon A_\infty \to \Pi\{A_n \mid n \in \omega\}$ be the diagonal map, i.e. $fx = \{x \mid n \in \omega\}$. By definition f is a uniform embedding.

Proof of (0) and (3). Countable products of analytic spaces are analytic, and so is any closed subspace. Thus(0) holds. If A has one of the properties in (3), then so has the property each A_n, hence $\Pi\{A_n\}$, and hence any closed set in the product (clearly $f[A_\infty]$ is closed).

Proof of (2). If $\{X_c \mid c \in C\}$ is discrete in A_∞, then $\{\pi_n[f[X_c]] \mid c \in C\}$ is discrete in $\Pi\{A_k \mid k \leqq n\}$ for some n, and hence $\{X_c\}$ is discrete in A_n, and hence σ-dd in A.

Proof of (1). The σ-algebra $f[\mathrm{Ba}\,(A)]$ on $f[A_\infty]$ is closed under discrete unions because $\mathrm{Ba}\,(A_n) = \mathrm{Ba}\,(A)$ for each n, and hence $\mathrm{Ba}\,(A) = \mathrm{Ba}\,(A_\infty)$.

Proof of (4). If $\mathscr{B} = \bigcup\{\mathscr{B}_n\}$ is a network for A, then $\mathscr{B}$ is a network for A_n for each n, hence $\mathscr{B}$ is a network for A_∞. But each set in $\mathscr{B}$ is closed and open in A_∞. Thus A_∞ is regular and has a σ-discrete basis for open sets, and hence is metrizable.

Proof of (5). Obvious: we put discrete topology on $\mathscr{B}_n$ and take the map $\varepsilon\colon A_\infty \to$ $\to \Pi\{\mathscr{B}_n \mid n \in \omega\}$ with $(\varepsilon x)_n$ the unique B in $\mathscr{B}_n$ such that $x \in B$.

Proof of (6). From (5) using the characterization of point-analyticity among analytic spaces in Theorem 0.

Since (7) is self-evident, the proof of Main Lemma is complete.

Remark 1. If A is completely metrizable then A_∞ from Main Lemma is not necessarily completely metrizable, by (3) it is point-Luzin, and so we can take a finer completely metrizable A' such that the identity $A' \to A_\infty$ is a Baire-isomorphism and σ-dd-isomorphism. However, we are loosing the control of the topology of sets from (7).

Remark 2. If A' is any uniform space between A_∞ and A then A' has all properties of A_∞ stated in (1)–(7).

Remark 3. Main Lemma has its most natural setting for σ-dd-simple spaces, see [Fro_3].

As a corollary we obtain the following fundamental

Theorem 1. *Assume that f is a Baire-measurable mapping of an analytic space A onto a metric space M. Then there exists an analytic space A' and a uniformly continuous bijection j: $A' \to A$ such that $f \circ j$ is continuous, and*

$$j[\text{Ba}(A')] = \text{Ba}(A)$$

$$j[\sigma\text{-dd}(A')] = \sigma\text{-dd}(A).$$

Moreover, if A is Luzin, point-analytic or point-Luzin, then so may be taken A'. In any case, for A' one can take the graph of f with the uniformity inherited from $A \times M$, and for j the restriction of the projection $A \times M \to A$.

Proof. For each n choose a σ-discrete partition $\mathscr{C}_n \subset \text{Ba}(M)$ of M such that the diameter of each C in $\mathscr{C}_n$ is $\leq 1/(n+1)$, and put $\mathscr{B}_n = f^{-1}[\mathscr{C}_n]$. Then each $\mathscr{C}_n$ is completely Ba (M)-additive, and hence each $\mathscr{B}_n$ is completely Ba (A)-additive. By non-trivial [F-H_1], Thm 1, each $\mathscr{B}_n$ is σ-dd in A, and let A_∞ be the space from Main Lemma. Obviously A_∞ has the properties of A'. If A' is the graph, then $\{x \to \langle x, fx \rangle\} : A_\infty \to A'$ is uniformly continuous because both f: $A_\infty \to M$, and the identity mapping $A_\infty \to A$ are uniformly continuous. Thus Remark 2 to Main Lemma applies.

The following is a partial generalization of the previous result:

Theorem 2. *Assume that f is a Baire-measurable compact-valued correspondence of an analytic space A onto a metric space M. There exists an analytic space A' and a uniformly continuous bijection j: $A' \to A$ such that*

$$\text{Ba}(A') = \text{Ba}(A)$$

$$\sigma\text{-dd}(A') = \sigma\text{-dd}(A)$$

and $f \circ j : A' \to M$ is lower semi-continuous (and Baire-measurable). Moreover, if A is Luzin, point-analytic or point-Luzin, then so may be taken A'.

Proof. Let $\mathscr{C}_n$ be locally finite collections of open sets such that $\mathscr{C} = \bigcup\{\mathscr{C}_n\}$ is a basis for open sets, and put $\mathscr{B}'_n = f^{-1}[\mathscr{C}_n]$. Since $\mathscr{B}'_n$ is point-finite (the values are compact and $\mathscr{C}_n$ are locally finite) and completely Ba (A)-additive, again by [F-H$_1$], Thm. 1, each $\mathscr{C}_n$ is σ-dd in A. Now let $\mathscr{B}_n$ be $\mathscr{B}'_n \cup \{A \setminus \bigcup\mathscr{B}'_n\}$, and apply Main Lemma.

Theorem 3. *Let f be a bi-measurable onto map of analytic metrizable spaces X and Y such that the preimages of points are compact. There exist analytic metrizable topological spaces X' finer then X, and Y' finer than Y such that*

$$\mathrm{Ba}(X') = \mathrm{Ba}(X), \quad \mathrm{Ba}(Y') = \mathrm{Ba}(Y),$$

$$\sigma\text{-dd}(X') = \sigma\text{-dd}(X), \quad \sigma\text{-dd}(Y') = \sigma\text{-dd}(Y),$$

$f: X' \to Y'$ is continuous and open, and the topology of the fibres is not changed (in particular, is compact). Moreover if X is Luzin, then so may be taken X', and if Y is Luzin, then Y' can be taken to be completely metrizable.

Proof. Using Theorems 1 and 2 define sequences of space $\{X_n\}$ and $\{Y_n\}$ such that X_{n+1} is finer than X_n, Y_{n+1} is finer than Y_n, the properties in Theorems are preserved (in particular, X and all X_n are Baire-equivalent, and σ-dd-equivalent, and similarly for Y and all Y_n) such that each

$f: X_{2n} \to Y_{2n}$ is continuous

and each

$f: X_{2n+1} \to Y_{2n+1}$ is open.

In the case of Y Luzin, we construct all Y_n completely metrizable. Let X' be the coarsest space finer than each X_n, and similarly, Y' be the coarsest space finer than each Y_n. It is easy to check that $f: X' \to Y'$ has all the properties required.

For the convenience of the reader let us state precisely what is needed to prove that $f: X' \to Y'$ is continuous and open: Observe that each $f: X' \to Y_{2n}$ is continuous, and each $f: X_{2n+1} \to Y'$ is open.

2. Point-analytic spaces. Point-analytic spaces were introduced and studied in [F-H$_3$]; the definition is recalled in the introduction. The properties of point-ω-analytic spaces have been studied for years, usually under the name Suslin spaces; here we refer just to two papers [Fre] and [T]; the former contains a long list of characterizations of point-ω-analytic spaces among all ω-analytic spaces (called K-analytic usually), and the latter contains a result saying that every point-ω-analytic space is a 1-1 continuous image of a metrizable analytic space. All these results will be proved here in the setting of analytic spaces (not necessarily "separable"). From the Main Lemma we obtain:

Theorem 4. *An analytic space A is point-analytic iff there exists a bijective continuous mapping $j: A' \to A$ such that A' is an analytic metrizable space (hence, point-analytic; see the introduction), and*

$$\mathrm{Ba}(A) = j[\mathrm{Ba}(A')], \quad \sigma\text{-dd}(A) = j[\sigma\text{-dd}(A')].$$

Proof. "If" is evident because we get a point-analytic parametrization of A by composing that of A' with j.

For "only if" choose any point-analytic parametrization $f: K^\omega \to A$ of A, i.e. a single-valued σ-dd-preserving usco (=upper semi-continuous) correspondence from the Baire space K^ω (K is cardinal with the discrete uniformity) onto A. For each n let $\mathscr{V}_n = \{B(s) \mid s \in K^n\}$ where $B(s) = \{\sigma \mid \sigma \upharpoonright n = s\}$. For $n = 0$, $\mathscr{V}_n = \{K^\omega\}$. Since each $\mathscr{V}_n$ is discrete, each family of analytic sets $f[\mathscr{V}_n]$ is σ-dd; choose any σ-discrete family $\mathscr{W}_n$ of analytic sets such that elements of $f[\mathscr{V}_n]$ are sub-unions of $\mathscr{W}_n$. Using the 1st Separation Principle choose Baire sets $\{B_W \mid W \in \mathscr{W}_n\}$ such that

$$W \subset B_W \subset \overline{W}$$

(since A is point-analytic, each open set is analytic). Clearly $\{B_W\}$ is σ-discrete. Now take any family $\{\mathscr{B}_n\}$ satisfying the requirements of Main Lemma such that each element of $\{B_W\}$ is a sub-union of $\bigcup\{\mathscr{B}_n\}$. Since f is single-valued and upper-semi-continuous it is clear that for each $x \in A$, and each neighborhood U of x there exists an $N \in \bigcup\{f[\mathscr{V}_n] \mid n \in \omega\}$ with $x \in N \subset \overline{N} \subset U$. It follows that $\{B_W\}$, and hence $\bigcup\{\mathscr{B}_n\}$, is a network for A. Now A_∞ is metrizable and analytic by Main Lemma.

Now we are ready to extend the list of Theorem 0.

Theorem 5. *Each of the following conditions on an analytic space A is necessary and sufficient for A to be point-analytic:*

(3) *There exists a metrizable topology coarser than that of A, and σ-dd and Baire equivalent to A.*

(4) *There exists a metrizable topology finer than that of A, and σ-dd and Baire equivalent to A.*

(5) *There exists a metrizable space M which is Baire isomorphic to A.*

(6) *There exists a σ-dd family of Baire sets in A which distinguishes the points of A.*

Proof of Theorem 5. Condition (4) is just the condition from Theorem 4. Condition (3) is sufficient by (2) of Theorem 0, and necessary, because the metric space in (2) of Theorem 0 is Baire-equivalent to A by the 1st Separation Principle.

Condition (6) is sufficient by Main Lemma (5) because then A_∞ admits a one-to-one continuous mapping onto a metric space, and hence is point-analytic by Condition (1) of Theorem 0. Self-evidently (4) implies (5). Finally (5) implies (6) by the non-trivial Theorem 2 of [F-H$_1$] used already in the proof of Theorems 1 and 2.

Remarks. (a) It is shown in [F-H$_3$], Ex. 3.1 that coarser in the topological sense can not be replaced by coarser in the uniform sense.

(b) In (4) finer in the topological sense can not be replaced by finer in the uniform sense. For example, let A be K^ω, K infinite, with the topological fine uniformity. One easily checks that there exists no metric d on the set A such that $\langle A, d\rangle$ is uniformly finer than A, and σ-dd-equivalent to A.

For the last list of characterizations of point-analytic spaces we recall the following well-known Lemma. In that Lemma paracompactness is understood in the usual way. Recall that a uniform space is said to be paracompact if every open cover has a σ-discrete (in the uniform sense!) refinement. So if X is paracompact then so is the induced topological space (and the converse is true for topologically fine uniformities). Every metric space is paracompact, and the image of a paracompact space under an usco-compact σ-dd-preserving correspondence is paracompact [F-H_2]. Hence each analytic space is paracompact, and each subspace of a point-analytic space is paracompact; thus point-analytic spaces are hereditarily paracompact.

Lemma. *The following two conditions on a paracompact topological space are equivalent*:

(1) *The diagonal Δ_X of $X \times X$ is a G_δ in $X \times X$.*

(2) *There exists a one-to-one continuous mapping of X onto a metric space.*

Proof. Evidently (2) implies (1) (without any assumption on X). Assume (1) and let $\Delta_X = \bigcap\{G_n \mid n \in \omega\}$ with all G_n open. For each n choose an open cover $\{U_x^n \mid x \in X\}$ of X such that $U_x^n \times U_x^n \subset G_n$. Since X is paracompact, for each $n \in \omega$ there is a continuous mapping f_n of X into a metric space M_n such that $\{f_n^{-1}m \mid m \in M_n\}$ refines $\{U_x^n\}$. Put $f = \{x \to \{f_n x\}\} : X \to \Pi\{M_n\}$. Clearly f is one-to-one because $f_n x = f_n y$ implies $\langle x, y\rangle \in G_n$.

Now we are ready to complete the list of various characterizations of point-analytic spaces.

Theorem 6. *Each of the following conditions is necessary and sufficient for an analytic space A to be point-analytic*:

(7) *$A \times A$ is hereditarily paracompact,*

(8) *$A \times A \setminus \Delta_A$ is paracompact,*

(9) *Δ_A is a G_δ in $A \times A$,*

(10) *Δ_A is a Baire set in $A \times A$ (and so is $A \times A \setminus \Delta_A$).*

Proof. If A is point-analytic then so is $A \times A$, and hence $A \times A$ is hereditarily paracompact; thus (7) is necessary. Self-evidently (7) implies (8). If an open subspace U is paracompact (in the subspace uniformity) then clearly U is an F_σ in the space; thus (8) implies (9). By Lemma, (9) implies that there exists a 1-1 continuous mapping onto a metric space, and hence A is point-analytic by Theorem 0. Each Baire set in an analytic space is analytic and hence paracompact, thus (10) implies (8). If $A \times A$ is point-analytic then each closed set in $A \times A$ is a Baire set, and hence (10) is necessary.

Remark. I do not know whether or not paracompactness in (8) can be weakened to paracompactness of the induced topology.

3. A problem on preservation of analyticity. Consider the following assertions (*)–(*v):

(*) If $f: A \to M$ is Baire measurable and surjective, A complete metric, M metric, and if $f[\text{open}(A)]$ has a σ-discrete base, then M is analytic;

(**) like (*) with A complete weakened to analytic;

(***) like (**) with Baire-measurability strengthened to continuity;

(*v) like (*) with Baire measurability strengthened to continuity.

Observation. (**) implies each of the conditions, and (*v) is implied by each of the other conditions. By $[H_1]$ or $[F\text{-}H_2]$ the assertion (v) holds.

Proposition. $(***) \Rightarrow (**)$.

Proof. Assume that $f: A \to M$ satisfies the assumptions in (**), and let A' be the graph with the subspace uniformity, and $\pi_1: A' \to A$, $\pi_2: A' \to M$ the restrictions of the corresponding projections. By Theorem 1 A and A' are Baire and σ-dd equivalent via π_1, and π_2 is continuous. Let $\mathscr{C}$ be a σ-discrete base for $f[\text{open}(A)]$. It is enough to check that if $\mathscr{B}$ is an appropriate base for open sets in M then $[\mathscr{B}] \cap [\mathscr{C}]$ is a base for $\pi_2[\text{open}(A')]$, because $\mathscr{B}$ can be taken σ-discrete.

First observe that $A' \cap ([\text{open}(A)] \times [\mathscr{B}])$ is an open base for $\text{open}(A')$, and hence

$$\pi_2(A' \cap ([\text{open}(A)] \times [\mathscr{B}])) = [f[\text{open}(A)]] \cap [\mathscr{B}]$$

is a base for $\pi_2[\text{open}(A')]$. Finally, $[\mathscr{C}] \cap [\mathscr{B}]$ is obviously a base for $[f[\text{open}(A)]] \cap \cap [\mathscr{B}]$, and hence for $\pi_2[\text{open}(A')]$.

Remark. For 1-1 maps the condition that $f[\text{open}(A)]$ has a σ-discrete base implies that f is σ-dd-preserving and hence everything is known.

Added in proofs. Recentely G. Gruenhage has proved that if K is compact and $K \times K \setminus \Delta_K$ is a paracompact topological space then K is metrizable. It follows, if X is analytic and $X \times X \setminus \Delta_X$ is a paracompact topological space then each compact subspace of X is metrizable, and hence by [Fre] it is consistent that the answer to the problem in Remark following Theorem 6 is yes for ω-analytic spaces.

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